Mathematics Paper 1 June 2016

FORM 4

Examiner:	A Gunning	**Moderators:**	P Denissen, C Mundy
Time:	2½ hours	**Marks:**	125

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY BEFORE ANSWERING THE QUESTIONS.

- This question paper consists of 6 pages, plus an information sheet. Please check that your question paper is complete.
- Read and answer all questions carefully.
- Number your answers exactly as the questions are numbered.
- It is in your own interest to write legibly and to present your work neatly.
- **All necessary working, which you have used in determining your answers, must be clearly shown.**
- Approved non-programmable calculators may be used except where otherwise stated. Where necessary give answers correct to 2 decimal places.
- Diagrams have not necessarily been drawn to scale.

Question 1

(a) Solve for x, leaving answers in surd form, if applicable. Show all relevant working detail. Do not just write down the answers.

(i) $x(x-3)-10=0$ (3)

(ii) $\frac{2x}{x-1}+\frac{1}{x+1}=1$ (5)

(iii) $-x+6x^2=2$ (4)

(iv) $3.9^{x-1}=27^{-x}$ (3)

(v) $2^{x+1}+2^{x+2}=48$ (4)

(vi) $x-2=\sqrt{8-x}$ (5)

(b) Solve for x by completing the square: $2x^2-6x-8=0$ (4)

(c) If $T=\frac{\sqrt{m-4}}{m-1}$, determine the values of m for which:

(i) T is undefined. (1)

(ii) T is not real. (2)

(d) Solve the following equations simultaneously, showing all relevant working details.

$x-3y=5$ and $x^2+xy+2y^2=4$ (7)

(e) Solve for x in each of these inequalities and illustrate your solution on a number line as well as giving the solution in interval notation.

(i) $x^2-2x-3>0$ (4)

(ii) $x^2\leq 100$ (4)

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Question 2

Simplify each of the following without using a calculator. You are required to show all relevant working detail.

(a) $\dfrac{16^{\frac{3}{4}}.\ \sqrt{3}.\ 4^{0,5}}{27^{-\frac{1}{2}}.\ 8^{\frac{2}{3}}}$ (4)

(b) $\dfrac{5^{2x}.\ 15^{x-1}}{125^{x}.\ 3^{x-2}}$ (4)

(c) $\sqrt{\dfrac{2^{x+3}+2^{x}}{2^{x}}}$ (3)

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Question 3

(a) Consider the sequences :

3; 6; 9; 12;….. and 3; 6; 11; 18; …….

For **each** of these 2 number sequences, write down:

(i) The next three terms (3)

(ii) An expression for the n^{th} term. (7)

(b) Consider the diagram given here

Determine the number of diamonds in the 20^{th} rectangular grid if the first three are given above. (5)

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Question 4

Given $g(x) = x^2 - 5x - 6$

(a) Where does the graph of this function intersect the y-axis? (1)

(b) Write down the **coordinates** of the x-intercepts of the graph. (2)

(c) Write down the equation of the axis of symmetry of the graph. (1)

(d) Hence give the coordinates of the turning point of the graph. (2)

(e) What is the range of g? (1)

(f) On the given set of axes (at the back of the front cover of the answer booklet), draw a rough sketch of $y = g(x)$ (3)

(g) On the same set of axes, draw the graph of the function f defined by $y - 2x = -6$. (3)

(h) Label on the graph, using A and B, where $f(x) = g(x)$ and calculate the coordinates of these points. Show all relevant working detail. (5)

(i) Hence, determine the value(s) of x for which $g(x) \leq f(x)$. (2)

(j) What can be said about the value of $b^2 - 4ac$, for g? (2)

[22]

Question 5

(a) Determine the equation of the parabola with a turning point $(1; 2)$ and which cuts the y-axis at the point $(0; 1)$ (4)

(b) In the sketch graph, f represents the function $f(x) = b^x$.

h is symmetrical to f about the y-axis.

The point $(2; 2\frac{1}{4})$ lies on the graph of f.

$(2;\ 2\frac{1}{4})$

(i) Find the value of b. (2)

(ii) Write down the domain and range of h. (2)

(iii) On the given set of axes (on the inside of the back cover of the answer booklet), sketch the graph of $f(x) + 3$, showing at least two points on the graph and the asymptote. (3).

(c) Sketched below is the graph of $y = \frac{2}{x-2} + 3$. A and B are the intercepts with the axes.

A

B

(i) Calculate the coordinates of A and B. (4)

(ii) Write down the equations of the asymptotes of the graph. (2)

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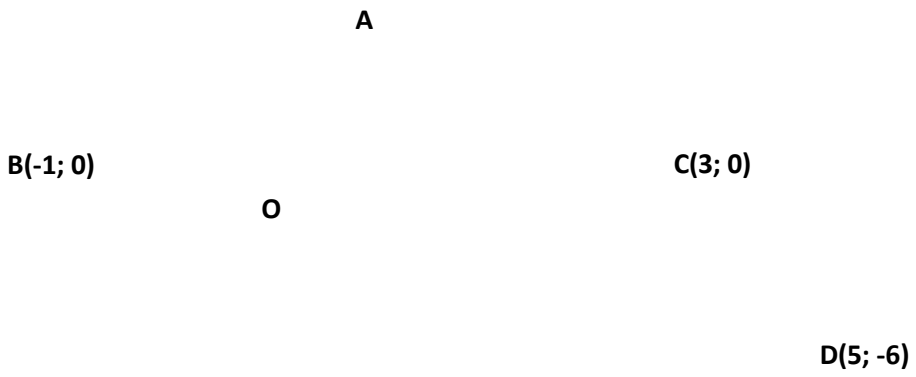
Question 6

Sketched below are the graphs of $f(x) = ax^2 + bx + c$ and $y = t^x + q$.

Both the graphs cut each other at A, the turning point of the parabola.

The parabola cuts the x-axis at $B(-1; 0)$ and $C(3; 0)$ and passes through the point $D(5; -6)$.

The exponential graph **also** passes through the origin.



(a) Determine the values of a, b and c. (4)

(b) Find the coordinates of A. (3)

(c) Find the values of t and q (3)

(d) Write down the range of the exponential graph. (2)

(e) Write down the coordinates of the turning point of $g(x)$ where $g(x)$ is the reflection of f about the x-axis. (2)

[14]

MATHEMATICS
INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\sum_{i=1}^{n} 1 = n \qquad \sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$T_n = a + (n-1)d \qquad S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$T_n = ar^{n-1} \qquad S_n = \frac{a(r^n - 1)}{r - 1};\ r \neq 1 \qquad S_\infty = \frac{a}{1 - r};\ -1 < r < 1$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad M\left(\frac{x_1 + x_2}{2};\ \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c \qquad y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad m = \tan\theta$$

$$(x - a)^2 + (y - b)^2 = r^2$$