F4 Maths Paper 1 June 2016. MEMO

Question 1.

a (i) $x(x-3) - 10 = 0$ ✓

$x^2 - 3x - 10 = 0$ ✓

$(x-5)(x+2) = 0$ ✓

$x = 5$ or -2 ✓ ✓ (3)

(ii) $\frac{2x}{x-1} + \frac{1}{x+1} = 1$ ✓

CD $= (x+1)(x-1)$

Rest. $x \neq \pm 1$ ✓

$2x(x+1) + (x-1) = (x-1)(x+1)$

$2x^2 + 2x + x - 1 = x^2 - 1$

$x^2 + 3x = 0$ ✓

$x(x+3) = 0$ ✓

$x = 0$ or -3 ✓ ✓ (5)

(iii) $-x + 6x^2 = 2$ ✓

$6x^2 - x - 2 = 0$ ✓

$(2x+1)(3x-2) = 0$ ✓

$x = -\frac{1}{2}$ or $x = \frac{2}{3}$ ✓ ✓ (4)

(10) Must show factors.

If they use their calculators will they get this mark.

(iv) $3 \cdot 9^{x-1} = 27^{-x}$

$3 \cdot 3^{2x-2} = 3^{-3x}$ ✓ ✓

$3^{2x-1} = 3^{-3x}$ ✓

$2x - 1 = -3x$ ✓

$5x = 1$

$x = \frac{1}{5}$ ✓ ✓ (3)

(v) $2^{x+1} + 2^{x+2} = 48$

$2^x(2 + 2^2) = 48$ ✓

$2^x(6) = 48$ ✓

$2^x = 8$ ✓

$2^x = 2^3$

$x = 3$ ✓ ✓ (4)

vi $x-2 = \sqrt{8-x}$

$x^2 - 4x + 4 = 8 - x$

$x^2 - 3x - 4 = 0$

$(x-4)(x+1) = 0$

$x = 4$ or -1

check:

Sub $x=4$

LHS = 2 RHS = 2

Sub $x=-1$

LHS = -3 RHS = 3

$x = 4$ (5)

vii $2x^2 - 6x - 8 = 0$

$x^2 - 3x = 4$

$\left(x - \frac{3}{2}\right)^2 = 4 + \frac{9}{4}$

$\left(x - \frac{3}{2}\right)^2 = \frac{25}{4}$

$x = \frac{3}{2} \pm \frac{5}{2}$

$= 4$ or -1. (4)

(b) $T = \frac{\sqrt{m-4}}{m-1}$

(i) T undefined if $m = 1$ (1)

(ii) T not real if $m - 4 < 0$

$m < 4$ (2)

(c) $x - 3y = 5$ (1)

$x^2 + 3xy + 2y^2 = 4$. (2)

from (1) $x = 5 + 3y$ (3)

Sub (3) into (2)

$(5+3y)^2 + y(5+3y) + 2y^2 - 4 = 0$

$25 + 30y + 9y^2 + 5y + 3y^2 + 2y^2 - 4 = 0$.

$14y^2 + 35y + 21 = 0$

$\div 7$ $2y^2 + 5y + 3 = 0$

$(2y + 3)(y + 1) = 0$

$2y = -3$

$y = \frac{-3}{2}$ or $y = -1$.

Sub into (3)

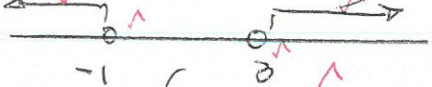
$x = 5 + 3\cdot\left(\frac{-3}{2}\right)$ or $x = 5 - 3$

$= \frac{1}{2}$ $= 2$

$\left(\frac{1}{2}, \frac{-3}{2}\right)$ ca $(2, -1)$ ca (7)

(d)(i) $x^2 - 2x - 3 > 0$.

$(x-3)(x+1) > 0$



$x \in (-\infty; -1)$ or $x \in (3; \infty)$ (4)

(ii) $x^2 \leq 100$

$(x-10)(x+10) \leq 0$

$x \in [-10; 10]$

if student writes $x \geq \pm 10$ no marks at all (4)

Question 2.

(a) $$\frac{16^{\frac{3}{4}} \cdot \sqrt{3}\, 4^{0,5}}{27^{-\frac{1}{2}} \cdot 8^{\frac{2}{3}}} = \frac{(2^4)^{\frac{3}{4}} \cdot 3^{\frac{1}{2}} \cdot 2^{2 \cdot \frac{1}{2}}}{(3^3)^{-\frac{1}{2}} \cdot (2^3)^{\frac{2}{3}}}$$

$$= \frac{2^3 \cdot 3^{\frac{1}{2}} \cdot 2}{2^2 \cdot 3^{-3/2}} = 2^2 \cdot 3^2$$

$$= 4 \cdot 9$$

$$= 36$$

(4)

(b) $$\frac{5^{2x} \cdot 15^{x-1}}{125^x \cdot 3^{x-2}} = \frac{5^{-2x} \cdot 3^{x-1} \cdot 5^{x-1}}{5^{3x} \cdot 3^{x-2}}$$

$$= 5^{-2x-1-3x} \cdot 3^{x-1-x+2}$$

$$= \frac{1}{5} \cdot 3^1$$

$$= \frac{3}{5}$$

(4)

(c) $$\sqrt{\frac{2^{x+3} + 2^x}{2^x}}$$

$$= \sqrt{\frac{2^x(2^3+1)}{2^x}}$$

$$= \sqrt{9} = 3$$

(3)

lose the last $\frac{1}{2}$ of part ± 3.

Question 3

(a) (i) 3; 6; 9; 12 ... 15; 18; 21

3; 6; 1; 18 ... 27; 38; 51

$2a = 2$

$a = 1$

$3a + b = 3$

$3 + b = 3$

$b = 0$

$1 + b + c = 3$

$c = 2.$

$T_w = w^2 + 2.$ (5)

are these ca?

(ii) $T_w = 3w.$ (2)

(3)

(b) 6; 12; 20

d_1 6 8

d_2 2

$2a = 2$

$a = 1$

$3 + b = 6$

$b = 3$

$1 + 3 + c = 6$

$c = 2$ ca

$T_w = w^2 + 3w + 2$

$T_{20} = 20^2 + 30(20) + 2$ sub

$= 462.$ ca

(5)

Question 4. [22]

$g(x) = x^2 - 5x - 6$.

(a) Y int $(0; -6)$ ✓ (1) no need to work as coord. $y = -6$ ✓ only.

(b) X int $0 = x^2 - 5x - 6$

$0 = (x-6)(x+1)$

$(6; 0)$ ✓ or $(-1; 0)$ ✓ (2) must be coords together 3rd mark.

(c) $x = \frac{5}{2}$

(half way between X int.)

OR $x = \frac{-b}{2a} = \frac{5}{2}$ ✓ (1)

* (d) TP $\left(\frac{5}{2}; -\frac{49}{4}\right)$ ✓

Sub $x = 5/2$ into original

$g\left(\frac{5}{2}\right) = \left(\frac{5}{2}\right)^2 - 5\left(\frac{5}{2}\right) - 6$ ✓

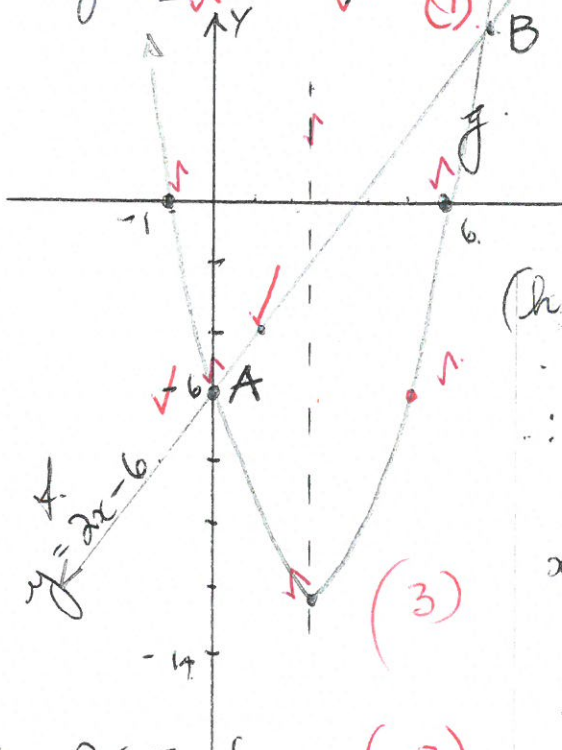
$= \frac{25}{4} - \frac{25}{2} - 6$

$= \frac{25 - 50 - 24}{4}$

$= -\frac{49}{4} = -12\frac{1}{4}$

(e) Range $y \in [-12\frac{1}{4}; \infty)$ ✓ (1)

(f)



(2) (3)

(g) $y - 2x = -6$

$y = 2x - 6$. (3)

(h) $y = 2x - 6$ (1)

$y = x^2 - 5x - 6$ (2)

$\therefore 2x - 6 = x^2 - 5x - 6$

$0 = x^2 - 7x$ ✓

$x(x - 7) = 0$

$x = 0$ or $x = 7$. ✓

$y = -6$ or $y = 8$

A $(0; -6)$ B $(7; 8)$ (5)

(i) $g(x) \leq f(x)$.

$x \in [0; 7]$ (2)

(j) $b^2 - 4ac > 0$ ✓✓ (2)

$b^2 - 4ac = 49$ a perf. square so roots real, rational & unequal.

Question 5. [17]

(a) $y = a(x-p)^2 + q$.

TP $(1; 2)$ $\quad y = a(x-1)^2 + 2$ ✓ ✓

1 pt $(0; 1)$ Sub $\quad 1 = a(-1)^2 + 2$ ✓

$-1 = a$

$\therefore y = -(x-1)^2 + 2$ ✓ ✓ (4)

$-x^2 + 2x + 1$

(b) (i) $f(x) = b^x$.

Sub. $\quad (2; \frac{9}{4})$ ✓ $\quad \frac{9}{4} = b^2$ ✓

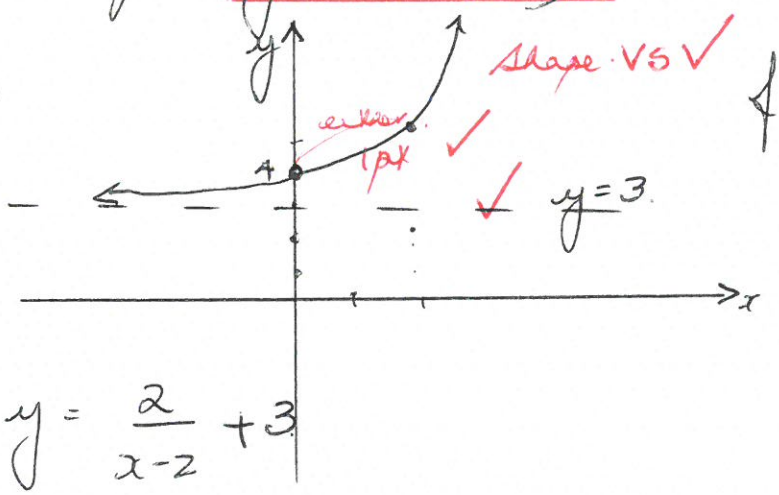
$b = \frac{3}{2}$ ✓ $\quad f(x) = \left(\frac{3}{2}\right)^x$.

(1)

(ii) Domain $x \in R$ ✓ (1)

Range $y \in (0; \infty)$ ✓ ✓ (1) (2)

(iii)



Shape VS ✓ ✓ ✓ $f(x) + 3$. (3)

(c) $y = \frac{2}{x-2} + 3$

(i) A Y int. $\quad y = \frac{2}{-2} + 3$

$= 2$ ✓ ✓ $\quad A(0; 2)$

B X int. $\quad 0 = \frac{2}{x-2} + 3$

$-3 = \frac{2}{x-2}$ ✓

$-3(x-2) = 2$

$-3x + 6 = 2$

$-3x = -4$

$x = \frac{4}{3}$ ✓ $\quad B\left(\frac{4}{3}; 0\right)$ (4)

Must be coordinates.

(ii) $x = 2$ ✓

$y = 3$ ✓ (2)

Question 6 [14]

(a) $f(x) = ax^2 + bx + c$

$y = a(x - x_1)(x - x_2)$ ✓

$y = a(x+1)(x-3)$

Sub $(5; -6)$ $-6 = a(5+1)(5-3)$

$-6 = 12a$ ✓

$-\frac{1}{2} = a$ ✓

$a = -\frac{1}{2}$

$y = -\frac{1}{2}(x^2 - 2x - 3)$

$= -\frac{1}{2}x^2 + x + \frac{3}{2}$ ← OR

$b = 1$ ✓

$c = \frac{3}{2}$ ✓ (4)

(b) Aq s $x = -\frac{b}{2a} = \frac{-1}{2(-\frac{1}{2})} = 1$ ✓

TP $(1; 2)$ ✓ must give coordinates /

Sub $x = 1$ into $f(x)$

$f(1) = -\frac{1}{2} + 1 + \frac{3}{2}$

$= 2$ ✓ (3)

(c) $y = t^x + q$

Y-int $(0; 0)$ $\therefore q = -1$ ✓

$y = t^x - 1$

sub $A(1; 2)$ ✓ $2 = t^1 - 1$

$3 = t$ ✓ (3)

(d) $y > -1$ ✓ ✓ or $y \in (-1; \infty)$ ✓ ✓ (2)

(e) TP of $g(x)$ $(1; -2)$ ✓ ✓ (2)