BALLITO

Mathematics Paper 1 June 2017

FORM 4

Examiner:	A Gunning	**Moderators:**	P Denissen
Time:	2½ hours	**Marks:**	125

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY BEFORE ANSWERING THE QUESTIONS.

- This question paper consists of 7 pages, which includes formulae. Please check that your question paper is complete.
- Read and answer all questions carefully.
- Number your answers exactly as the questions are numbered.
- It is in your own interest to write legibly and to present your work neatly.
- **ALL NECESSARY WORKING, WHICH YOU HAVE USED IN DETERMINING YOUR ANSWERS, MUST BE CLEARLY SHOWN.**
- Approved non-programmable calculators may be used except where otherwise stated. Where necessary give answers correct to 2 decimal places.
- Diagrams have not necessarily been drawn to scale.

Ques No	1	2	3	4	5	6	7	8	9	Total	%
Out of	50	13	8	11	22	6	5	5	5	125	100
Mark											

Question 1

(a) Solve for x in each of the following. You may not use a calculator to solve the equations and must show all relevant working details.

(i) $x(6x-10)-4=0$ (3)

(ii) $2x-3=\frac{2}{x}$ (4)

(iii) $9^{2x-1} = 27^{4-x}$ (4)

(iv) $4.3^x - 3^{x+1} = 81$ (4)

(v) Give your answers in simplest form, in terms of a. $x^2-ax+(a-1)=0$

You will need to use the formula. (5)

(b) Consider the equation $\sqrt{x}+3x=2$

(i) Show that, when solving this equation it can be reduced to

$(9x-4)(x-1)=0$ (3)

(ii) Determine the solutions (1)

(iii) and determine which (if any) of these solutions are invalid (2)

(c) Given $2x^2-7x-15 \geq 0$

(i) Solve for x if $2x^2-7x-15 \geq 0$ giving your final answer in interval notation.

(4)

(ii) Hence, or otherwise, determine for which positive values of x the expression

$\frac{\sqrt{2x^2-7x-15}}{x-8}$ will be real. (2)

(d) Given $M = \sqrt{(x+1)^2 - 4}$ where M is a real number,

(i) Solve for x if $M = 4$ (showing all relevant working detail, and without using a calculator, leave your answers in simplest surd form) (4)

(ii) Write down the minimum value of M. (1)

(e) Consider $\frac{3}{x+2} + 1 = \frac{1}{x-3}$

(i) Rewrite this equation in standard form, showing all relevant working detail. (3)

(ii) Hence solve for x by completing the square. (4)

(f) Solve for x and y simultaneously given

$6 - 4x - y = 0$ and $12 - 2x^2 - y = 0$ (6)

[50]

Question 2

(a) Simplify, without using a calculator. Remember to show all relevant working detail.

(i) $\frac{(5^{2x})^{-2}.20^{x+1}.125^{x-1}}{2^{1+2x}}$ (5)

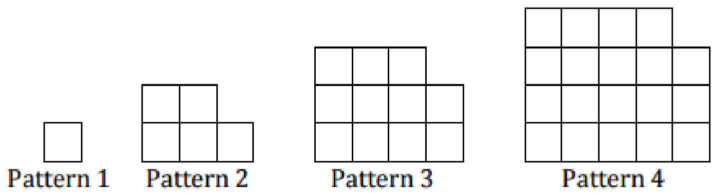
(ii) $\frac{x^2}{1+x}$ if $x = 1 + \sqrt{3}$ (4)

(b) Prove that the equation $\left(1 - \frac{2}{\sqrt{x}}\right)\left(1 + \frac{2}{\sqrt{x}}\right) = 3$ has no solution (4)

[13]

Question 3

The sequence $1; 5; 11; 19; \ldots\ldots$ gives the number of squares in each pattern below.



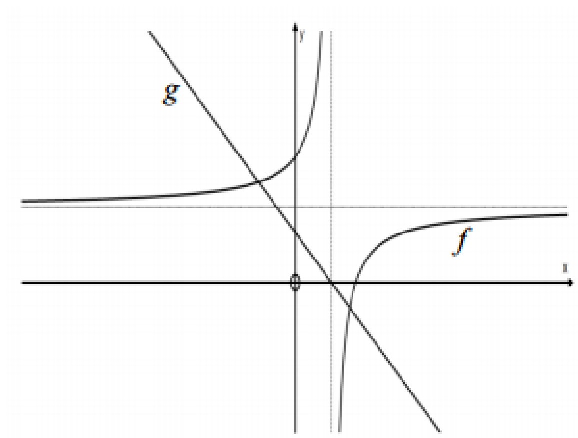
(a) Calculate a formula for the n^{th} pattern of the sequence, in its simplest form. (4)

(b) Calculate the smallest pattern number, n, for which the number of squares will be greater than 505. (4)

[8]

Question 4

Given $f(x) = \dfrac{-4}{x-2} + 3$ and $g(x) = -x + 2$



(a) Write down the equations of the asymptotes of f. (2)

(b) What is the domain of f? (2)

(c) Find the values of x for which $f(x) = g(x)$. Show all relevant working detail.

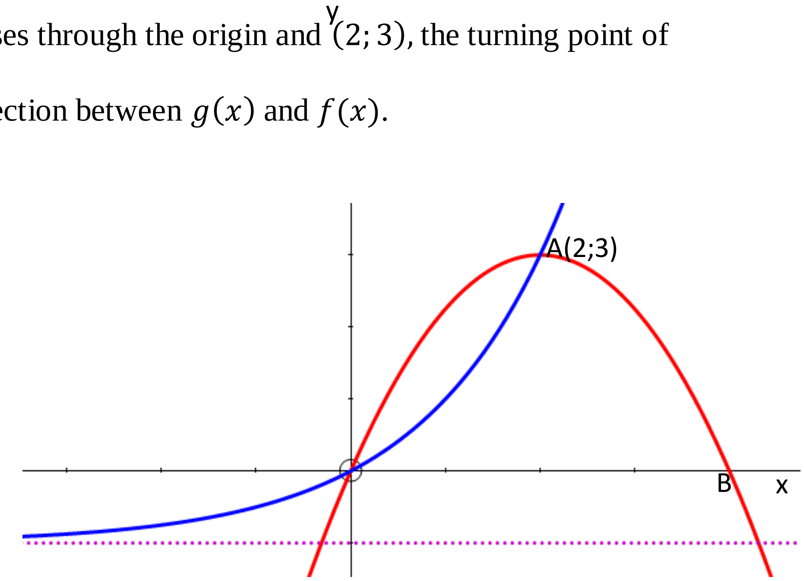
(7)

[11]

Question 5

In the diagram, the graphs of $f(x) = ax^2 + bx + c$ and $g(x) = p^x + q$ are represented. $y = -1$ is an asymptote to $g(x)$. $f(x)$ passes through the origin and $(2;3)$, the turning point of $f(x)$. $(2;3)$ is also the point of intersection between $g(x)$ and $f(x)$.

Write down:



(a) The equation of the axis of symmetry of f. (1)

(b) The coordinates of B. (1)

(c) The values of x for which $f(x)$ is decreasing. (1)

(d) The values of x for which $f(x) > 0$ (2)

Determine, showing all relevant working detail:

(e) The equation of $f(x)$ (3)

(f) the equation of $g(x)$. (3)

State:

(g) The range of $g(x) + 1$ (1)

(h) The coordinates of the turning point of $f(x)$ if it undergoes a transformation of $f(x + 4) + 5$ (2)

(i) the equation of $k(x)$ if $k(x)$ is the reflection of f about the x-axis. (2)

(j) the nature of the roots of $f(x) = 0$ (1)

(k) (1) what transformation needs to happen so that $f(x) = 0$ would have equal roots (1)

(2) What are these roots? (1)

(l) The value of k if $ax^2 + bx + c = k$, has no real roots. (1)

(m) The x values for which $g(x) \leq f(x)$ (2)

[22]

Question 6

On the set of axes provided on the inside back cover of your answer booklet, sketch the graph

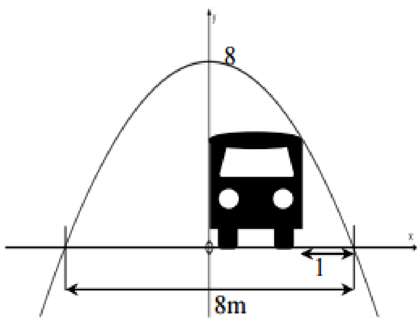
of $f(x) = \frac{1}{x+4} - 2.$

(You must show all relevant details clearly ie any asymptotes, intercepts with the axes etc.) (6)

[6]

Question 7

A tunnel has a parabolic cross-section with a maximum height of 8 meters and a width of 8 meters.



(a) Give the equation of the parabola. (3)

(b) The roof of the bus just touches the top edge of the tunnel when its wheels are 1 metre from the side of the tunnel. Determine the height of the bus. (2)

[5]

Question 8

Consider $2x^2 - 4x + 6$

(a) Does $2x^2 - 4x + 6$ have a maximum or minimum value? What is this value? (3)

(b) Hence write down the maximum value of the expression $y = \frac{1}{\sqrt{2x^2-4x+6}}$ (2)

[5]

Question 9

Draw sketch graphs (each on its own set of axes) of the curves which satisfy the conditions specified. Indicate the cuts on the x-axis, the equation of the axis of symmetry and asymptotes if applicable.

(a) $f(x) = ax^2 + bx + c$ where $f(x) \geq 0$ for $-5 \leq x \leq 1$ and $a < 0$ (2)

(b) $g(x) = a.2^x + q$ where $a < 0$ and $q > 0$. (3)

[5]

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$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\sum_{i=1}^{n} 1 = n \qquad \sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$T_n = a + (n-1)d \qquad S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$T_n = ar^{n-1} \qquad S_n = \frac{a(r^n - 1)}{r - 1}\ ;\ r \neq 1 \qquad S_\infty = \frac{a}{1-r}\ ;\ -1 < r < 1$$