ASHTON INTERNATIONAL COLLEGE
BALLITO

MEMO

Mathematics
Paper 1
FORM 4
6 June 2018
Session 1

TIME: 2 hours | **TOTAL: 100 marks**

Examiner: Miss M. Eastes | **Moderator: Mrs. D. Algie**

NAME:

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY BEFORE ANSWERING THE QUESTIONS.

- This question paper consists of 16 pages. Formulae are given on page 2.
  Please check that your question paper is complete.
- Answer all questions on your question paper.
- Read and answer all questions carefully.
- It is in your own interest to write legibly and to present your work neatly.
- All necessary working which you have used in determining your answers **must** be clearly shown.
- Approved non-programmable calculators may be used except where otherwise stated. Where necessary give answers correct to **2 decimal places** unless otherwise stated.
- Ensure that your calculator is in DEGREE mode.
- Diagrams have not necessarily been drawn to scale.
- State all restrictions where necessary.

PA ME

Questions	1	2	3	4	5	6
Out of	27	6	9	5	5	12
Mark			AG			
Question	**7**	**8**	**9**	**10**	**11**	**TOTAL**
Out of	4	5	18	4	5	100
Mark						

SECTION A

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $\quad T_n = a + (n-1)d$

QUESTION 1 [27]

Solve for x without using the calculator:
(you can use the calculator to check your answers if necessary)

a) $x(3x - 1) = 2 \; ; \; x \in Z$ (4)

$3x^2 - x = 2$ Va

$3x^2 - x - 2 = 0$ Vca

$(x-1)(3x+2) = 0$

$x \neq 1 \quad x \neq -\frac{2}{3}$ Vca

b) $\frac{1}{x+1} + \frac{3}{x-2} = \frac{x+2}{x+1}$ (6)

$\frac{1}{x+1} + \frac{3}{x-2} = \frac{x+2}{x+1}$

$(x-2)$ Va $+ 3(x+1)$ Va $= (x+2)(x-2)$ Vca

$x - 2 + 3x + 3 = x^2 - 4$

$0 = x^2 - 4x - 5$ Vm

$x \neq 5$ or $x \neq -1$ Vca $\quad x \neq -1 \quad x \neq 2$ Vca

c) $2^{-2x} = \frac{1}{32}$ (3)

$2^{-2x} = 2^{-5}$ Va

$-2x = -5$ Vm

$x = \frac{5}{2}$ Vca

d) $x + \sqrt{x-2} = 4$ (5)

$\sqrt{x-2} = 4 - x$ Va

$x - 2 = 16 - 8x + x^2$ Vm

$0 = x^2 - 9x + 18$

$0 = (x-6)(x-3)$ Va

$x \neq 6$ Vca or $x = 3$ Vca

e) $5x^{\frac{-2}{3}} = 80$ (5)

$5x^{-\frac{2}{3}} = 80$

$x^{-\frac{2}{3}} = 16$ Va

$x = (16)^{-\frac{3}{2}}$ Vm

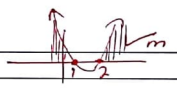
$x = (2^4)^{-\frac{3}{2}}$

$x = 2^{-6}$ Vca

$x = +\frac{1}{64}$ Va Vca

f) $x^2 - 3x + 2 \geq 0$ (4)

$(x-2)(x-1) \geq 0$ Va



Vm

$x \geq 2$ or $x \leq 1$ Va Va

QUESTION 2 [6]

Consider the sequence - 5; - 2; 1; 4; 7; ...

a) Write down the next two terms of this sequence. (2)

10 ; 13
✓a ✓a

b) Determine the formula of the n^{th} term of this sequence in its simplest form. (2)

$T_n = -5 + (n-1)(3)$ ✓a
$= 3n - 8$ ✓ca

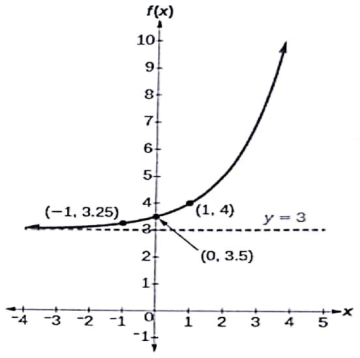
c) Determine the value of the 25^{th} term. (2)

$T_n = 3(25) - 8$ ✓a
$= 67$ ✓ca

QUESTION 3 [9]

Determine the equations of the following graphs:

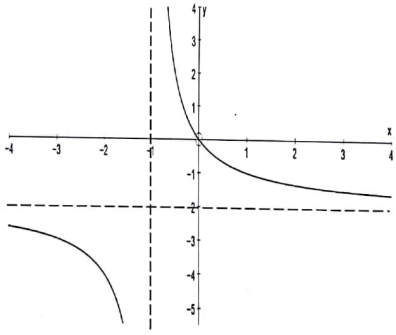
a) $y = 2^{x-p} + q$ (5)



$y = 2^{x-p} + 3$ ✓a
$4 = 2^{1-p} + 3$ ✓m
$1 = 2^{1-p}$
$2^0 = 2^{1-p}$ ✓a
$0 = 1 - p$ ✓m
$1 = p$ ✓ca

$y = 2^{x-1} + 3$

b) $f(x) = \frac{a}{x+p} + q$ (4)



$y = \frac{a}{x+1} - 2$ ✓a ✓a

$0 = \frac{a}{0+1} - 2$ ✓m

$2 = \frac{a}{1}$

$2 = a$ ✓ca

$y = \frac{2}{x+1} - 2$

SECTION B

QUESTION 4 [5]

Solve for x: $\sqrt{4^{x+1} + 2^{2x+5}} = 3.2^{2-x}$ (5)

$\sqrt{2^{2x+2} + 2^{2x+5}} = 3.2^{2-x}$ ✓q

$\sqrt{2^{2x}(2^2 + 2^5)}$ ✓ca $= 3.2^{2-x}$

$\sqrt{2^{2x}.36} = 3.2^{2-x}$

$2^x.6 = 3.2^{2-x}$ ✓q

$2^x.2 = 2^{2-x}$

$2^{x+1} = 2^{2-x}$ ✓ca

$x+1 = 2-x$

$2x = 1$

$x = \frac{1}{2}$ ✓ca

QUESTION 5 [5]

A pupil solves a quadratic equation by using the quadratic formula.

Her solution is: $x = \frac{-5 \pm \sqrt{25 - p^2}}{2}$

a) For which value(s) of p will the roots be equal? (2)

$25 - p^2 = 0$ ✓q

$-p^2 = -25$

$p = \pm 5$ ✓ca

b) Discuss the nature of the roots if p = -3. (3)

$\Delta = 25 - (-3)^2$ ✓m

$= 16$ ✓a

∴ square nr ∴ rational + real ✓ca roots

QUESTION 6 [12]

Simplify: * without using a calculator.

a) $\frac{3^x - 3^{x-2}}{6.3^x - 4.3^{x-2}}$ (5)

$= \frac{3^x(1 - 3^{-2})}{3^x(6 - 4.3^{-2})}$ ✓m ✓q

$= \frac{1 - \frac{1}{9}}{6 - \frac{4}{9}}$ ✓ca

$= \frac{4}{25}$ ✓ca

b) $\frac{\sqrt[3]{(a-b)^3} \times \sqrt[3]{(a-b)^3}}{a^2 - b^2}$ (3)

$= \frac{(a-b)(a-b)}{(a+b)(a-b)}$ ✓a

$= \frac{a-b}{a+b}$ ✓ca

c) $\left(16^{\frac{1}{4}} + 32^{-\frac{2}{5}}\right)^{\frac{1}{2}}$ (4)

$= \left[(2^4)^{\frac{1}{4}} + (2^5)^{-\frac{2}{5}}\right]^{\frac{1}{2}}$ ✓m

$= (2 + 2^{-2})^{\frac{1}{2}}$ ✓ca

$= \left(2 + \frac{1}{4}\right)^{\frac{1}{2}}$

$= \left(\frac{9}{4}\right)^{\frac{1}{2}}$ ✓m

$= \frac{3}{2}$ ✓ca

QUESTION 7 [4]

For which real values of k does the following function have real roots?

$2x^2 - 8x + 6 = k$ (4)

$\Delta \geq 0$ ✓m

$(-8)^2 - 4(2)(6-k) \geq 0$ ✓a

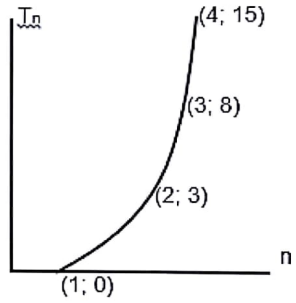
$64 - 48 + 8k \geq 0$ ✓ca

$k \geq -2$ ✓ca

QUESTION 8 [5]

The graph below depicts the relationship between n and T_n.

Determine the general term. (Tn in terms of n) (5)



0; 3; 8; 15

3 5 7 ✓m

2 2

$2a = 2$ $3a + b = 3$ $a + b + c = 0$

$a = 1$ ✓a $b = 0$ ✓a $1 + 0 + c = 0$

$c = -1$ ✓a

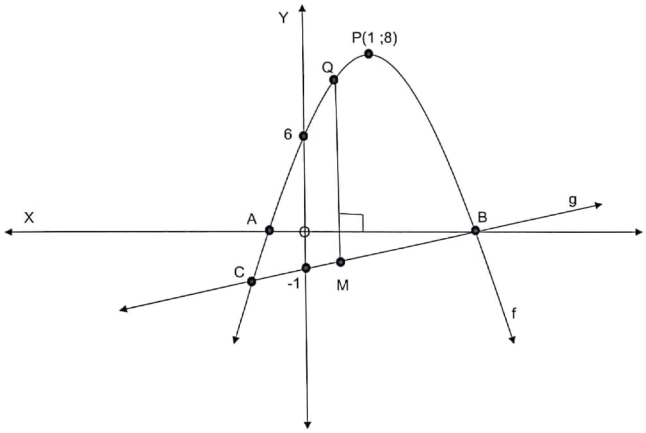
$T_n = n^2 - 1$ ✓ca

QUESTION 9 [18]

Below is a sketch graph of a parabola, *f*, and a straight line, *g*.
P(1; 8) is the turning point of *f*.

$g(x) = \frac{1}{3}x - 1$

The graph of *f* cuts the y-axis at (0;6), g cuts the y-axis at (0;-1).
B is a point on the x-axis and QM is perpendicular to the x-axis.



a) Show that $f(x) = -2x^2 + 4x + 6$ (4)

$y = a(x-1)^2 + 8$ ✓a
$6 = a(0-1)^2 + 8$ ✓m
$-2 = a$ ✓a

$y = -2(x-1)^2 + 8$
$y = -2(x^2 - 2x + 1) + 8$ ✓a
$y = -2x^2 + 4x + 6$

b) Determine the coordinates of point C (6)

$-2x^2 + 4x + 6 = \frac{1}{3}x - 1$ ✓m
$-2x^2 + \frac{11}{3}x + 7 = 0$
$6x^2 - 11x - 21 = 0$ ✓a
$x = \frac{-7}{6}$ ✓a
$y = \frac{1}{3}\left(\frac{-7}{6}\right) - 1$ ✓a
$y = \frac{-25}{18}$ ✓ca
$C\left(\frac{-7}{6}; \frac{-25}{18}\right)$ ✓ca

c) Determine the maximum length of QM between the graph of *f and g*. QM ⊥AB. (5)

$QM = -2x^2 + 4x + 6 - \frac{1}{3}x + 1$ ✓a
$= -2x^2 + \frac{11}{3}x + 7$

$x = \frac{-(\frac{11}{3})}{2(-2)}$ ✓a
$= \frac{11}{12}$ ✓ca
$QM = -2(\frac{11}{12})^2 + \frac{11}{3}(\frac{11}{12}) + 7$ ✓m
$= \frac{625}{72}$ ✓ca

d) Find values of x for which: $f(x).g(x) > 0$ (3)

$x < -1$ ✓ca

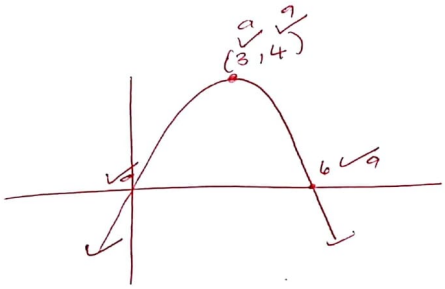
$A(-1, 0)$ ✓ca
$0 = -2x^2 + 4x + 6$ ✓a
$x \neq 3 \quad x = -1$

QUESTION 10 [4]

$p(x) = ax^2 + bx + c$. You are given the following information about p:

- the roots differ by 6
- the value of $x = \frac{-b}{2a}$ is 3.
- The range is $y \leq 4$.

Draw a sketch graph of p below, indicating the x-intercepts and the co-ordinates of the turning point. (4)



QUESTION 11 [5]

One of our soccer players kicks the ball so that it follows the path described by the equation

$h = \frac{-1}{20}(d - 15)^2 + \frac{45}{4}$,

where h is the height and d is the horizontal distance, in metres.

She claims that she can kick the ball a horizontal distance of 40m.

Is that true?

Prove your answer by showing all calculations. (5)

$-\frac{1}{20}(d-15)^2 + \frac{45}{4} = 0$ ✓a

$-\frac{1}{20}(d-15)^2 = -\frac{45}{4}$ ✓ca

$(d-15)^2 = 225$ ✓a

$d - 15 = \pm 15$

$d = 30$ ✓a

not true max distance is 30m ✓ca