education

Department:
Education
PROVINCE OF KWAZULU-NATAL

NATIONAL SENIOR CERTIFICATE

GRADE 11

MATHEMATICS P1

COMMON TEST

JUNE 2018

MARKS: 100

TIME: 2 hours

This question paper consists of 8 pages.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions:

1. This question paper consists of 6 questions.
2. Answer ALL the questions.
3. Number the answers correctly according to the numbering system used in this question paper.
4. Clearly show ALL calculations, diagrams, graphs, et cetera, which you have used in determining the answers.
5. Answers only will NOT necessarily be awarded full marks.
6. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
7. If necessary, round off answers to TWO decimal places, unless stated otherwise.
8. Diagrams are NOT necessarily drawn to scale.
9. Write neatly and legibly.

QUESTION 1

1.1 Solve for x in each of the following:

1.1.1 $(2x+1)(x-1)=0$ (2)

1.1.2 $2x^2+11=3x+21$ (correct to TWO decimal places) (4)

1.1.3 $\sqrt{2x-1}+5=\dfrac{14}{\sqrt{2x-1}}$ (5)

1.2 Solve simultaneously for x and y:
$y=2x+1$ and $3x^2-5xy+4y^2=24$ (5)

1.3 Consider the equation $kx^2+kx+2=0$

1.3.1 Solve for x in terms of k. (2)

1.3.2 For which values of k will the equation $kx^2+kx+2=0$ have non-real roots? (3)

[21]

QUESTION 2

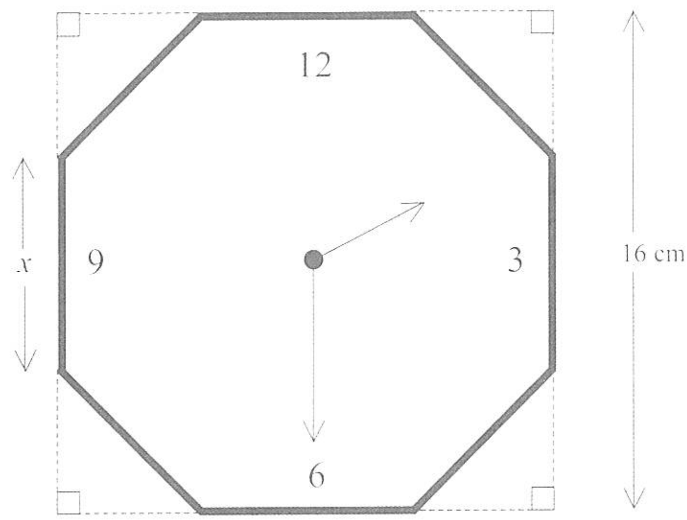
2.1 Evaluate, WITHOUT using a calculator: $\dfrac{6^{\frac{1}{2}n}\times 12^{n+1}\times 27^{-\frac{1}{2}n}}{32^{\frac{1}{2}n}}$ (4)

2.2 Calculate the value of $\dfrac{\sqrt{10^{2016}}}{\sqrt{10^{2018}}-\sqrt{10^{2014}}}$ WITHOUT using a calculator.
Show ALL your calculations. (4)

[8]

QUESTION 3

Jabu manufactures wall clock faces in the shape of a regular octagon (eight-sided shape having sides of equal length). He cuts off identical isosceles triangles from the corners of a square sheet of glass having sides 16 cm long.



If the length of a side of the clock face is x, calculate the value of x correct to TWO decimal places.

[7]

QUESTION 4

4.1 Consider the quadratic number pattern: $0 ; -9 ; -16 ; -21 ; -24$

4.1.1 Determine the n^{th} term (T_n) of the pattern. (4)

4.1.2 Calculate the 30^{th} term of the pattern. (2)

4.1.3 Determine which term of the pattern will be equal to 200. (4)

4.2 The first three figures in a pattern of grey and white squares are shown below.

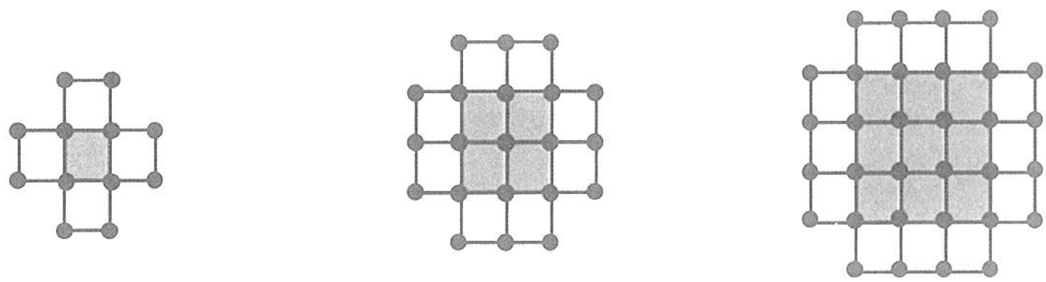


Figure 1 Figure 2 Figure 3

4.2.1 Considering Figure 4 in the pattern, determine:

(a) The number of grey squares in this figure. (1)

(b) The number of white squares in this figure. (1)

(c) The number of dots in this figure. (1)

4.2.2 Considering the n^{th} figure, determine an expression in terms of n for:

(a) The number of grey squares. (1)

(b) The number of white squares. (1)

(c) The number of dots. (2)

4.2.3 There are 320 dots in one of the figures. Determine the number of grey squares in this figure. (5)

[22]

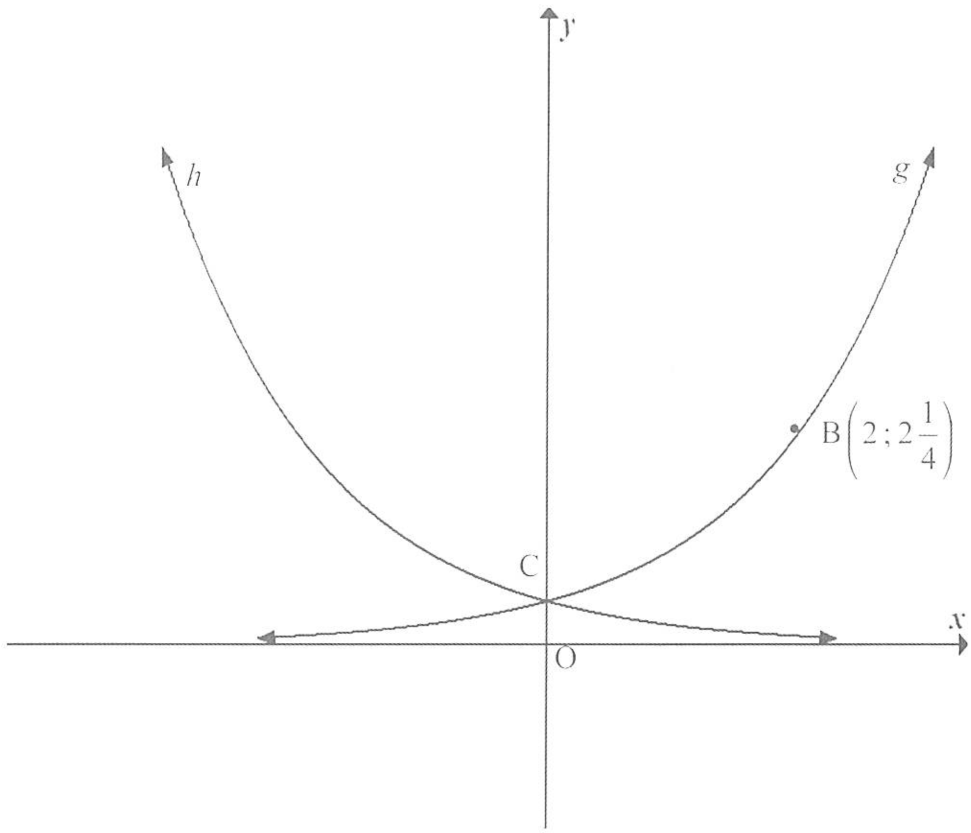
QUESTION 5

5.1 Given: $f(x)=\dfrac{-6}{x+3}+2$.

5.1.1 Draw a neat sketch graph of f, indicating the asymptote(s) and intercept(s) with the axes. Show all your calculations. (6)

5.1.2 An axis of symmetry of f has the equation $y=mx+5$. Write down the value of m. (1)

5.2 In the diagram below, g represents the function $g(x)=a^x, a>0$. The graph h is symmetrical to g about the y – axis. The point $B\left(2;2\frac{1}{4}\right)$ lies on the curve of g and C is the y-intercept of both g and h.



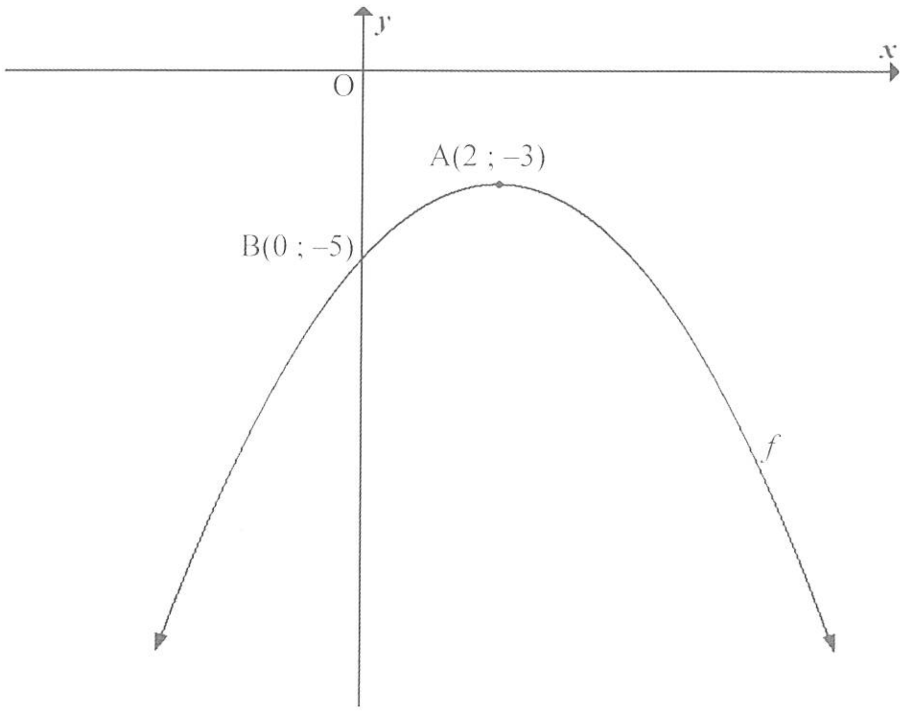
5.2.1 Write down the range of g. (1)

5.2.2 Calculate the value of a. (2)

5.2.3 Determine the equation of h in the form $y=b^x$. (2)

5.2.4 B′ is the reflection of B in the y-axis. Calculate the average gradient between B′ and C. (4)

5.3 The diagram below shows the graph of a parabola f, with turning point A(2 ; –3) and y – intercept B(0 ; –5).



5.3.1 Show that the equation of the parabola can be written as $y = -\frac{1}{2}x^2 + 2x - 5$. (5)

5.3.2 Use the graph to determine the value(s) of k for which the equation $-\frac{1}{2}x^2 + 2x - 5 = k$ will have real and unequal roots. (2)

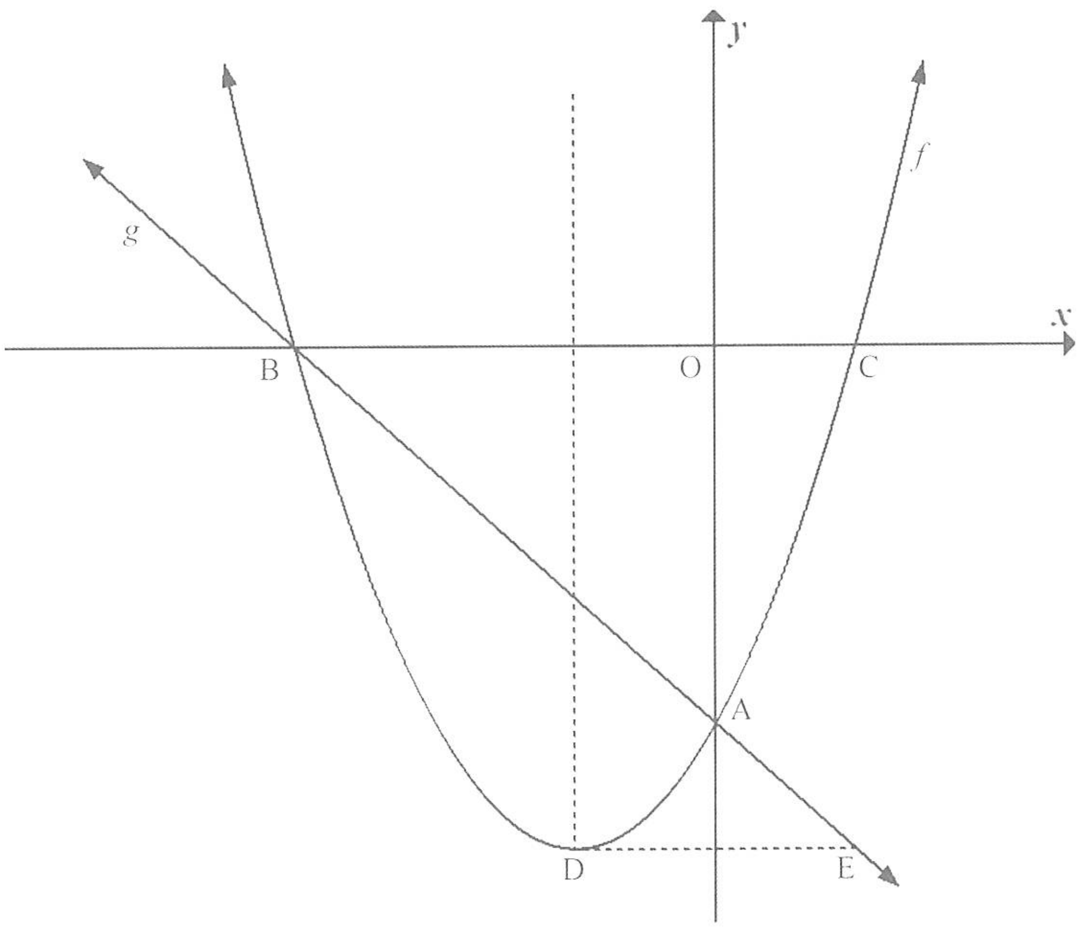
5.3.3 The parabola f is shifted vertically until the new y – intercept is at the origin. Determine the new turning point of the parabola. (2)

[25]

QUESTION 6

The diagram below represents the functions: $f(x) = \frac{3}{2}x^2 + 3x - \frac{9}{2}$ and $g(x) = mx + c$.

- A, B and C are the points at which f intersects the axes.
- D is the turning point of f.
- g passes through A and B.
- E is a point on g.



6.1 Determine the coordinates of A, B and C. (5)

6.2 Determine the coordinates of D. (3)

6.3 Use the graphs to determine the values of x for which $f(x) \geq g(x)$. (2)

6.4 Determine the equation of g in the form $g(x) = mx + c$. (3)

6.5 Determine the length of DE, if DE is perpendicular to the y-axis. (4)

[17]

TOTAL MARKS: 100

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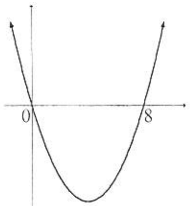
MARKING GUIDELINE

MARKS: 100

This marking guideline consists of 9 pages.

QUESTION 1

1.1.1	$(2x+1)(x-1)=0$ $x=-\frac{1}{2}$ or $x=1$	✓ $x=-\frac{1}{2}$ ✓ $x=1$ (2)
1.1.2	$2x^2+11=3x+21$ $2x^2-3x-10=0$ $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ $x=\frac{-(-3)\pm\sqrt{(-3)^2-4(2)(-10)}}{2(2)}$ $x=3{,}11$ or $x=-1{,}61$	✓ standard form ✓ substitution into quadratic formula ✓ answer ✓ answer (4)
1.1.3	$\sqrt{2x-1}+5=\frac{14}{\sqrt{2x-1}}$ Let $\sqrt{2x-1}=k$ $k+5=\frac{14}{k}$ $k^2+5k-14=0$ $(k+7)(k-2)=0$ $k=-7$ or $k=2$ $\sqrt{2x-1}=-7$ or $\sqrt{2x-1}=2$ No solution $\therefore 2x-1=4$ $\therefore x=2\frac{1}{2}$	✓ standard form ✓ factors ✓ both answers ✓ $\sqrt{2x-1}\neq-7$ ✓ answer (5)
1.2	$3x^2-5x(2x+1)+4(2x+1)^2=24$ $3x^2-10x^2-5x+4(4x^2+4x+1)=24$ $3x^2-10x^2-5x+16x^2+16x+4=24$ $9x^2+11x-20=0$ $(9x+20)(x-1)=0$ $x=-\frac{20}{9}$ or $x=1$ $y=2\left(-\frac{20}{9}\right)+1$ or $y=2(1)+1$ $y=-\frac{31}{9}$ or $y=3$	✓ substitution ✓ standard form ✓ factors ✓ both x-values ✓ both y-values (5)

1.3.1	$kx^2 + kx + 2 = 0$ $x = \dfrac{-k \pm \sqrt{k^2 - 4(k)(2)}}{2k}$ $x = \dfrac{-k \pm \sqrt{k^2 - 8k}}{2k}$	✓ substitution into quadratic formula ✓ answer (2)
1.3.2	$k^2 - 8k < 0$ $k(k-8) < 0$  $0 < k < 8$	✓ $k^2 - 8k < 0$ ✓ factors ✓ answer (3)
		[21]

QUESTION 2

2.1	$\dfrac{6^{\frac{1}{2}n} \times 12^{n+1} \times 27^{-\frac{1}{2}n}}{32^{\frac{1}{2}n}}$ $= \dfrac{(3\times2)^{\frac{1}{2}n} \times (3\times2^2)^{n+1} \times (3^3)^{-\frac{1}{2}n}}{(2^5)^{\frac{1}{2}n}}$ $= \dfrac{3^{\frac{1}{2}n} \times 2^{\frac{1}{2}n} \times 3^{n+1} \times 2^{2n+2} \times 3^{-\frac{3}{2}n}}{2^{\frac{5}{2}n}}$ $= 2^{\frac{1}{2}n+2n+2-\frac{5}{2}n} . 3^{\frac{1}{2}n+n+1-\frac{3}{2}n}$ $= 3^1 . 2^2$ $= 12$	✓ writing as prime bases ✓ simplification using laws ✓ simplification using laws ✓ answer **NOTE: If a calculator is used, then NO marks will be awarded.** (4)

2.2	$\dfrac{\sqrt{10^{2016}}}{\sqrt{10^{2018}} - \sqrt{10^{2014}}}$ $= \dfrac{10^{1008}}{10^{1009} - 10^{1007}}$ $= \dfrac{10^{1008}}{10^{1007}(10^2 - 1)}$ $= \dfrac{10}{10^2 - 1}$ $= \dfrac{10}{99}$ OR $\dfrac{\sqrt{10^{2016}}}{\sqrt{10^{2018}} - \sqrt{10^{2014}}}$ $= \dfrac{\sqrt{10^{2014}}.\sqrt{10^2}}{\sqrt{10^{2014}}(\sqrt{10^4} - 1)}$ $= \dfrac{10}{10^2 - 1}$ $= \dfrac{10}{99}$ or 0,10	✓ determining square roots ✓ factorising denominator ✓ $\dfrac{10^{1008}}{10^{1007}} = 10$ ✓ answer (4) OR ✓ writing numerator as $\sqrt{10^{2014}}.\sqrt{10^2}$ ✓ factorizing denominator ✓ simplification ✓ answer (4)
		[8]

QUESTION 3

The length of the side of the clock is x $\therefore$ the length of each corner cut out is $\dfrac{16-x}{2}$ $\left(\dfrac{16-x}{2}\right)^2 + \left(\dfrac{16-x}{2}\right)^2 = x^2$ [Theorem of Pythagoras] $\dfrac{256 - 32x + x^2}{4} + \dfrac{256 - 32x + x^2}{4} = x^2$ $512 - 64x + 2x^2 = 4x^2$ $2x^2 + 64x - 512 = 0$ $x^2 + 32x - 256 = 0$ $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $x = \dfrac{-32 \pm \sqrt{(32)^2 - 4(1)(-256)}}{2(1)}$ $x = 6,63 \because x \neq -38,63$ the length of the side of the clock is 6,63 cm	✓ $\dfrac{16-x}{2}$ ✓ equation ✓ simplification ✓ standard form ✓ substitution into quadratic formula ✓ both values of x ✓ answer [7]

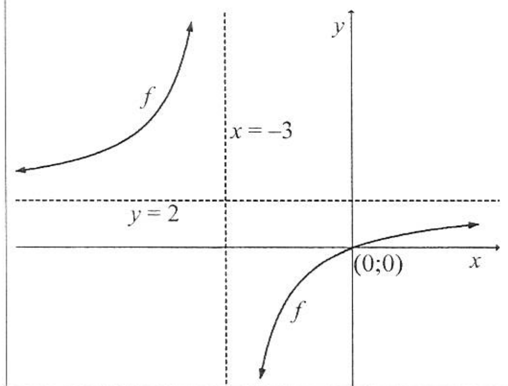
QUESTION 4

4.1.1	0 ; −9 ; −16 ; −21 first differences: −9 ; −7 ; −5 second differences: 2 ; 2 ; 2 $2a=2$ $a=1$ $3(1)+b=-9$ $b=-12$ $1-12+c=0$ $c=11$ $\therefore T_n=n^2-12n+11$	✓ calculate a ✓ calculate b ✓ calculate c ✓ answer (4)
4.1.2	$\therefore T_{30}=(30)^2-12(30)+11$ $=551$	✓ substitution of 30 for n into T_n ✓ answer (2)
4.1.3	$200=n^2-12n+11$ $n^2-12n-189=0$ $(n-21)(n+9)=0$ $n\neq-9\ \therefore n=21$	✓ equating T_n to 200 ✓ standard form ✓ factors or quadratic formula ✓answer (4)

4.2.1(a)	Number of grey squares: $1;4;9;\ldots = 1^2;2^2;3^2\ldots$ Number of grey squares in Figure 4 = $4^2=16$	✓ 16 (1)
4.2.1(b)	Number of white squares: $4;8;12;\ldots = 1(4);\ 2(4);\ 3(4);\ldots$ Number of white squares in Figure 4 = $4(4)=16$	✓ 16 (1)
4.2.1(c)	Number of dots: $12;21;32;\ldots = 2^2+4(2);\ 3^2+4(3);\ 4^2+4(4);\ldots$ Number of dots in Figure 4 = $5^2+5(4)=25+20=45$ OR Number of dots: 12 ; 21 ; 32 ; 9 11 Number of dots in Figure 4 = 32+13 = 45	✓45 (1) OR ✓45 (1)

4.2.2(a)	n^2	✓ answer (1)
4.2.2(b)	$4n$	✓ answer (1)
4.2.2(c)	$(n+1)^2+4(n+1)$ **OR** $2a=2$ $a=1$ $3(1)+b=9$ $b=6$ $1+6+c=12$ $c=5$ $\therefore n^2+6n+5$ **OR** $(n+3)^2-4$	✓ $(n+1)^2$ ✓ $+4(n+1)$ (2) **OR** ✓ calculating a, b and c ✓ answer (2) **OR** ✓ ✓ answer (2)
4.2.3	$n^2+6n+5=320$ $n^2+6n-315=0$ $(n+21)(n-15)=0$ $n\neq-21\ \therefore n=15$ number of grey squares = 15^2 $=225$	✓ equating answer of 4.2.2(c) to 320 ✓ standard form ✓ factors or quadratic formula ✓ value of n ✓ answer (5)
		[22]

QUESTION 5

5.1.1	For y-intercept, substitute $x = 0$: $y = \frac{-6}{0+3} + 2$ $= 0$ OR For x-intercept, substitute $y = 0$: $0 = \frac{-6}{x+3} + 2$ $\frac{6}{x+3} = 2$ $x + 3 = 3$ $x = 0$ The graph therefore goes through the origin: (0 ; 0) 	✓ substitution ✓ answer ✓ horizontal asymptote: $y = 2$ ✓ vertical asymptote: $x = -3$ ✓ passing through the origin ✓ shape	(6)
5.1.2	$m = 1$	✓ answer	(1)
5.2.1	$y > 0$ **OR** $y \in (0 ; \infty)$	✓ answer **OR** ✓ answer	(1)
5.2.2	$\frac{9}{4} = a^2$ $a = \frac{3}{2}$	✓ substitution ✓ answer	(2)
5.2.3	$y = \left(\frac{3}{2}\right)^{-x}$ $\therefore y = \left(\frac{2}{3}\right)^{x}$	✓ change sign of x ✓ answer	(2)
5.2.4	$B'\left(-2 ; 2\frac{1}{4}\right)$ C(0 ; 1) Average gradient $= \frac{2\frac{1}{4} - 1}{-2 - 0}$ $= -\frac{5}{8}$	✓ coordinates of B' ✓ coordinates of C ✓ substitution into average gradient formula ✓ answer	(4)

5.3.1	$y = a(x+p)^2 + q$ $y = a(x-2)^2 - 3$ $-5 = a(0-2)^2 - 3$ $-2 = 4a$ $a = -\frac{1}{2}$ $y = -\frac{1}{2}(x-2)^2 - 3$ $y = -\frac{1}{2}(x^2 - 4x + 4) - 3$ $y = -\frac{1}{2}x^2 + 2x - 5$	✓ substitution of turning point, A ✓ substitution of point B ✓ value of a ✓ substitution of a ✓ simplification	(5)
5.3.2	$k < -3$	✓✓ answer	(2)
5.3.3	(2 ; 2)	✓ x–coordinate ✓ y–coordinate	(2)
			[25]

QUESTION 6

6.1	$A\left(0;-4\frac{1}{2}\right)$ $\frac{3}{2}x^2+3x-\frac{9}{2}=0$ $x^2+2x-3=0$ $(x-1)(x+3)=0$ $x=1$ or $x=-3$ B(−3 ; 0) and C(1 ; 0)	✓ coordinates of A ✓ let $y=0$ ✓ factors ✓ coordinates of B ✓ coordinates of C (5)
6.2	Axis of symmetry: $x=\frac{-3+1}{2}$ OR $x=\frac{-3}{2\left(\frac{3}{2}\right)}$ $\therefore x=-1$ Minimum value: $y=\frac{3}{2}(-1)^2+3(-1)-\frac{9}{2}$ $y=-6$ $\therefore D(-1;-6)$	✓ method of determining the axis of symmetry ✓ x–coordinate ✓ y–coordinate (3)
6.3	$x\leq-3$ or $x\geq0$	✓✓ answer (2)
6.4	gradient of AB: $=\frac{0-(-4,5)}{-3-0}$ $=-\frac{3}{2}$ $y=-\frac{3}{2}x-\frac{9}{2}$	✓ substitution in gradient formula ✓ value of gradient ✓ equation of g (3)
6.5	$-6=-\frac{3}{2}x-\frac{9}{2}$ $x=1$ at point E. $\therefore$ DE $=1-(-1)$ DE = 2 units	✓✓ equating y_E to y_D ✓ value of x ✓ answer (4)
		[17]

TOTAL: 100