Mathematics Paper 1 November 2016

FORM 4

Examiner:	A Gunning	**Moderators:**	P Denissen, C Mundy
Time:	3 hours	**Marks:**	150

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY BEFORE ANSWERING THE QUESTIONS.

- This question paper consists of 11 pages, which includes the information sheet. Please check that your question paper is complete.
- Read and answer all questions carefully.
- Number your answers exactly as the questions are numbered.
- It is in your own interest to write legibly and to present your work neatly.
- **All necessary working, which you have used in determining your answers, must be clearly shown.**
- Approved non-programmable calculators may be used except where otherwise stated. Where necessary give answers correct to 2 decimal places.
- Diagrams have not necessarily been drawn to scale.

SECTION A

Question 1

(a) Solve for x: (Leave answers in surd form where necessary)

(i) $2 = bx$ (Give your answer in terms of b) (1)

(ii) $2(x^2 - 6) = 3(x - 1)$ (3)

(iii) $-3(x + 1)(x - 3) < 0$ (4)

(iv) $x + 5 = \sqrt{3 - 3x}$ (5)

(v) $3^{1-x^2} = \frac{27^{-1}}{3}$ (4)

(b) Given the quadratic equation: $px^2 + 6x - p = 0$

(i) Solve the equation, in terms of p, using the quadratic formula. Leave your answer in simplest surd form. (4)

(ii) Hence, state for which value(s) of p the equation will have real roots. (2)

(c) Solve simultaneously for x and y: $2x + 6 - y = 0$ and $y + 3x^2 = 8x + 3$ (5)

(d) Given that $4^x = 8^{2-x}$, find the value of x. (3)

(e) The roots of the quadratic equation $g(x) = 0$ are given by:

$$x = \frac{-20 \pm \sqrt{400 - 48h}}{24}$$

(i) Determine the roots if $h = 1$ (2)

(ii) For which value(s) of h will the roots of $g(x)$ be real? (2)

(iii) Write down the largest possible integral value of h for which the roots will be rational. (2)

(iv) If $x = \frac{-20 \pm \sqrt{400-48h}}{24}$

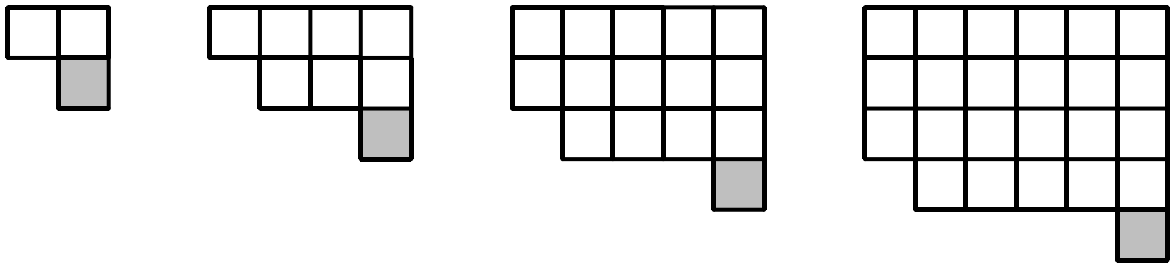
1) For which value of h will one of the roots be 0? (2)

2) Now write down the other root. (2)

[41]

Question 2

Consider the following picture pattern which continues in the same way indefinitely.



Calculate the number of square blocks in the 50^{th} pattern. (5)

[5]

Question 3

(a) R12 000 is invested at an interest rate of 13% per annum, compounded monthly. Calculate the value of the investment after 5 years. (3)

(b) R10 000 is invested at 12% per annum, compounded monthly. Calculate the effective interest rate. (3)

(c) A man invests R200 000 at 12% per annum compounded monthly. After 2 years he withdraws R50 000 and after another 3 years he withdraws another R30 000. Calculate the value of his investment after 8 years. (Hint: Use a timeline.) (6)

(d) A firm buys computers. After 5 years the value of the computers will be R12 000. Calculate the initial value of the computers if the computers decreased at a rate of 12% p.a. on a reducing basis. (3)

(e) Money is invested at an interest rate of 10,5% per annum, compounded quarterly. After 6 years the interest on the investment amounts to R8 624,14. What was the initial amount? (5)

[20]

Question 4

(This question is to be answered on the inside back cover of the Answer Booklet.)

Given: $f(x)=\frac{3}{x-2}-1$ and $g(x)=x^2-4x$

(a) Write down the **equations** of the asymptotes of f. (2)

(b) Determine the x and y intercepts of f. (3)

(c) Write down the coordinates of the turning point of g. (3)

(d) Determine the x and y intercepts of g . (2)

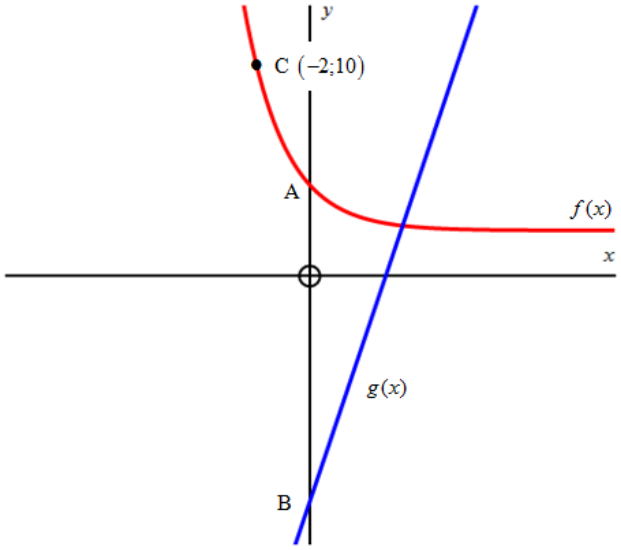
(e) Draw neat sketch graphs of f and g on the same system of axes on the set of axes printed in the answer booklet. Clearly indicate asymptotes, turning points and intercepts with the axes. (7)

(f) What transformation must g undergo to have no real roots? (2)

[19]

SECTION B

Question 5



The graphs of $f(x)=3^{-x}+p$ and $g(x)=3x-5$ are drawn above.

C is the point $(-2;10)$ and A and B are y intercepts.

(a) Determine the value of p. (2)

(b) Determine the length of AB. (3)

(c) Determine the equation of h if h is formed by reflecting g about the x axis and then translating it 2 units to the right. (3)

[8]

Question 6

(a) Without using a calculator, find the sum of the digits in the product $4^9.5^{12}$ (3)

(b) Consider the expression x^2-2x

(i) Does this have a maximum or minimum value? Determine this value. (2)

(ii) Hence determine the minimum value of 2^{x^2-2x} (1)

(c) If d and e are elements of the set $\{1\ ;\ 2\ ;\ 3\}$, find the number of equations of the form $x^2 + dx + e = 0$ which have real roots. (4)

[10]

Question 7

Dr Rink sets up a sinking fund to cover the cost of replacing her practice's medical equipment. She makes the same annual payment into a fund offering 10% p.a.. The first payment is made at the end of the first year, and the last, at the end of the 4th year.

What must her annual payments be in order to cover the replacement costs anticipated to be R1 200 000 at that time? (6)

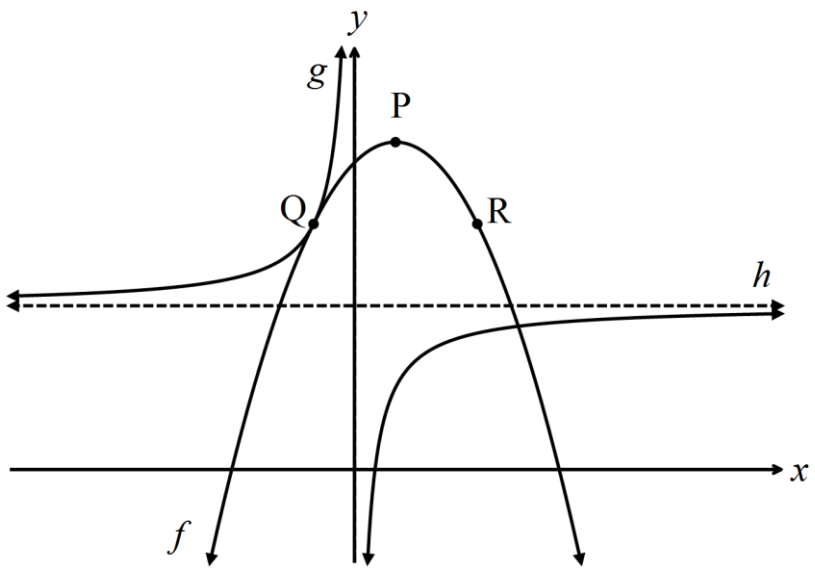
[6]

Question 8

The graphs of $f(x) = -\frac{1}{2}x^2 + x + c$, $g(x) = -\frac{2}{x} + q$ and $h(x) = q$ are sketched below.

Q and R are symmetrical to each other about the symmetry line of f.

P is the turning point of f and $f(-1) = g(-1) = 6$.



(a) Write down the coordinates of Q. (1)

(b) Show that $c = \frac{15}{2}$ and $q = 4$. (4)

(c) Determine the range of f. (5)

(d) One of the axes of symmetry of g is a decreasing function. State the equation of this axis of symmetry. (2)

(e) You are given $f(-3) = f(5) = g\left(\frac{1}{2}\right) = 0$,

a. What does this mean? (1)

b. Hence, or otherwise state the values of x for which $f(x).g(x) \geq 0$. (3)

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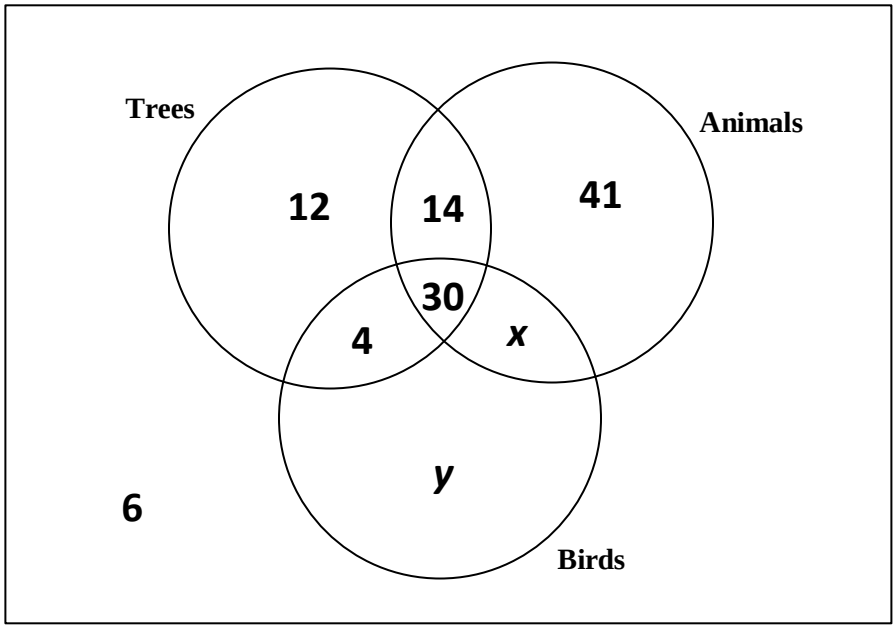
Question 9

A survey was conducted of 200 visitors to the Kruger Game Reserve, to ask them what interested them most in the game reserve, the Trees, the Animals or the Birds.
The following results were obtained:



(a) If 156 visitors said they were interested in Animals, what are the values of x and y? (4)

(b) How many visitors were NOT interested in Trees at all? (1)

(c) What is the probability that a visitor was only interested in one of the sights of Kruger? (2)

[7]

Question 10

At an Athletic event, athletes are tested for steroids using two different tests. The first test has a 93% probability of giving accurate results while the second test is accurate 87% of the time.

Using a tree diagram to help you, answer the following questions.

For a sample that does contain steroids, what is the probability that:

(a) Neither test shows that steroids are present? (3)

(b) Both tests show that steroids are present? (3)

(c) At least one of the tests detects the steroids? (3)

[9]

Question 11

The first term of a quadratic sequence is 4, the third term is 34 and the constant second difference is 10.

(a) Determine the 4^{th} term of the sequence. (5)

(b) Hence determine an expression for the n^{th} term of the quadratic sequence. (4)

[9]

MATHEMATICS INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\sum_{i=1}^{n} 1 = n \qquad \sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$T_n = a + (n-1)d \qquad S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$T_n = ar^{n-1} \qquad S_n = \frac{a(r^n - 1)}{r - 1};\ r \neq 1 \qquad S_\infty = \frac{a}{1 - r};\ -1 < r < 1$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$A = P(1 + ni) \qquad A = P(1 - ni)$$

$$A = P(1 + i)^n \qquad A = P(1 - i)^n$$

$$F = x\left[\frac{(1+i)^n - 1}{i}\right] \qquad P = x\left[\frac{1 - (1+i)^{-n}}{i}\right]$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad M\left(\frac{x_1 + x_2}{2};\ \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c \qquad y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad m = \tan\theta$$

In ΔABC:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc.\cos A$$

$$area\ \Delta ABC = \frac{1}{2}ab.\sin C$$

$$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta \qquad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$$

$$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta \qquad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases} \qquad \sin 2\alpha = 2\sin\alpha.\cos\alpha$$

$$\bar{x} = \frac{\sum fx}{n} \qquad \sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)} \qquad P(A\ or\ B) = P(A) + P(B) - P(A\ and\ B)$$