Form 4 Maths Paper 1 November 2016.

Section A

Question 1

(a) (i) $2 = bx$

$x = \frac{2}{b}$ ✓ ✓ (1)

(ii) $2(x^2-6) = 3(x-1)$

$2x^2 - 12 - 3x + 3 = 0$ ✓

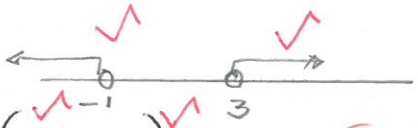
$2x^2 - 3x - 9 = 0$ ✓

$(2x+3)(x-3) = 0$ no need to show factors.

$x = -\frac{3}{2}$ or $x = 3$ ✓ ✓ (3)

(iii) $-3(x+1)(x-3) < 0$

$(x+1)(x-3) > 0$ ✓



$x \in (-\infty; -1)$ or $x \in (3; \infty)$ ✓ ✓ ✓ ✓ (4)

if whole $x > -1$ or $x > 3$ max 2/4.

(iv) $(x+5)^2 = (\sqrt{3-3x})^2$ √m of squaring.

$x^2 + 10x + 25 = 3 - 3x$ ✓a

$x^2 + 13x + 22 = 0$

$(x+11)(x+2) = 0$

$x = -11$ or $x = -2$ ✓ ca.

check: $x \neq -11$ ✓a

only solution $x = -2$ (5)

(v) $3^{1-x^2} = \frac{27^{-1}}{3}$

$3^{1-x^2} = \frac{3^{-3}}{3}$ ✓m

$3^{1-x^2} = 3^{-4}$ ✓m

$1 - x^2 = -4$ ✓m + ca.

$-x^2 = -5$

$x^2 = 5$ ✓ ca.

$x = \pm\sqrt{5}$ ✓ ca. P1 if do not put ± (4)

(b) $px^2+6x-p=0$.

(i) $$x = \frac{-6 \pm \sqrt{36+4p^2}}{2p}$$ ✓a.

$$= \frac{-6 \pm \sqrt{4(9+p^2)}}{2p}$$

$$= \frac{-6 \pm 2\sqrt{9+p^2}}{2p} = \frac{2(-3 \pm \sqrt{9+p^2})}{2p}$$

$$= \frac{-3 \pm \sqrt{9+p^2}}{p}$$ ✓ ca.

but must be simplest form which involves working with surds!

(4)

(ii) Real roots if $9+p^2 > 0$ ✓m.

$9+p^2$ is always ≥ 0

thus $p \in R$. ✓ Where $p \neq 0$ ✓.

(2)

(c) $2x+6-y=0$

$y = 2x+6$ (i)

$y+3x^2 = 8x+3$

$y = -3x^2+8x+3$ (2)

(1) = (2) $2x+6 = -3x^2+8x+3$ ✓m.

$3x^2-6x+3=0$.

$x^2-2x+1=0$ ✓a

$x=1$ ✓ ca.

Sub into $y=2x+6$

$y=8$ ✓ca.

(5)

(d) $4^x = 8^{2-x} \implies 2^{2x} = 2^{6-3x}$

$2x = 6-3x$ ✓ca

$5x = 6$

$x = \frac{6}{5}$ ✓ ca.

(3)

(a) $x = \dfrac{-20 \pm \sqrt{400 - 48h}}{24}$

(i) Sub $h = 1$ ✓m

$x = \dfrac{-20 \pm \sqrt{400 - 48}}{24}$ ✓a $= \dfrac{-5 \pm \sqrt{22}}{6}$ ✓ca (2)

(ii) real roots if $400 - 48h \geq 0$ ✓m

$400 \geq 48h$

$\dfrac{400}{48} \geq h$

$h \leq \dfrac{25}{3}$ ✓ca (2)

(iii) Roots rational if

$400 - 48h$ = perf square ✓m

$(h \leq \frac{25}{3})$

$\Rightarrow h = 8$ ✓ca (2)

(iv) 1) $x = \dfrac{-20 \pm \sqrt{400}}{24}$

$= \dfrac{-20 + 20}{24}$ ✓ $h = 0$ ✓

$= 0$ ✓ (2)

2) $x = \dfrac{-20 - 20}{24} = \dfrac{-40}{24} = \dfrac{-5}{3}$ ✓ (2)

[41]

Question 2.

Block 1	Block 2	Block 3	Block 4
1	1	1	1
2	7	14	23 ✓
d_1	5	7	9 ✓ ✓
d_2		2	2

$2a = 2$ ✓
$a = 1$ ✓

$3a + b = 2$
$3 + b = 2$
$b = -1$ ✓

$a + b + c = 2$
$1 - 1 + c = 2$
$c = 2$ ✓

$T_n = n^2 - n + 2$ ✓ ✓
$T_{50} = 50^2 - 50 + 2 = 2452$ ✓
no of squares in $T_{50} = 2453$ ✓

(5)

[5]

3	8	15	24 ✓
d_1	5	7	9 ✓
d_2		2 ✓	

$2a = 2$
$a = 1$ ✓

$3 + b = 5$
$b = 2$ ✓

$a + b + c = 3$
$c = 0$ ✓

$T_n = n^2 + 2n$ ✓ ✓
$T_{50} = 50^2 + 2(50)$
$= 2500 + 100$ ✓
$= 2600$ ✓

(5)

PI for not 2dp.

Question 3

(a) $A = P(1+i)^n$ ✓ ✓

$= 12\,000\left(1 + \frac{13\%}{12}\right)^{5\times 12}$ ✓ ✓

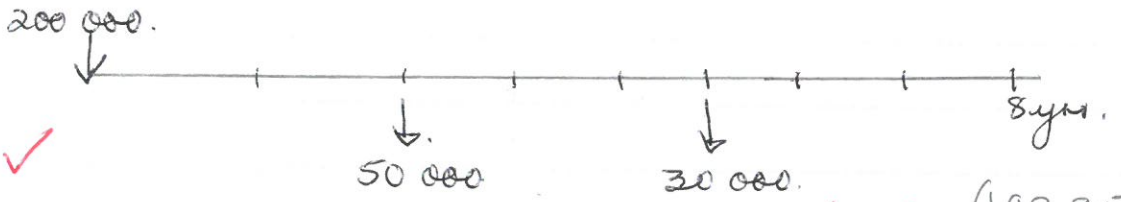
$= \underline{R\ 22\,906,28}$ ✓ (3)

(b) $1 + i_{eff} = \left(1 + \frac{i_{nom}}{m}\right)^m$ ✓m

$i_{eff} = \left(1 + \frac{12\%}{12}\right)^{12} - 1$ ✓a

$i_{eff} = \underline{12,68\%}$ ✓ ca. (3)

(c)



$i = \frac{12\%}{12}$ ✓

final $A = 200\,000\left(1 + \frac{12\%}{12}\right)^{12\times 8}$ ✓a − $50\,000\left(1 + \frac{12\%}{12}\right)^{6\times 12}$ ✓a − $30\,000\left(1 + \frac{12\%}{12}\right)^{3\times 12}$ ✓a.

Sto A (519854,5851) Sto B (102 354,9656) Sto C. 42 923,0635

final amt = $A - B - C$ ✓m.

$= \underline{R.374\,576,56}$ ✓ca. (6)

(d) $A = P(1-i)^n$ ✓

$12\,000 = P(1 - 12\%)^5$ ✓

$P = \frac{12\,000}{(1-12\%)^5}$ ✓

$= \underline{R22738,82}$ ✓ (3)

Question 6.

(a) $4^9 . 5^{12} = 2^{18} . 5^{12}$ Prime factors ✓.

$= (2^{12} . 5^{12}) . 2^6$ ✓.

$= (10)^{12} . 64$ ✓.

Total of digits

$= 10$ ✓

(3)

(b) $x^2 - 2x$.

(i) minimum value ✓

$x^2 - 2x + 1 - 1$

$= (x-1)^2 - 1$ ✓ Min $= -1$ ✓

(2)

(ii) min 2^{x^2-2x}

$= 2^{-1}$ ✓

$= \frac{1}{2}$ ✓

(1)

(c) $\{1; 2; 3\}$ $x^2 + dx + e = 0$.

Real roots

$\Rightarrow d^2 - 4e > 0$ ✓ ✓.

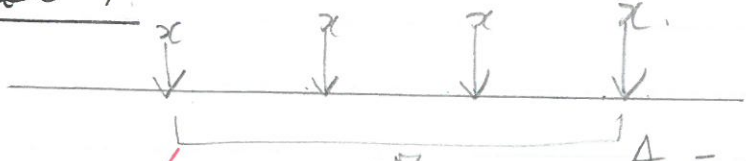
$d \neq 1$ ✓	$d = 2$	$d = 3$ ✓	$d = 3$ ✓
	$e = 1$ ✓	$e = 1$ ✓	or $e = 2$

3 equations ✓.

(4)

[10]

Question 7



$A = 1\,200\,000$.

$$1\,200\,000 = x(1+10\%)^3 + x(1+10\%)^2 + x(1+10\%)^1$$

$$1\,200\,000 = x\left[(1+10\%)^3 + (1+10\%)^2 + (1+10\%)^1 + 1\right] + x.$$

$$\frac{1\,200\,000}{[\quad]} = x$$

$$x = R258\,564,96$$

(6) [6]

Question 8

$f(x) = -\frac{1}{2}x^2 + x + c$ $\quad g(x) = \frac{-2}{x} + q$. $\quad h(x) = q$.

(a) coords of Q $(-1; 6)$ ✓✓ (1)

(b) Sub $(-1; 6)$ into $f(x) = -\frac{1}{2}x^2 + x + c$ ✓m

$6 = -\frac{1}{2} - 1 + c$ ✓a

$7\frac{1}{2} = c$

Sub $(-1; 6)$ into $g(x) = \frac{-2}{x} + q$ ✓m

$6 = 2 + q$ ✓a

$4 = q$ (4)

(c) $f(x) = -\frac{1}{2}x^2 + x + 7\frac{1}{2}$

Ad S $x = -\frac{b}{2a} = \frac{1}{-1} = 1$ ✓m ✓a.

Sub into original $y = -\frac{1}{2} + 1 + 7\frac{1}{2}$ ✓cao ✓cao

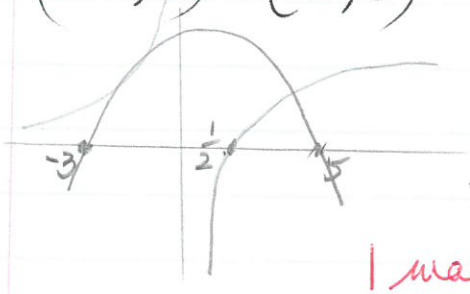
$= 8$.

Range $y \in (-\infty; 8]$ ✓✓ ✓ca. (5)

(d) $y = -x + 4$ ✓✓ (2)

(e) $f(-3) = f(5) = g\left(\frac{1}{2}\right) = 0$.

$(-3; 0)$ $(5; 0)$ $\left(\frac{1}{2}; 0\right)$ ✓ x intercepts (1)



$f(x)g(x) \geq 0$

$x \in [-3; 0]$ or $x \in [\frac{1}{2}; 5]$ ✓✓ ✓

1 mark for all square brackets (3)

[16]

Question 9

(a) $n(\text{animals}) = 156$ m

$14 + 20 + 41 + x = 156$ ✓a

$x = 71$ ✓a

$n(\text{all}) = 200$. ✓ca

$60 + 41 + 71 + 6 + y = 200$ ✓a

$y = 22$ ✓ca (4)

(b) $n(\text{not Trees}) = n(T') = 41 + 71 + 22 + 6$

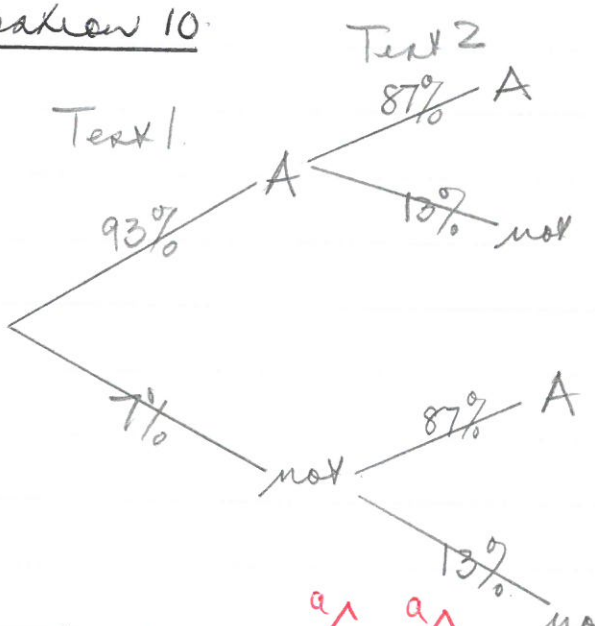
$= 140$. ✓ (1)

(c) $P(\text{only 1}) = \frac{12}{200}$ ✓ $+ \frac{41}{200}$ ✓ $+ \frac{22}{200}$ ✓

$= \frac{75}{200} = \frac{3}{8}$ ✓ (3)

[7]

Question 10



AA = 93% × 87% ①

AN = 93% × 13% ②

NA = 7% . 87% ③

NN = 7% × 13%

(a) $P(NN) = 7\% \times 13\% = \frac{91}{10000}$ ✓a ✓a ✓m ✓ca (3)

(b) $P(AA) = 93\% \times 87\% = \frac{8091}{10000}$ ✓a ✓m ✓a ✓ca (3)

(c) $P(\text{at least 1}) = 1 - P(NN) = 1 - \frac{91}{10000}$ ✓m

$= \frac{9909}{10000}$ ✓a (3)

or add ① + ② + ③.

[9]

Question 11

(a) $4 \quad x \quad 34$

$x-4 \quad 34-x$

$34-x-x+4 = 10$ ✓

$-2x = -28$

$x = 14$ ✓

$4 \; ; \; 14 \; ; \; 34 \; ; \; 64$ ✓

$10 \quad 20 \quad 30$

$10 \quad 10$

$T_4 = 64$ ✓ (5)

(b) $2a = 10$ $\quad 3a+b = 10$ $\quad a+b+c = 4$

$a = 5$ ✓ $\quad b = -5$ ✓ $\quad 5-5+c = 4$

$c = 4$ ✓

$T_n = 5n^2 - 5n + 4$ ✓ (4)

[9]

Question 4

Given: $f(x) = \frac{3}{x-2} - 1$ and $g(x) = x^2 - 4x$

(a) Write down the **equations** of the asymptotes of f. (2)

$x = 2$ ✓ $\quad y = -1$ ✓ (2)

(b) Determine the x and y intercepts of f. (3)

✓a

X int $\quad 0 = \frac{3}{x-2} - 1$ $\quad$ Y int $(0; -\frac{5}{2})$

$1 = \frac{3}{x-2}$ ✓m $\quad$ no need for coordinates.

$x - 2 = 3$

$x = 5$ ✓a $\quad (5;0)$ (3)

(c) Write down the coordinates of the turning point of g. (3)

AoS $x = \frac{-b}{2a}$ $\quad$ Sub $x = 2$ into $g(x)$

$= \frac{4}{2}$ $\quad g(2) = 4 - 8$

$= -4$ ✓ca

$x = 2$ ✓ma $\quad$ TP $(2; -4)$ ✓ (3)

(d) Determine the x and y intercepts of g (2)

X int $\quad 0 = x^2 - 4x$ $\quad$ Y int

$0 = x(x-4)$ $\quad (0;0)$ ✓

$x = 0$ or $x = 4$

X int $(0;0)$ ✓ $(4;0)$ ✓ (2)

(e) Draw neat sketch graphs of f and g on the same system of axes on the set of axes.

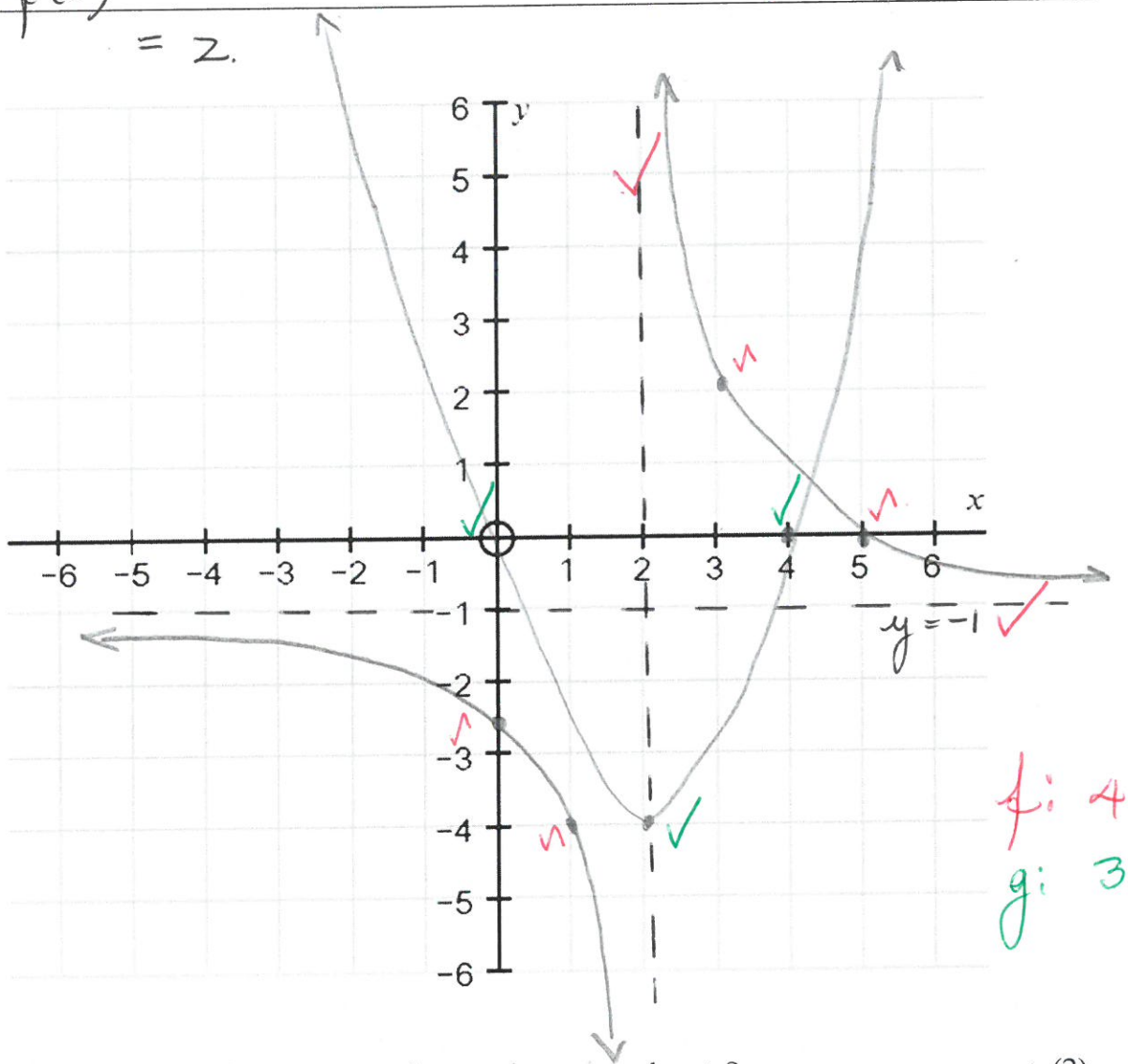
Clearly indicate asymptotes, turning points and intercepts with the axes, as well as least 2 additional points on each arm of $f(x)$. (7)

$f(x) = \frac{3}{x-2} - 1$

$f(1) = -3 - 1 = -4$

$f(3) = 3 - 1$

$= 2.$



f: 4 marks.

g: 3 marks.

(f) What transformation must g undergo to have no real roots? (2)

$g(x) = x^2 - 4x$

Vertical shift bigger than 4. (2)

[19]

(a) let $P = x$

$A = x + 8624,14$

$x + 8624,14 = x\left(1 + \frac{10,5\%}{4}\right)^{6\times 4}$ ✓m ✓m

$8624,14 = x\left(1 + \frac{10,5\%}{4}\right)^{24} - x$ ✓a.

$8624,14 = x\left(\left(1 + \frac{10,5\%}{4}\right)^{24} - 1\right)$ ✓ca

$x = \frac{8624,14}{(\quad)}$ ✓ca

$= R10\,000,00$ (5)

[20]

Question 4 (answered on booklet back cover) [19]

Question 5

(a) $f(x) = 3^{-x} + p$

Sub. $(-2; 10)$ ✓m

$10 = 3^{2} + p$ ✓a

$1 = p$ ✓ca (2)

(b) Y-val of $f(x)$

$3^{-0} + 1 = 2$. $A(0; 2)$ ✓a.

Y-val of $g(x)$ is -5 $B(0; -5)$ ✓a

$\therefore AB = 7$ ✓ca (3)

(c) $g(x) = 3x - 5$

Reflect about X-axis: y becomes negative.

2 units right → horizontal shift

$-y = 3(x-2) - 5$ ✓ ✓ ✓

$-y = 3x - 11$

$y = -3x + 11$ ⟹ $h(x) = -3x + 11$ ✓ ✓

(3) [8]