Mathematics Paper 1 November 2017

FORM 4

Examiner	A Gunning	**Moderator**	C Mundy
Time	3 hours	**Marks**	150

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY BEFORE ANSWERING THE QUESTIONS.

- This question paper consists of 9 pages. Please check that your question paper is complete.
- You have also been given an information sheet
- Read and answer all questions carefully.
- Number your answers exactly as the questions are numbered.
- It is in your own interest to write legibly and to present your work neatly.
- **All necessary working, which you have used in determining your answers, must be clearly shown.**
- Approved non-programmable calculators may be used except where otherwise stated. Where necessary give answers correct to 2 decimal places.
- Diagrams have not necessarily been drawn to scale.

SECTION A

Question 1

(a) Solve each of the following. Remember to show all relevant working which you have used to get your answers.

(i) $x = -3 + \frac{4}{x}$ (4)

(ii) $x(x-2) = 2$, giving your answers in the simplest surd form. (4)

(iii) $3\sqrt{x-1} - x = 1$ (4)

(b) Consider $(x-1)(x+2) \leq 4$.

(i) Solve for x giving your answer in interval notation. (4)

(ii) The sum of all of the x values within the interval calculated in (b)(i) is k. Given that $x \in \mathbb{Z}$, find the value of k. (1)

(c) Consider $3x^2 + 2x - m = 0$

(i) Solve for x giving your answer in terms of m and in the simplest surd form. (4)

(ii) Hence or otherwise give at least one integral value of m for which the roots are real and rational. (3)

(d) Solve the following equations simultaneously

(i) $y = 2x^2 + 3x - 1$ and $2y + 4x - 4 = 0$ (5)

(ii) $81^x = 3.27^y$ and $2x + 3y = 5$ (5)

(e) Show by completing the square that $2m^2 - 4m + 5$ has a minimum value of 3. (3)

(f) Write down a suggested quadratic equation in the form $ax^2 + bx + c = 0$ for each of the following

(i) The roots are 0 and -1. (2)

(ii) The roots of the equation are non-real and $c > 0$ (3)

(iii) One root is 3 and $b^2 = 4ac$ with $a > 0$. (3)

[45]

Question 2

(a) Simplify the following expressions fully, without the use of a calculator, and showing all relevant working detail.

(i) $8^{x+1}.16^{-x}.2^{x}$. Leave your answer in exponential form. (2)

(ii) $\dfrac{4^{x-1}+4^{x+1}}{17.12^{x}}$ (4)

(iii) $\sqrt{\left(t\sqrt{27}-\sqrt{2t^{2}}\right)}\sqrt{\left(t\sqrt{27}+\sqrt{2t^{2}}\right)}$ (3)

(b) Solve for x: $2.8^{x-3}=32^{-x}$ (3)

[12]

Question 3

(a) The n^{th} term of a quadratic sequence is $T_n=2n^2-n+1$.

(i) Write down the first 3 terms of the sequence. (3)

(ii) Is 3161 a term in this sequence? Show all relevant working detail to justify the answer you give. (3)

(b) You are given the following quadratic sequence.

$$0;\quad 5;\quad p;\quad 18;\quad q$$

Find the values of p and q. (5)

[11]

SECTION B

Question 4 (Answers to all these questions must be given correct to 2 decimal places.)

(a) Find the annual rate of depreciation if a car which cost R120 000 on 1st January 2015 was sold for R74 500 at the end of June 2017. (3)

(b) What is the effective annual interest rate which equates to an interest rate of 13,9% p.a. compounded quarterly? (3)

(c) Mrs G wants to go to England in 5 years' time. The current price of a return airfare to London, Heathrow is R11 900. Inflation is expected to remain at a constant annual rate of 4,8% p.a. for the next 5 years.

(i) What should Mrs G plan on spending on her ticket in 4,5 years' time? (2)

(ii) She sets up a savings account on 1 November 2017 and deposits R12 500 immediately. The bank offers her 8,97% p.a. compounded quarterly but after 2 years, the interest rate drops to 6,42% p.a. compounded monthly. Regrettably she has medical bills to pay after three years and has to withdraw R 2 200. Will she have sufficient money in the account at the end of June 2022 to be able to pay cash for the ticket at that time? Show all working details to justify the answer you give. (6)

(d) What initial investment was made if after 7,5 years, Henry was able to withdraw R20 750 from his account? The bank offered interest to be calculated at 12% p.a. compounded half yearly for the first 5 years but decreased it to 9,7% p.a. to be compounded monthly for the remaining years. (4)

[18]

Question 5

The graphs of $f(x) = \frac{2}{4-x} + 1$ and $g(x) = 4 - x$ are shown in the diagram below.

A

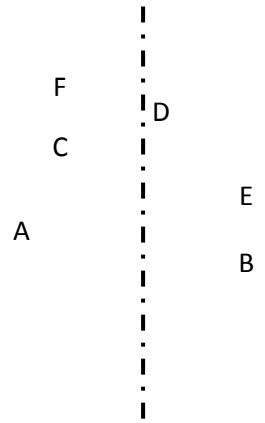
B

Showing all relevant working detail, find the length of AB, where A and B are the points of intersection of the 2 graphs. Give your answer in the simplest surd form. (7)

[7]

Question 6

The graphs of $f(x)$ and $g(x)$ are drawn below. $f(x) = -x^2 + 3x + 4$ and $g(x) = \frac{a}{x-p} + q$



The x and y intercepts of f are A, B, and C and it turns at D.

$g(x)$ passes through the point E(4; 2) and has asymptotes as indicated on the diagram.

(a) Determine the length AB. (3)

(b) Write down the equations of the asymptotes of $g(x)$. (2)

(c) And hence, or otherwise, find the equation of $g(x)$. (3)

(d) Write down the equation of the axis of symmetry of $g(x)$ which has a positive gradient. (2)

(e) Find the length FC, where F is the y intercept of g. (2)

(f) Write down <u>the x coordinate</u> (only) of the turning point of the function $f(x-2)$ describing briefly how you arrived at this answer. (2)

(g) Describe the transformation of $g(x)$ to become $g(-x) - 1$. Hence state the domain of this new function. (3)

[17]

Question 7

(This question is to be answered in FULL on the inside front cover of your answer booklet.)

(a) On the set of axes given in your answer booklet,

sketch the graph of $h(x) = -3.3^x + 1$ showing clearly the x and y intercepts (if any)

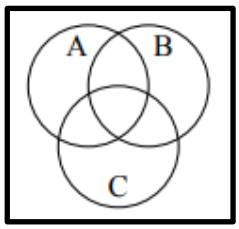
and any relevant asymptotes. (5)

(b) Write down the range of h. (2)

[7]

Question 8

(a) Consider the following Venn Diagram.



<u>In the answer booklet</u>, shade the relevant region for each of the following:

(i) $(A \cup B) \cap C$ (2)

(ii) $(A \text{ and } C \text{ and } B)'$ (2)

(b) A Class of 40 students completed a survey on what pets they like. The choices were:

Cats, Dogs, and Birds. Everyone liked at least one pet.

10 students liked Cats and Birds but not dogs

2 students liked Dogs and Birds but not Cats

12 students liked Cats and Birds

8 students liked Cats and Dogs

All together 28 students liked Cats, 19 liked Dogs and 15 liked Birds.

(i) Represent this information on a Venn Diagram. (4)

(ii) What is the probability that a person chosen at random will have all three pets? (Answers must be simplified where possible.) (2)

(iii) Write down the probability that a person chosen at random will have 2 or more pets. (2)

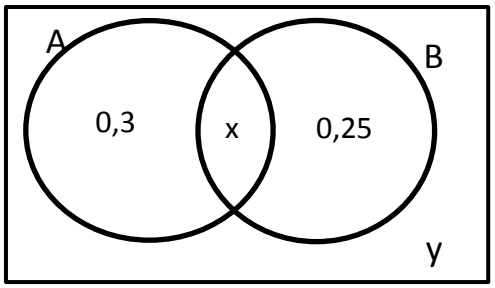
(c) The probability that Samantha receives a phone-call every Wednesday from her friend in Johannesburg is $0{,}35$ and the probability that her brother phones from UCT that same night is $0{,}43$. What is the probability that:

(i) Both will phone her on any Wednesday night. (2)

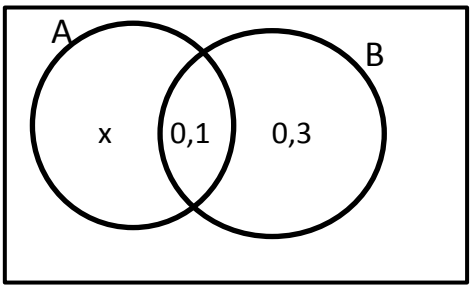
(ii) She will receive a call from **at least** one of them on a Wednesday night. (3)

(d) Consider the Venn Diagrams given below.

(i) Find the values of x and y given that A and B are mutually exclusive. (2)



(i) Find the value of x given that A and B are independent. (4)



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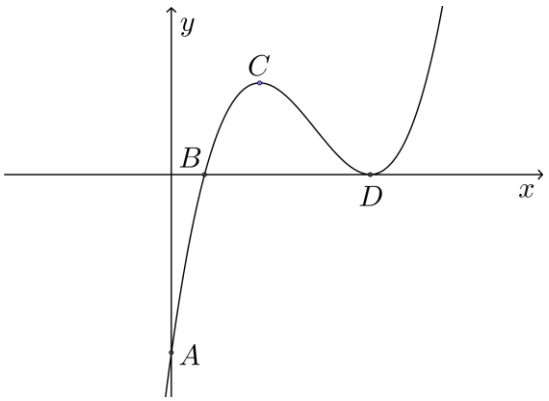
Question 9

Consider the cubic function shown in the sketch below.

$f(x) = 2x^3 - 13x^2 + 24x - 9$

A, B and D are intercepts of f with the axes. C is a local maximum.



(a) Write down the co-ordinates of A. (1)

(b) Given the equation $f(x) = 2x^3 - 13x^2 + 24x - 9$,

a. How many x-intercepts would you expect? (1)

b. Is this evident on the graph? Explain. (1)

(c) If $g(x)$ is the image of f translated three units to the left, and 1 unit vertically up, give the equation defining g. (You do not need to multiply out the brackets.) (2)

[5]

Question 10

A bridge is built in the shape of a parabolic arch. The bridge arch has a span of 196 metres and a maximum height of 30 metres.

Find the height of the arch at 15 metres from its centre. (Correct to 1 decimal place) (5)

[5]