Section A

Question 1

(a) (i) $x = -3 + \frac{4}{x}$ $\quad x \neq 0$

$x^2 = -3x + 4$

$x^2 + 3x - 4 = 0$ ✓✓

$(x+4)(x-1) = 0$

$x = -4$ ✓ or $x = 1$ ✓ (4)

(ii) $x(x-2) = 2$.

$x^2 - 2x - 2 = 0$ ✓

$x = \frac{2 \pm \sqrt{4+8}}{2}$ ✓ or complete the sq. $= \frac{2 \pm \sqrt{12}}{2}$ ✓

$= \frac{2 \pm 2\sqrt{3}}{2} = 1 \pm \sqrt{3}$ ✓

only 2 out of 4 if used calc. (4)

(iii) $3\sqrt{x-1} - x = 1$

$3\sqrt{x-1} = x + 1$

$9(x-1) = x^2 + 2x + 1$ ✓ squaring.

$x^2 - 7x + 10 = 0$ ✓

$(x-5)(x-2) = 0$

$x = 5$ ✓ or 2 ✓

no marks awarded for checking. (4)

check: Sub $x = 5$ $\quad 3\sqrt{5-1} - 5 = 1$

$x = 2$ $\quad 3\sqrt{1} - 2 = 1$

both valid (4)

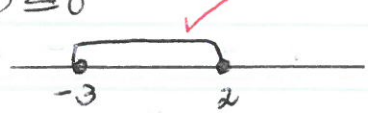
(b) (i) $(x-1)(x+2) \leq 4$

$x^2 + x - 2 - 4 \leq 0$

$x^2 + x - 6 \leq 0$ ✓

$(x+3)(x-2) \leq 0$

CV $x = -3$ or 2 ✓



$x \in [-3; 2]$ ✓ (4)

(ii) $-3 + -2 + -1 + 0 + 1 + 2 = k$ ✓

$k = -3$ ✓ (2)

(c) $3x^2 + 2x - m = 0$.

(i) $x = \dfrac{-2 \pm \sqrt{4 + 12m}}{6}$ ✓✓ or complete the square method

$= \dfrac{-2 \pm \sqrt{4(1 + 3m)}}{6}$ ✓

$= \dfrac{-2 \pm 2\sqrt{1 + 3m}}{6} = \dfrac{-1 \pm \sqrt{1 + 3m}}{3}$ ✓ (4)

(ii) Roots real & rational
$\Rightarrow$ $1 + 3m$ is a perfect ✓ square.
Sub $m = 1$ ✓ $\therefore \Delta = 4$. (2)

(d) (i) $y = 2x^2 + 3x - 1$ $\quad 2y + 4x - 4 = 0$
$-2x + 2 = 2x^2 + 3x - 1$ $\quad y = -2x + 2$ ✓
$2x^2 + 5x - 3 = 0$ ✓
$(2x - 1)(x + 3) = 0$.
$x = \frac{1}{2}$ ✓ or $x = -3$ ✓
$y = -2(\frac{1}{2}) + 2$ or $y = -2(-3) + 2$
$= 1$ ✓ $= 8$ (6)

(ii) $81^x = 3.27^y$
$3^{4x} = 3.3^{3y}$ ✓ ✓
$4x = 3y + 1$ ✓ ① and $2x + 3y = 5$ $\times 2$
$4x = -6y + 10$ ✓ ② $\Rightarrow$ $4x = -6y + 10$.
$0 = 9y - 9$ ① − ②
$y = 1$ ✓
$4x = 4$ ✓
$x = 1$. $y = 1$ (5)

(e) $2m^2 - 4m + 5$
$= 2\{m^2 - 2m + \frac{5}{2}\}$ ✓
$= 2\{(m - 1)^2 - 1 + \frac{5}{2}\}$ ✓
$= 2(m - 1)^2 + \frac{3}{2}$ ✓
Max value is 3. (3)

(f) (i) $x = 0$ or $x = -1$ $\quad x(x+1) = 0$

$x^2 + x = 0$. (2)

(ii) roots not real and $c > 0$.

$y = (x-1)^2 + 2$. $\quad b^2 - 4ac < 0$.

$= x^2 - 2x + 3 = 0$. (3)

or anything that holds true!

(iii) $x = 3$ $\quad b^2 - 4ac = 0$

$\Rightarrow$ roots equal.

$y \quad (x-3)^2 = 0 \quad a > 0$.

$x^2 - 6x + 9 = 0$. (3)

[45]

Question 2

(a) (i) $8^{x+1} \cdot 16^{-x} \cdot 2^x$

$= 2^{3x+3} \cdot 2^{-4x} \cdot 2^x$

$= 2^{3x+3-4x+x} = 2^3$ (2)

(iii) $\sqrt{(t\sqrt{27} - \sqrt{12t^2})(\sqrt{t\sqrt{27}} + \sqrt{12t^2})}$

$= \sqrt{t^2 27 - 2t^2}$ diff of 2 squares

$= \sqrt{25t^2} = 5t$ (3)

(ii) $\dfrac{2^{2x-2} + 2^{2x+2}}{17 \cdot 2^{2x} \cdot 3^x} = \dfrac{2^{2x}(2^{-2} + 2^2)}{17 \cdot 2^{2x} \cdot 3^x}$

$= \left(\frac{1}{4} + 4\right) \div 17 \cdot 3^x$

$= \frac{17}{4} \times \frac{1}{17 \cdot 3^x} = \dfrac{1}{4 \cdot 3^x}$ (4)

(b) $2 \cdot 8^{x-3} = 32^{-x}$

$2 \cdot 2^{3x-9} = 2^{-5x}$

$3x - 8 = -5x$

$8x = 8$

$x = 1$.

if $8^{x-3} = 16^{-x}$ awarded zero.

[12] (3)

Question 3.

(a) $T_n = 2n^2 - n + 1$.

(i) $T_1 = 2 - 1 + 1 = 2$ ✓
$T_2 = 2.2^2 - 2 + 1 = 7$.
$T_3 = 2.9 - 3 + 1 = 16$.
Sub. n ✓
2; 7; 16 ... ✓ (3)

(ii) $2n^2 - n + 1 = 3161$ ✓
$2n^2 - n - 3160 = 0$ ✓
$n = 40$ ✓ or $-\frac{79}{2}$ ✗
Yes it is (the 40th term) ✓
(Must show working as above to get first 2 marks) (3)

(b) 0; 5; p; 18; q.

d_1	5	$p-5$	$18-p$	$q-18$ ✓
d_2	$p-10$ ✓	$18-p-p+5$ ✓		$q-18-18+p$ ✓
	$p-10 = 23-2p$ ✓			$q+p-36$ ✓
	$3p = 33$ ✓			$11-10 = q+11-36$
	$p = 11$. ✓			$11-10+36 = q$
				$26 = q$ ✓ (5)

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Section B.

Question 4

(a) $74500 = 120000(1-i)^{2\frac{1}{2}}$ ✓

$\sqrt[2\frac{1}{2}]{\frac{745}{1200}} - 1 = -i$ ✓

$-0,1736 = -i$

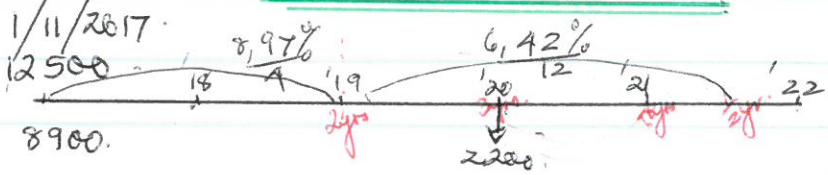
$i = 17,36\%$ ✓ (3)

(b) $1 + i_{eff} = \left(1 + \frac{13,9\%}{4}\right)^4$ ✓

$i_{eff} = \left(1 + \frac{13,9\%}{4}\right)^4 - 1$ ✓

$i_{eff} = 14,64\%$ ✓ (3)

(c)



(i) ticket cost in $4\frac{1}{2}$ years $= 11900\left(1 + \frac{4,8\%}{4}\right)^{4,5}$ ✓ ✓

$= 14695,11$ ✓ (2)

14695,11

14926,63

(ii) $A = 12500\left(1 + \frac{8,97\%}{4}\right)^{8}\left(1 + \frac{6,42\%}{12}\right)^{22} = 17705,77$ ✓ ✓ ✓

17705,77 — Sho A

$B = 2200\left(1 + \frac{6,42\%}{12}\right)^{20} = 2447,76$ ✓ Sho B

2200

15505,77

final amt $= A - B$ ✓ $= 15258,01$ ✓

Yes ✓ she will have enough money.

(6)

(d) $20750 = P\left(1 + \frac{12\%}{2}\right)^{10}\left(1 + \frac{9,7\%}{12}\right)^{30}$ ✓ ✓ ✓ ✓ m

$P = 9100,52$ ✓ (4)

[18]

OR

4.4 $12500\left(1+\frac{8,97\%}{4}\right)^{8}\left(1+\frac{6,42\%}{12}\right)^{12}$

$= 14926,63\left(\quad\right)$

$15913,62$

$-\ 2200$

$= 13713,62\left(1+\frac{6,42\%}{12}\right)^{20}$

$= 15258,01$

(6)

Question 5

$f(x) = \frac{2}{4-x} + 1 \qquad g(x) = 4 - x$

$\frac{2}{4-x} + 1 = 4 - x$ ✓

$\frac{2}{4-x} = 3 - x$

$2 = (4-x)(3-x)$ ✓

$2 = 12 - 7x + x^2$

$x^2 - 7x + 10 = 0$ ✓

$(x-5)(x-2) = 0$

$x = 2$ or 5 ✓

$y = 4 - 2$ or $y = 4 - 5$

$= 2$ ✓ $\qquad = -1$ ✓

$(2; 2) \qquad (5; -1)$ (5)

$\therefore AB = \sqrt{(5-2)^2 + (-1-2)^2}$ ✓

$= \sqrt{9 + 9}$ ✓

$= 3\sqrt{2}$ (2)

[7]

Question 6.

(a) $f(x) = -x^2 + 3x + 4$

x intercepts $\quad x^2 - 3x - 4 = 0$ ✓

$(x-4)(x+1) = 0$

$x = 4$ or -1 ✓

$\therefore AB = 5$ ✓ (3)

(b) Horizontal asymptote $\quad y = 4$ ✓ (Y int of parabola)

Axis of symmetry $\quad x = \frac{-b}{2a} = \frac{-3}{-2} = \frac{3}{2}$ ✓

Vertical asymptote $\quad x = \frac{3}{2}$ (2)

(c) $g(x) = \frac{a}{x - \frac{3}{2}} + 4$ ✓ ✓

Sub. $(4; 2) \quad 2 = \frac{a}{4 - \frac{3}{2}} + 4$ ✓

$-2\left(\frac{5}{2}\right) = a$

$a = -5$ ✓ (3)

$g(x) = \frac{-5}{x - \frac{3}{2}} + 4$

(d) $y = \left(x - \frac{3}{2}\right) + 4$

$y = x + \frac{5}{2}$ (2)

(e) y int of $y = \frac{-5}{x - \frac{3}{2}} + 4$.

$y = -5 \div -\frac{3}{2} + 4$

$= \frac{10}{3} + 4 = 7\frac{1}{3}$.

$\therefore$ FC $= 7\frac{1}{3} - 4 = 3\frac{1}{3}$. (2)

(f) x value TP of $f(x)$ is $x = \frac{3}{2}$

$\therefore$ x value of $f(x-2)$ is $x = 3\frac{1}{2} = \frac{7}{2}$ (2)

(g) $g(-x) - 1$

$g(x)$ reflected about the Y axis and shifted down by 1. (vertical shift)

Domain $x \in R$ but $x \neq -\frac{3}{2}$ (3)

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Question 7

(This question is to be answered in FULL on the inside back cover of your answer booklet.)

(a) On the set of axes given in your answer booklet,

sketch the graph of $h(x) = -3.3^{-x} + 1$ showing clearly the x and y intercepts (if any)

and how you calculated them, and any relevant asymptotes. (5)

x int. $0 = -3.3^{+x} + 1$

$-1 = -3.3^{+x}$

$\frac{1}{3} = 3^{+x}$

$3^{-1} = 3^{+x}$

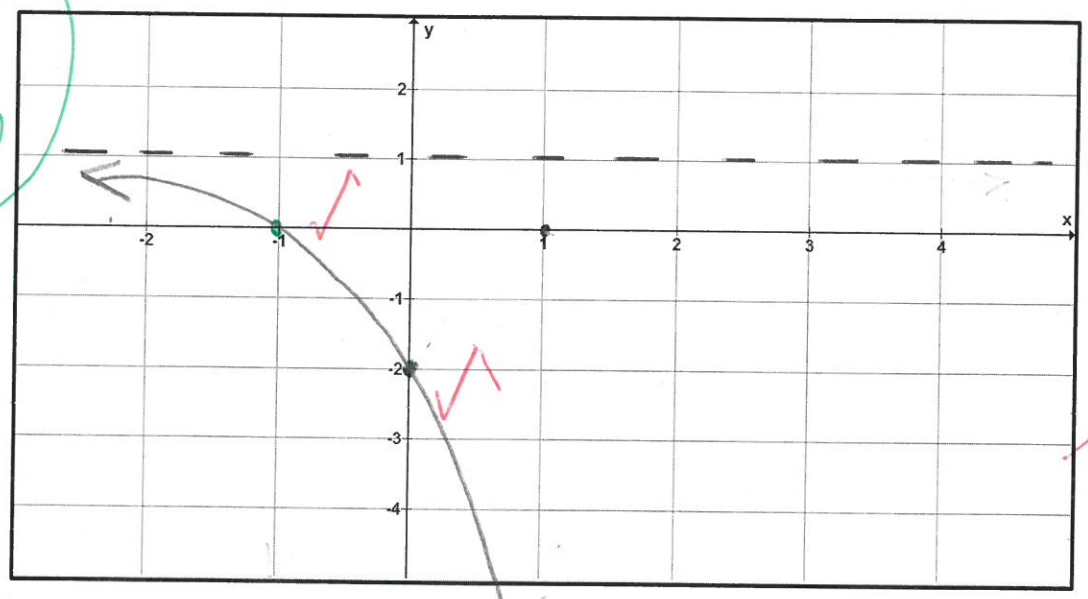
$x = -1.$

y int. $y = -3.3^{0} + 1$

$= -2$

asymptote $y = 1$

changed self axes



$y = 1$

shape

(b) Write down the range of h. (2)

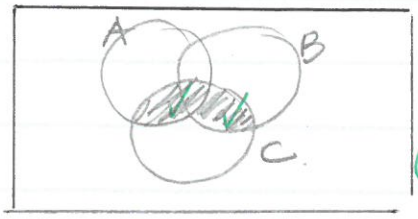
$y \in (-\infty; 1)$

(1; -8)

[23]

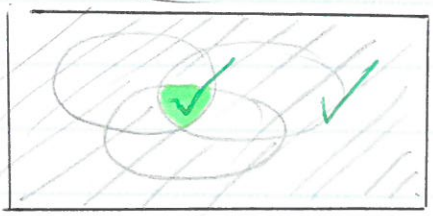
Question 8.

(a) (i) $(A \text{ or } B) \text{ and } C$



(2)

(ii) $(A \text{ and } C \text{ and } B)'$

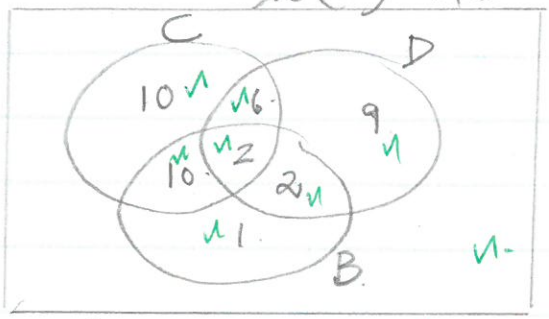


(2)

everything except $A \cap B \cap C$. (green highlight)

(b) (i)

$n(S) = 40$.



$n(C) = 28$
$n(D) = 19$
$n(B) = 15$

(4)

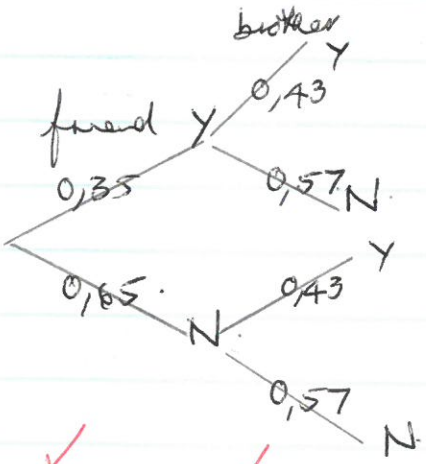
(ii) $P(\text{all 3}) = \frac{2}{40} = \frac{1}{20}$

(2)

(iii) $P(\text{2 or more}) = \frac{6+2+10+2}{40} = \frac{1}{2}$

(2)

(c)



YY (i) $P(\text{both}) = 0,35 \times 0,43$
$= \frac{301}{2000} = 0,15$

(2)

YN

NY (ii) $P(\text{at least 1}) = 1 - P(\text{none})$
$= 1 - 0,65 \times 0,57$
$= 1 - \frac{741}{2000}$
$= \frac{1259}{2000} = 0,63$

NN

OR $0,35 + 0,43 + 0,15 = 0,93$ (3)

OR 0,78 gets 1 mark.

(d) (i) $x = 0$ A and B mutually exclusive (3)

$y = 0,45$ $\therefore P(A \cap B) = 0$

(ii) $P(A \text{ and } B) = P(A) \cdot P(B)$ look for formula. (2)

$0,1 = (x + 0,1)(0,4)$ look for $x + 0,1$ $0,4$ $\Rightarrow x = \frac{1}{4} - 0,1$

$x = \frac{3}{20} = 0,15$ (4)

Question 9.

(a) $f(x) = 2x^3 - 13x^2 + 24x - 9$

$A(0; -9)$ ✓ must be coordinates (1)

(b) (i) 3 x intercepts (x^3) ✓ (1)

(ii) not evident as only 2 x intercepts but at D, 2 equal roots ✓ (1)

$\therefore$ 3

(c) $g(x) = 2(x+3)^3 - 13(x+3)^2 + 24(x+3) - 8$ ✓ ✓ ✓ ✓

$g(x+3)+1$ gets ½ mark.

[5] (2)

Question 10

Correct sketch gets 1 mark. (2)

TP $(0; 30)$ ✓ or x int $(-98; 0)$ ✓ $(98; 0)$

$y = ax^2 + 30$ ✓ OR $y = a(x+98)(x-98)$ ✓

sub. $(98; 0)$ ✓ sub. $(0; 30)$ ✓

$0 = 98^2 a + 30$ $30 = -98^2 a$

$-\frac{30}{98^2} = a$ $-\frac{30}{98^2} = a$

$y = -\frac{15}{4802}x^2 + 30$ ✓ etc.

sub $x = 15$

$y = 29{,}297$

$= 29{,}3$ m. ✓

(5)