SEDULIS PARAESTANTIA
ASHTON
INTERNATIONAL COLLEGE
EXCELLENCE BELONGS TO THE ZEALOUS
BALLITO

Mathematics Paper 2 June 2016

FORM 4

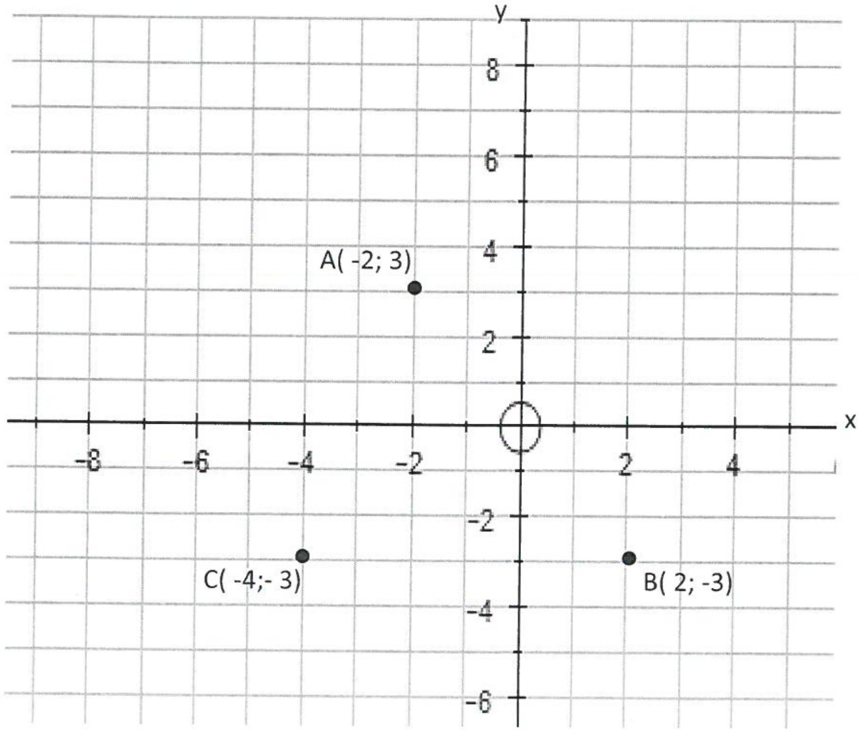
Examiner:	A Gunning	**Moderators:**	P Denissen, C Mundy
Time:	2½ hours	**Marks:**	125

NAME: MEMO

Ques No	1	2	3	4	5	6	7	8	TOTAL	%
Out of	16	14	40	10	10	6	20	9	125	100
Mark										

- All questions are to be answered in this booklet.
- This question paper consists of 17 pages. Included in this, is a list of useful formulae. Please check that your question paper is complete.
- Read and answer all questions carefully.
- It is in your own interest to write legibly and to present your work neatly.
- **All necessary working which you have used in determining your answers must be clearly shown.**
- Approved non-programmable calculators may be used except where otherwise stated. Where necessary give answers **correct to 2 decimal places**.
- Diagrams have not necessarily been drawn to scale.

Question 1



$A(-2;3)$, $B(2;\ -3)$ and $C(-4;\ -3)$ are the vertices of a triangle.

(a) Calculate the gradients of AC and BC (3)

$m_{AC} = \frac{3-(-3)}{-2-(-4)}$ $\quad m_{BC} = \frac{-3-(-3)}{2+4}$

$= \frac{6}{2}$ $\quad = 0$

$= 3$

(3)

(b) Calculate the inclination of the lines AC and BC (3)

$\tan\theta = 3$ $\quad$ inclination of BC

$\theta = 71,57°$ $\quad = 0°$

(3)

(c) Find the coordinates of the midpoint of AC. Label this point M. (2)

midpt AC $= \left(\frac{-4-2}{2} ; \frac{-3+3}{2}\right)$

$= (-3 ; 0)$

(2)

(d) Find the equation of the line which is perpendicular to AC and which passes through M.

(4)

$y - y_1 = m(x - x_1)$

$m_{AC} = -\frac{1}{3}$ $(-3 ; 0)$

$y - 0 = -\frac{1}{3}(x+3)$

$y = -\frac{1}{3}x - 1$

(4)

(e) Write down the coordinates of the x and y intercepts of the line found in (d) (2)

$Y(0 ; -1)$ int

$0 = -\frac{1}{3}x - 1$

$1 = -\frac{1}{3}x$

$-3 = x$

$(-3 ; 0)$ Xint

(no need to write (2) coordinates)

(f) Find the area of triangle ABC. (2)

area $\triangle ABC = \frac{1}{2} BC \times 6$

$= \frac{1}{2} 6 . 6$

$= 18 u^2$

(2)

or area $\triangle ABC$

$= \frac{1}{2} AC . BM$

$AC = \sqrt{40}$ $BM = \sqrt{34}$

$\therefore$ area $= \frac{1}{2} \sqrt{40} . \sqrt{34} = 18{,}44 u^2$

[16]

Question 2

(a) Determine the equation of a straight line which passes through the points B(-2; 4) and C(2; 2) (4)

$y - y_1 = m(x - x_1)$

1 pt $(2;2)$ ✓ ✓

$m_{BC} = \frac{4-2}{-2-2} = \frac{2}{-4} = -\frac{1}{2}$ ✓

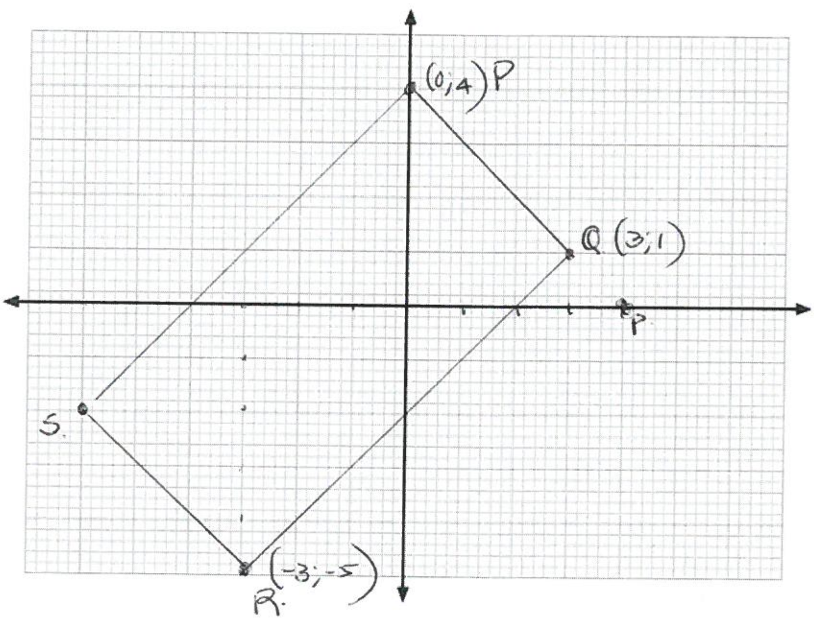
$y - 2 = -\frac{1}{2}(x - 2)$ ✓

$y = -\frac{1}{2}x + 3$ ✓

(4)

(b) You are given quadrilateral PQRS with coordinates $P(0; 4)$, $Q(3; 1)$, $R(-3; -5)$ and $S(-6; -2)$.

(i) Plot each of these points on the given set of axes. (2)



(ii) How would you prove that PQRS is a rectangle? (3)

* Prove PQRS is a parm ✓ ⇒ diag bisect each other ✓

Then prove $m_{PQ} \cdot m_{RQ} = -1$ ✓ ie prove 1 angle $= 90°$ 3

* alternate methods are: ⇒ prove opp sides //

⇒ prove opp sides =

⇒ prove 1 pr opp sides = & //.

(allocate 3 marks accordingly)

(iii) Using the method you specified, prove this. (5)

$M_{SQ}\left(\frac{-6+3}{2}; \frac{-2+1}{2}\right)$ $\quad$ $M_{PR}\left(\frac{0-3}{2}; \frac{4-5}{2}\right)$

$=\left(\frac{-3}{2}; \frac{-1}{2}\right)$ ✓ $\quad = \left(\frac{-3}{2}; \frac{-1}{2}\right)$ ✓

∴ PQRS is a parm. (diag bisect each other)

$m_{PQ} = \frac{4-1}{0-3} = -1$ ✓ $\quad m_{RQ} = \frac{1-(-5)}{3-(-3)} = \frac{6}{6} = 1$ ✓

$m_{PQ} \cdot m_{RQ}$

$= -1 . 1$

$= 1$ ✓

∴ $\hat{Q} = 90°$

∴ PQRS is a rectangle.

(5)

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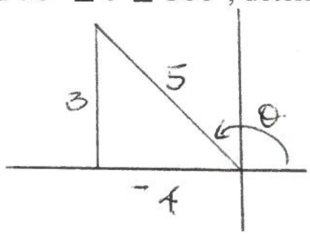
Question 3

(a) If $5\,sin\theta = 3$ and $90° \leq \theta \leq 360°$, determine with the aid of a sketch, and without the use of a calculator,



Quad ✓

x value ✓

(i) $\frac{1}{\tan\theta}$ (4)

$= 1 \div \frac{3}{-4}$ ✓

$= \frac{-4}{3}$ ✓

(4)

(ii) $\frac{5\cos\theta}{2}$ (2)

$\frac{\cancel{5}}{2} \cdot \frac{-4}{\cancel{5}}$ ✓ ✓ $= -2$ ✓

(2)

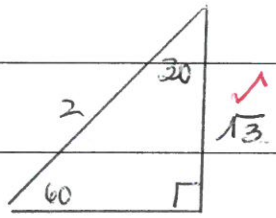
(b) Using the special angle triangles, determine the value of each of the following, without using a calculator. Show all relevant steps needed to determine the answers.

(i) $\sin 150° + \cos 120°$ (3)

$= \sin 30 - \cos 60$

$= \frac{1}{2} - \frac{1}{2}$

$= \underline{0}$



(3)

Must show working to get 3/3.

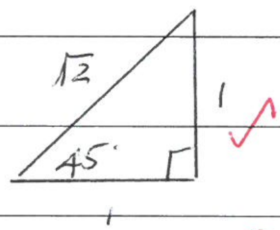
(ii) $sin^2 225° - \tan 135°$ (3)

$= (-\sin 45)^2 - (-\tan 45)$

$= \sin^2 45 + 1$

$= \left(\frac{1}{\sqrt{2}}\right)^2 + 1$

$= \frac{1}{2} + 1 \quad = \underline{\frac{3}{2}}$



(3)

(c) Simplify without the use of a calculator. Show all relevant steps needed to determine the answers.

(i) $\frac{\sin(-20°)}{\cos 250°} = \frac{-\sin 20}{-\cos 70}$ (3)

$\sin 20 = \cos 70$ (comp ∠'s)

$= \underline{1}$

(3)

(ii) $1 - sin^2\theta - cos^2\theta$ (2)

$= \cos^2\theta - \cos^2\theta$

$= \underline{0}$.

(2)

(iii) $\dfrac{\cos(360°-x).\cos(90°-x).\tan(180°-x)}{\cos(180°+x).\sin(360°-x)}$ (7)

$= \dfrac{\cos x . \sin x . -\tan x}{-\cos x . -\sin x}$

$= -\tan x$

(7)

(iv) $\dfrac{\cos(90°-\alpha).\tan(180°+\alpha)}{\tan(180°-\alpha).\sin(180°+\alpha)} + \dfrac{\cos(90°+\alpha)}{\sin(360°-\alpha)}$ (7)

$= \dfrac{\sin\alpha . \tan\alpha}{-\tan\alpha . -\sin\alpha} + \dfrac{-\sin\alpha}{-\sin\alpha}$

$= 1 + 1$

$= 2$

(7)

(v) $\dfrac{\sin(-338°).\cos(300°)}{\cos 248°.tan135°}$ (6)

$= \dfrac{\sin 22 . \cos 60}{-\cos 68 (-\tan 45)} =$

$\sin 22 = \cos 68$

$= +\frac{1}{2} \div 1$

$= \frac{1}{2}$

(6)

(d) If $sin18° = p$ determine the following in terms of p.

(i) $sin198°$ (1)

$= -\sin 18 = -p$

(1)

(ii) $\cos(-108°)$ (2)

$= -\cos 72$

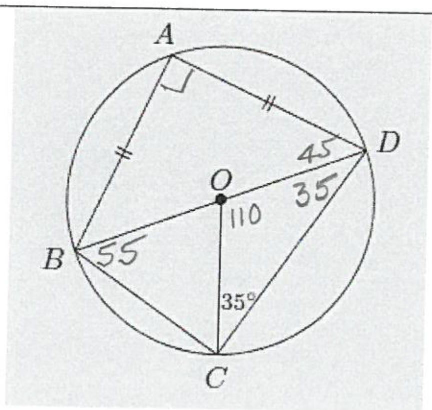
$= -\sin 18$

$= -p$

(2)

[40]

Question 4



BOD is a diameter of the circle with centre O. AB = AD and $O\hat{C}D = 35°$,

Calculate the value of the following angles, giving all relevant reasons.

(a) $O\hat{D}C$ (2) $O\hat{D}C = 35°$ ✓	∠'s in isos △ ✓ (equal radii). (2)
(b) $C\hat{O}D$ (2) $C\hat{O}D = 110°$ ✓	∠'s in △ ✓ (2)
(c) $C\hat{B}D$ (2) $C\hat{B}D = 55°$ ✓	∠ at centre = 2 ∠ on circum ✓ (2) Many other possible reasons.
(d) $B\hat{A}D$ (2) $B\hat{A}D = 90°$ ✓	∠'s in semi circle ✓ (2)
(e) $A\hat{D}B$ (2) $A\hat{D}B = 45°$ ✓	∠'s in isos △ ✓ (2)

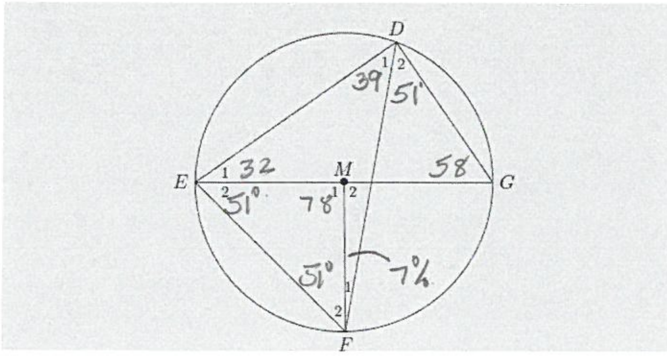
[10]

[10]

Question 5

D, E, F and G are points on the circle with centre M.

$\widehat{F_1} = 7°$ and $\widehat{D_2} = 51°$. Determine the value of each of the following angles. Give all relevant reasons.



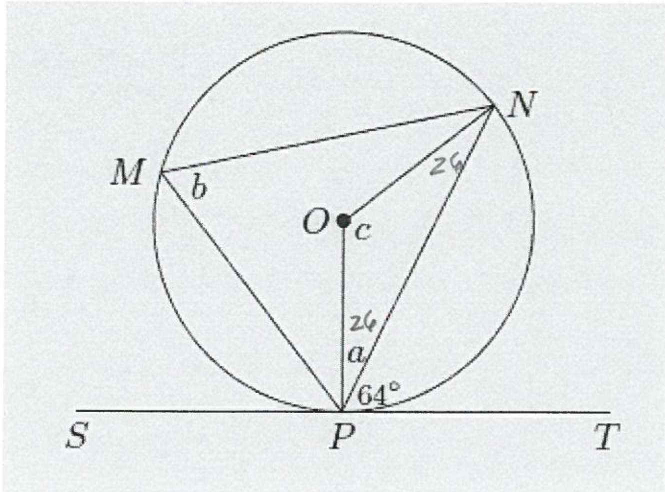
(a) $\widehat{D_1}$ (2) $\hat{D} = 90°$ $\therefore D_1 = 39°$	∠ in semi circle. Must give step 1 to get (2) marks (2)
(b) $\widehat{M_1}$ (2) $\hat{M}_1 = 78°$	∠ at centre = 2 ∠ at circum (2)
(c) $\widehat{F_2}$ (2) $\hat{F}_2 = 51°$	∠'s in isos △ (2) other possible reasons.
(d) $\hat{G}$ (2) $\hat{G} = 51° + 7°$ $= 58°$	∠'s in same segment. (chord DE) (2)
(e) $\widehat{E_1}$ (2) $\hat{E}_1 = 32°$	∠'s of △. (2)

[10]

[10]

Question 6

O is the center of the circle and SPT is a tangent. Determine the values of a, b and c. Give all relevant reasons.



(a) Determine a (2) OP ⊥ PT ✓ ∴ $a = 26°$ ✓	rad ⊥ tangent. (2)
(b) Determine b (2) $b = 64°$ ✓	tan chord Th ✓ (2) Many other possible reasons.
(c) Determine c (2) $c = 128°$ ✓	∠ at centre ✓ = 2 ∠ on circumf (2) could use ∠'s in Δ. (proved above)

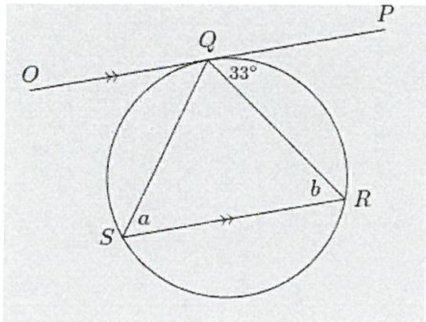
[6]

[6]

Question 7

Determine, with all relevant reasons, the values of the unknown in each of the following.

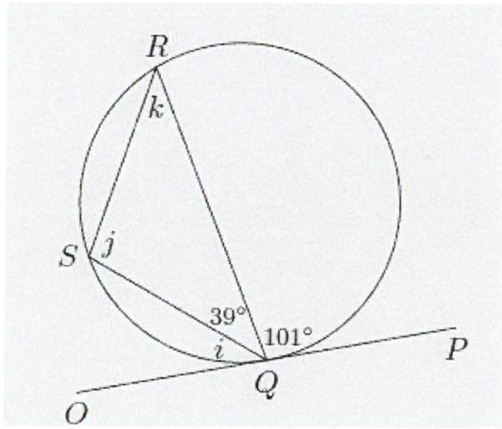
(a) You are given that OQP is a tangent to the circle QRS, at Q.



(4)

$a = 33°$ ✓ $b = 33°$ ✓	tan chord Theorem ✓ alt ∠'s =; OP // SR ✓ must state which lines are parallel to get full mark. (4)

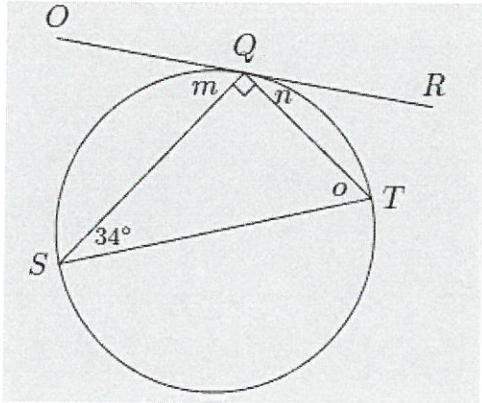
(b) You are given that OQP is a tangent to the circle QRS, at Q



(3)

$j = 101°$ ✓ $i = 40°$ ✓ $k = 40°$ ✓	tan chord Theorem ✓ ∠'s on str line ✓ ∠'s in △ ✓ (or tan chord Th) (3)

(c) OQR is a tangent to circle QST.

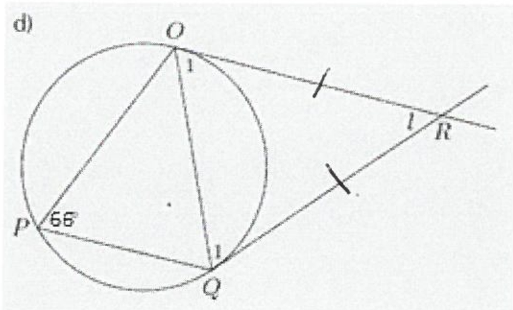


(3)

$n = 34°$ ✓ $m = 56°$ ✓ $o = 56°$. ✓	tan chord theorem ✓ ∠'s on str line ✓ tan chord Th ✓ (or ∠'s in △)

(3)

(d) OR and QR are tangents to circle OPQ. Find the value of l.



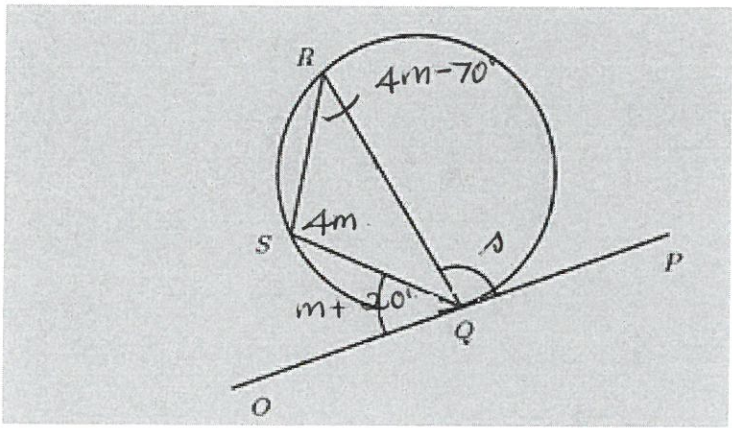
(5)

$\hat{O}_1 = 68°$ ✓ △ ORQ is isos $\therefore \hat{Q}_1 = 68°$. ✓ $\therefore l = 44°$ ✓	tan chord Th ✓ 2 tang. from same pt = ✓ isos △. ∠'s in isos △. ✓

(5)

(e) OQP is a tangent to circle RSQ. $\hat{R} = 4m - 70°$, $R\hat{Q}P = s$, $O\hat{Q}S = m + 20°$, $\hat{S} = 4m$

Find the values, with reasons, of m and s.



(5)

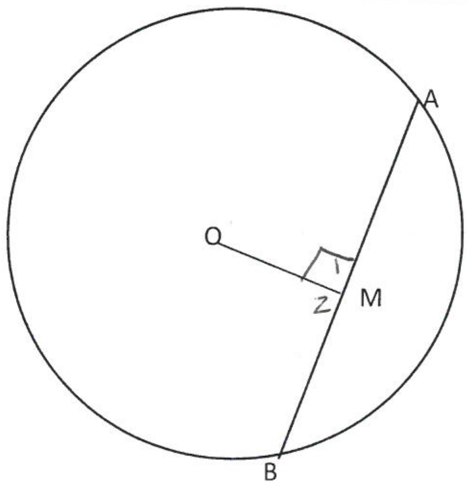
$m + 20 = 4m - 70$ $90 = 3m$ $30° = m$ $s = 4m$ $= 120°$	tan chord Th. tan chord Th (5)

[20]

[20]

Question 8

(a)In the diagram below, you are given a circle center O. A line is drawn from O perpendicular to a chord AB. Complete the statements below to prove the theorem which states that this line will bisect the chord AB. (4)



Given: $OM\ is\ perpindicular\ to\ AB$

Required to prove: BM = MA

Proof:

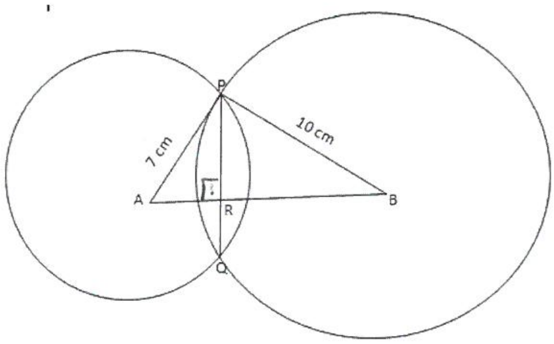
Join OA and OB.

In ΔOMB and $\Delta\ OMA$

1. OB = OA	equal radii
2. $\hat{M}_2 = \hat{M}_1$	given OM $\perp$ AB.
3. OM common	
$\therefore\ \Delta OMB \equiv \Delta\ OMA$	RHS.
$\therefore$ BM = AM.	congr. Δ's.

[4]

(b) Two circles with centers A and B, as shown below, have radii of 7 cm and 10 cm respectively. AB is perpendicular to PQ. The diagram has NOT been drawn to scale. If the length of the common chord PQ is 8 cm, what is the length of AB? Remember to show all relevant working and state all reasons. (5)



In $\triangle APQ$	
$R_1 = 90^\circ$	give
$\therefore BR = RQ = 4$ cm	line from centre $\perp$ to chord
$AR^2 = 7^2 - 4^2$	Pythagoras
$= 49 - 16$	
$AR = \sqrt{33}$	
In $\triangle PRB$	
$RB^2 = 10^2 - 4^2$	Pythag.
$= 100 - 16$	
$RB = \sqrt{84}$	
$\therefore AB = \sqrt{33} + \sqrt{84}$	can be left in surd form
$= 14,91$ cm	

[9]

[5]

Useful formulae

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$M\left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c$$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan\theta$$