ASHTON INTERNATIONAL COLLEGE

BALLITO

Mathematics Paper 2 June 2017

FORM 4

Examiner:	A Gunning	**Moderator:**	P Denissen
Time:	2½ hours	**Marks:**	125

NAME: MEMO.

Ques No	1	2	3	4	5	6	7	8
Out of	11	11	6	8	5	19	14	10
Mark								

Ques No	9	10	11	12	13	TOTAL	%
Out of	5	8	9	8	11	125	100
Mark							

- All questions are to be answered in this booklet.
- This question paper consists of 20 pages. Included in this, is a list of useful formulae. Please check that your question paper is complete.
- Read and answer all questions carefully.
- It is in your own interest to write legibly and to present your work neatly.
- **ALL NECESSARY WORKING WHICH YOU HAVE USED IN DETERMINING YOUR ANSWERS MUST BE CLEARLY SHOWN.**
- Approved non-programmable calculators may be used except where otherwise stated. Where necessary give answers **correct to 2 decimal places**.
- Diagrams have not necessarily been drawn to scale.

Question 1

Given points $R(1;3)$, $S(3;7)$ and T (-1; -1)

(a) Determine the gradient of RS. (2)

$m_{RS} = \frac{7-3}{3-1} = \frac{4}{2} = 2$ ca

(2)

(b) The length of ST is $w\sqrt{5}$. Find the value of w. Show all relevant working detail. (3)

$ST = \sqrt{(3+1)^2 + (7+1)^2} = w\sqrt{5}$

$16 + 64 = w^2 . 5$

$80 = w^2 . 5$

$w^2 = 16$

$w = 4$

(3)

(c) Determine the equation of the line through T parallel to RS, giving your answer in standard form. (2)

$y - y_1 = m(x - x_1)$

$m = 2$ ca 1 pt T $(-1;-1)$

$y + 1 = 2(x+1)$ $y = 2x + 1$. ca

(2)

(d) Determine the equation of a line which is perpendicular to RS AND which bisects RS. (4)

$m_\perp = -\frac{1}{2}$ ca

midpt RS $\left(\frac{1+3}{2}; \frac{7+3}{2}\right) = (2;5)$.

$y - 5 = -\frac{1}{2}(x-2)$

$y = -\frac{1}{2}x + 6$

(4)

[11]

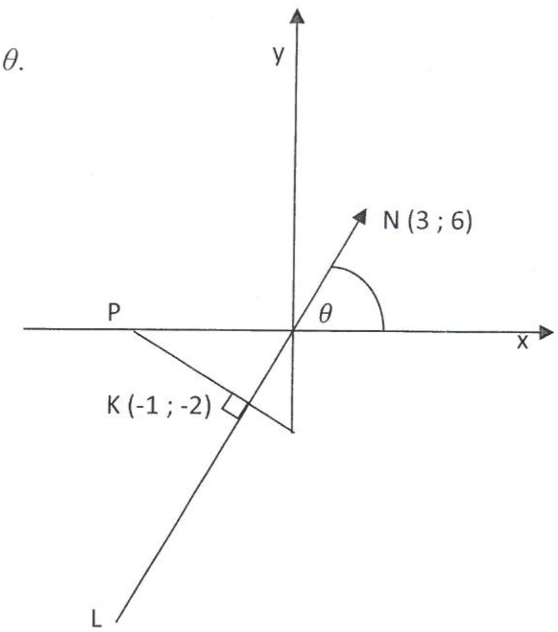
Question 2

This diagram has not been drawn to scale.

In the diagram, $K(-1;\ -2)$ is the midpoint of LN with $N(3;6)$.

PK $\perp$ LN with P on the x-axis.

The angle of inclination of NL is θ.



Determine

(a) the gradient of LN (2)

$$m_{LN} = \frac{6-(-2)}{3-(-1)} = \frac{8}{4} = 2$$ (2)

(b) the size of θ, correct to 1 decimal place (2)

$\tan\theta = 2$

$\theta = \tan^{-1}(2) = 63{,}4^\circ$ (2)

(c) the co-ordinates of L (4)

(no need to write as coords)

K midpt $(-1;-2)$ N $(3;6)$

L $(x;y)$ $-1 = \frac{x+3}{2}$ $-2 = \frac{y+6}{2}$

$x = -5$ $y = -10.$

(4)

(d) the length of NK, leaving your answer in the simplest surd form. (3)

$NK = \sqrt{(3-(-1))^2 + (6-(-2))^2}$

$= \sqrt{16+64}$

$= \sqrt{80} = 4\sqrt{5}.$

(3)

[11]

Question 3

The points $A(3;1)$, $B(2;\ -2)$ and $C(2;3)$ are given.

(a) If CD is perpendicular to AB with $D(-3;t+1)$, determine the value of t. (3)

$CD \perp AB \quad m_{CD} \cdot m_{AB} = -1.$

$\frac{t+1-3}{-3-2} \cdot \frac{1-(-2)}{3-2} = -1.$

$\frac{(t-2)}{-5} \cdot 3 = -1 \Rightarrow \quad 3t-6 = 5$

$3t = 11$

$t = \frac{11}{3}$

3,67

(3)

(b) If A, B and E are collinear, given $E(r;4)$, evaluate r. (3)

$A(3;1) \quad B(2;-2) \quad E(r;4)$

$m_{AB} = m_{BE}$

$3 = \frac{4-(-2)}{r-2}$

$3r-6 = 6$

$3r = 12$

$r = 4$

(3)

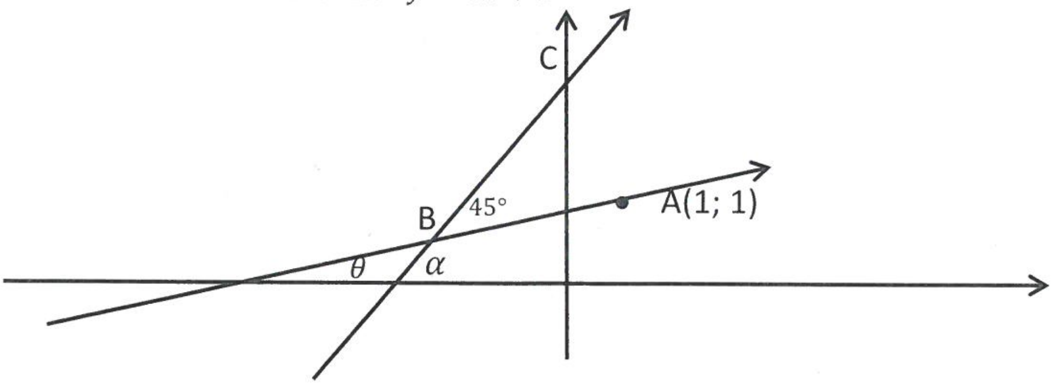
[6]

Question 4

In the given sketch (not drawn to scale) A(1; 1) is a point on the line AB. And $A\hat{B}C = 45°$

The line BC makes an angle α with the x-axis and the line AB makes an angle θ with the x-axis.

Points B and C lie on the line $y = 3x + 2$



(a) Determine the inclination of the line BC . (2)

line BC $y = 3x + 2$ $\tan\alpha = 3$

$m_{BC} = 3$ $\alpha = \tan^{-1}(3)$

$\alpha = 71,6°$

(2)

$(71,57°)$

(b) Determine the value of θ, the inclination of the line AB. (2)

$\theta = 71,6 - 45°$ ext ∠ △.

$\therefore \theta = 26,6°$

(2)

$(26,57°)$

(c) Determine the equation of the line AB. (4)

$m_{AB} = \tan 26,6' = \frac{1}{2}$

A(1;1) $y - 1 = \frac{1}{2}(x - 1)$

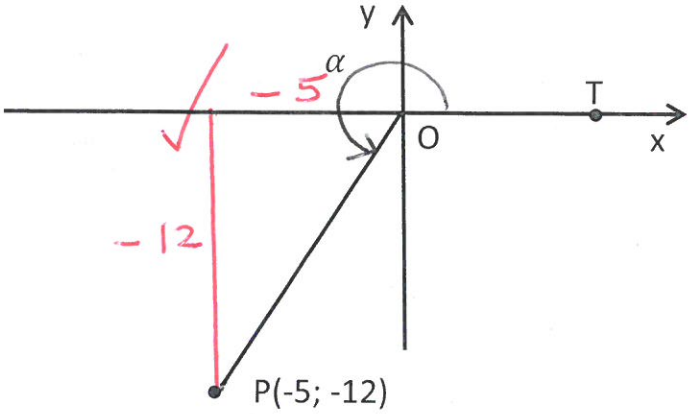
$y = \frac{1}{2}x + \frac{1}{2}$

(4)

[8]

Question 5

In the diagram below, reflex $T\hat{O}P = \alpha$ and P has coordinates $(-5;\ -12)$



Determine the value of each of the following, WITHOUT USING A CALCULATOR.

(a) $\cos\alpha$ (3)

$OP = \sqrt{25+144}$
$= 13$

$\cos\alpha$
$= \frac{-5}{13}$

(3)

(b) $\tan(180° - \alpha)$ (2)

$= -\tan\alpha$
$= -\frac{12}{5}$

(2)

[5]

Question 6

Without the use of a calculator, simplify each of the following.

(a) $\frac{\sin 100°}{\cos(350°)}$ (2)

$= \frac{\sin 80}{\cos 10}$

These are equal as they are complementary

$= 1$

(2)

(b) $sin^2(-20°) + cos^2(200°)$ (4)

$(-\sin 20)^2 + (-\cos 20)^2$

$= \sin^2 20 + \cos^2 20$

$= 1$

(4)

(c) $\frac{\cos 130° - \sin(90° - \theta)}{\sin 400° + \cos(-\theta)}$ (5)

$= \frac{-\cos 50 - \cos\theta}{\sin 40 + \cos\theta} = \frac{-(\cos\theta + \cos 50)}{\cos\theta + \sin 40}$

$\cos 50 = \sin 40.$

$\therefore$ ans $= -1.$

(5)

(d) $\dfrac{\cos 140°.\tan(-320°)}{\sin 220°}$ (4)

$= \dfrac{-\cos 40 . \tan 40}{-\sin 40}$

$= +\dfrac{\cancel{\cos 40}}{\cancel{\sin 40}} . \dfrac{\cancel{\sin 40}}{\cancel{\cos 40}}$

$= 1.$

(4)

(e) Evaluate $\sqrt{4^{\sin 150°}.2^{3\tan 135°}}$ without the use of a calculator. (4)

$= \sqrt{2^{2\sin 30} . 2^{3.-\tan 45}}$

$= \sqrt{2^{2.\frac{1}{2}} . 2^{-3}}$ $\sin 30 = \frac{1}{2}$

$\tan 45 = 1.$

$= \sqrt{\frac{1}{4}}$

$= \frac{1}{2}.$

(4)

[19]

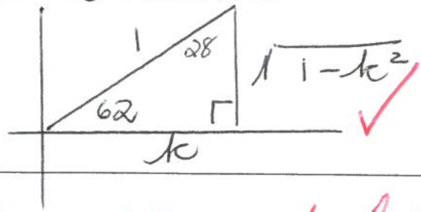
Question 7

(a) If $\cos 62° = k$, determine the value of each of the folling in terms of k.

(i) $\sin 28°$ $= \cos 62$ (2)

$= \underline{k}$ ✓

OR



no need for this, really.

hence ✓✓ for correct answer. (2)

(ii) $\cos 242°$ (2)

$= -\cos 62°$ ✓

$= \underline{-k}$ ✓ (2)

(b) Simplify to a single ratio

$$\frac{\tan(360°-x).\sin(90°+x)}{\sin(-x)}$$ (5)

$= \dfrac{-\tan x.\cos x}{-\sin x}$ ✓✓✓ $= \dfrac{\sin x}{+\cos x}$ ✓ $\cdot \dfrac{\cos x}{\sin x}$

$= +1.$ ✓

(5)

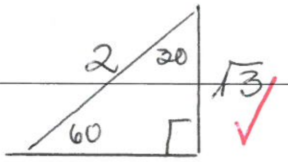
(c) Consider $4sin^2x - 3 = 0$ for $0° \leq x \leq 270°$,

(i) In how many quadrants will there be solutions for this equation? (1)

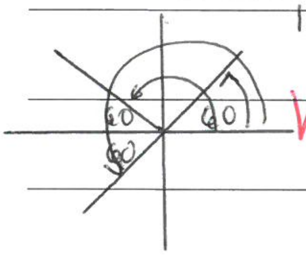
$\sin^2 x = \frac{3}{4}$

$\sin x = \pm \frac{\sqrt{3}}{2}$ answers in 3 quadrants. (1)

(ii) Without the use of a calculator, solve for x, giving all possible values of x (4).



Ref $\angle = 60°$



answers $60°$; $120°$; $240°$

(4)

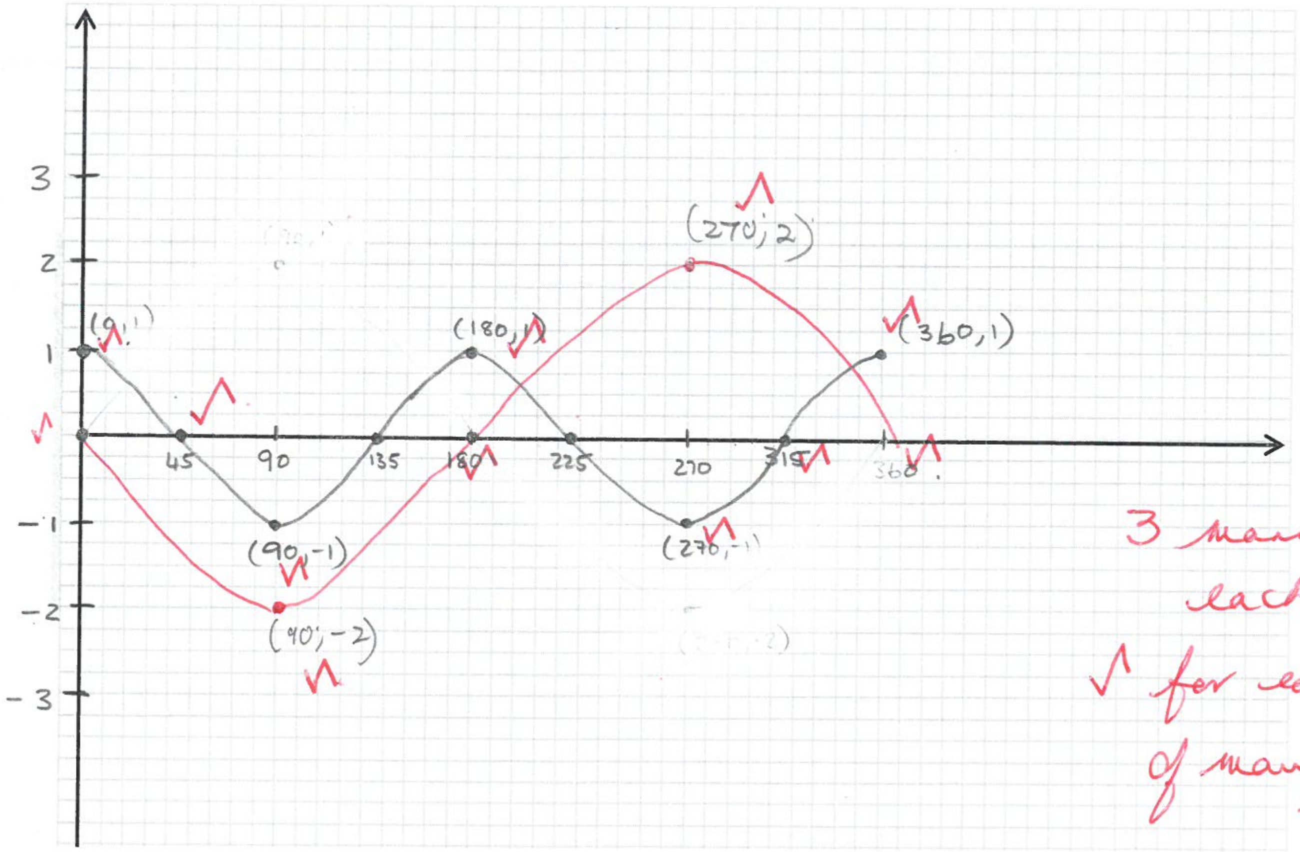
[14]

Question 8

(a) On the same set of axes below, draw sketch graphs of each of

$f(x) = -2\sin x$ and $g(x) = \cos 2x$ for the domain $x \in [0°; 360°]$.

You must show clearly any intercepts with the axes as well as the coordinates of the maximum and minimum values for each graph. (6)



3 marks each

✓ for each of main points

(b) Without doing any working, write down the period of the graph $k(x) = \sin 3x$ (1)

120° ✓ (1)

(c) State the amplitude of the graph $f(x) + 1$ (1)

2 ✓ (1)

(d) What is the range of $g(x) = \cos 2x$? (1)

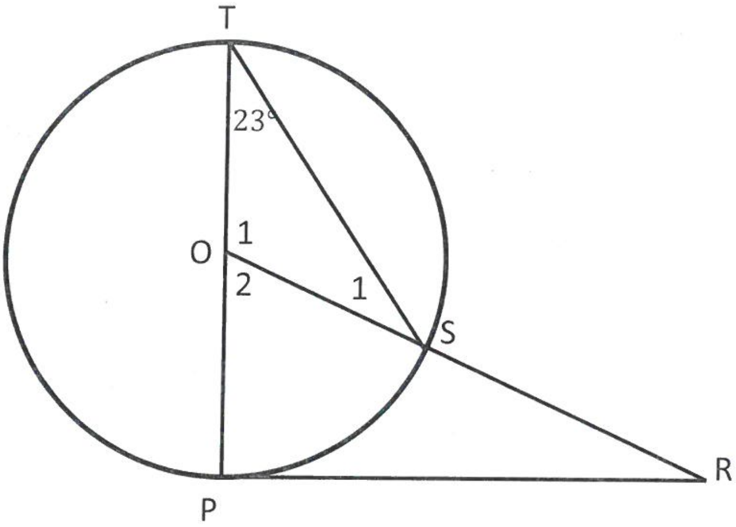
$y \in [-1; 1]$ ✓ (1)

(e) If $f(x)$ has been shifted 30° to the right, write down the equation of the new graph. (1)

$-2\sin(x - 30)$ ✓ (1)

[10]

Question 9



O is the center of the circle.

TOP is the diameter and PR is a tangent.

If $\widehat{T} = 23°$, to determine the size of $\hat{R}$, the following steps have been given. For each of the statements below, fill in the relevant reason.

(Remember, you are building a logically sequenced proof.)

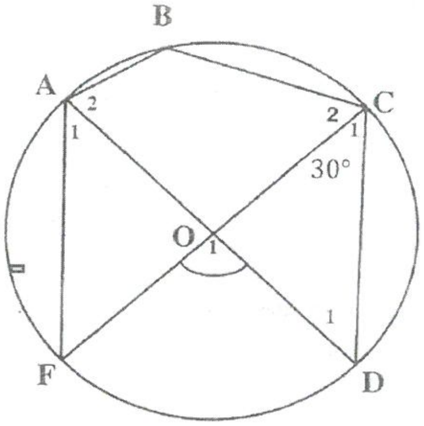
STATEMENTS	REASONS
TO = OS	equal radii
$\hat{S}_1 = 23°$	isos △.
$\hat{O}_2 = 46°$	either ∠ at centre = 2∠ on circum OR ext ∠ △ OR ∠'s in △ and ∠'s on str line.
$\hat{P} = 90°$	radius ⊥ tang.
$\hat{R} = 44°$	∠'s in △.

(5)

[5]

Question 10

In the given diagram, O is the centre of the circle. $\widehat{C_1} = 30°$



(a) Name with reasons, three other angles each equal to 30° (4)

$D_1 = 30°$ ∠'s in isos △.

$A_1 = C_1 = 30°$ ∠'s in same segment chord FD.

$\hat{F} = \hat{D}_1 = 30°$ ∠'s in same segment chord AC

must have all 3 reasons to get ✓.

(b) Calculate, with reasons, the following angles.

(i) $\hat{B}$ = 150° opp ∠'s cyclic quad suppl. (2)

(ii) $\hat{O}_1$ = 60° ~~1 of~~ ∠ at centre = 2∠ at circum (2)

(2)

OR – ext ∠ △.

OR – ∠'s in △ and ∠'s on str line. (BOTH)

(2)

[8]

Question 11

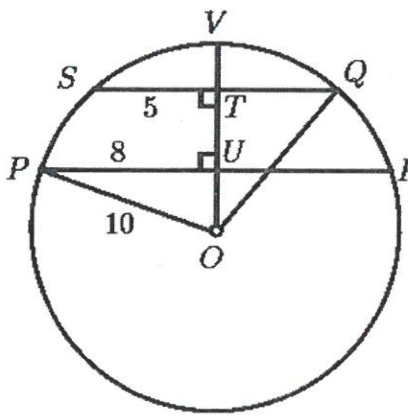
(a) Complete the following statement: (1)

A line drawn from the center of a circle perpendicular to the chord bisects the chord.

(1).

(b) In the circle with centre O, $OT \perp SQ$, $OT \perp PR$, $OP = 10$ units and $PU = 8$ units.

Determine TU. (8)



In $\triangle PUO$

$OU^2 + SU^2 = 10^2$ Pythag.

$OU^2 = 36$

$OU = 6$.

In $\triangle STO$

$OT^2 + ST^2 = 10^2$ Pythag.

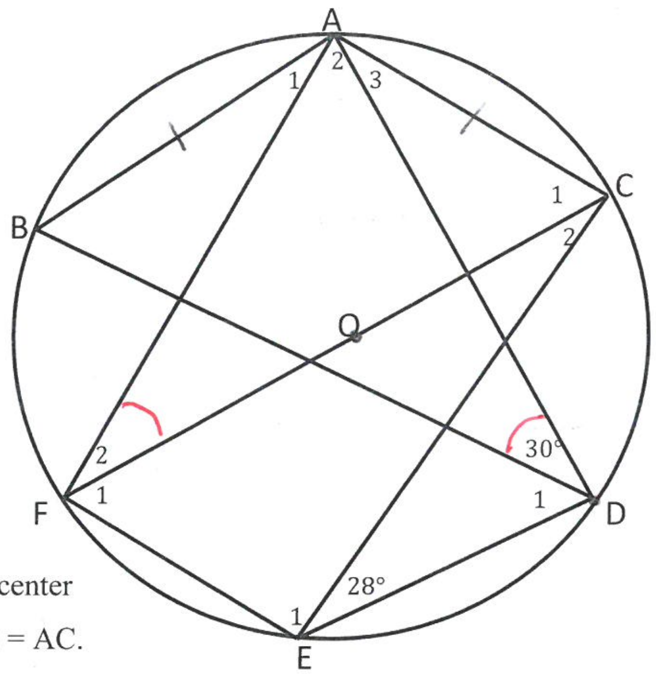
$OT^2 = 100 - 25$

$OT = \sqrt{75} = 5\sqrt{3}$.

$\therefore TU = 5\sqrt{3} - 6$

(8)

Question 12



In the diagram, O is the center of the circle. Chords AB = AC.

$C\hat{E}D = 28°$ and $A\hat{D}B = 30°$.

Calculate with reasons, the sizes of each of the following angles.

(a) $\hat{E}_1$ (2)

$= 90°$ ∠ in semicircle. (2)

(b) $\hat{F}_2$ (2)

$= \hat{D} = 30°$ equal chords subtend equal ∠'s (2)

(c) $\hat{A}_2$ (4)

$\hat{A}_3 = 28° = E$ ∠'s in same segment chord CD.

$\hat{A}_2 + \hat{A}_3 = 90°$ ∠ in semicircle.

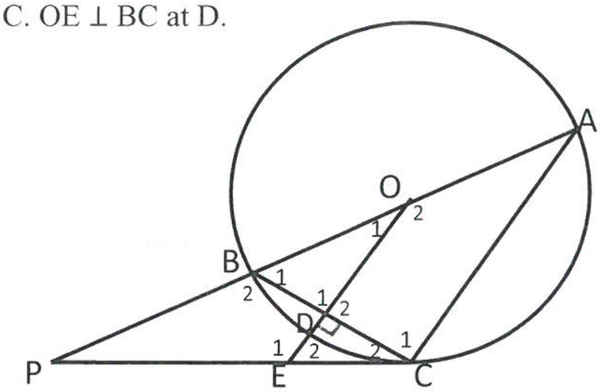
$\therefore \hat{A}_2 = 62°$

[8]

(4)

Question 13

In the diagram, AB is the diameter of the circle, center O. AB is produced to P. PC is a tangent to the circle at C. OE ⊥ BC at D.



(a) Prove with reasons that $EO \parallel CA$. (4)

$\hat{C}_1 = 90°$ ∠ in semi circle.

$D_1 = 90°$ given

$\therefore \hat{C}_1 = \hat{D}$

These are equal corresp ∠'s

$\therefore EO \parallel CA$

(4)

(b) If $\hat{C}_2 = x$, name with reasons, two other angles also equal to x. (4)

$\hat{C}_2 = \hat{A} = x$ tan chord Th

$\hat{A} = \hat{O}$ corresp ∠'s =; $OE \parallel AC$

(4)

(c) Calculate, with reasons, the size of $\hat{P}$ in terms of x (3)

$\hat{P} = 180 - \hat{A} - \hat{C}_1 - \hat{C}_2$ ∠'s in △

$= 180 - x - x - 90$

$= 90 - 2x$

[11] (3)