SECTION A

Question 1

The heights of 20 children were measured (in centimetres) and the results were recorded. The data collected is given in the table below.

127	129	131	134	134
137	139	141	142	144
128	130	133	134	136
138	140	142	143	145

a. Write down the median height measured. (1)

136,5 ✓ answer

b. Determine:

1. The mean height (2)

$\frac{2728}{20} = 136,\cancel{4}$ 35 ✓ • Answer

2. The range (1)

145 − 127 ✓
= 18 • Answer

3. The interquartile range (3)

LQ = 132 ✓
UQ = 14,5 ✓
IQR = 9,5 cm ✓

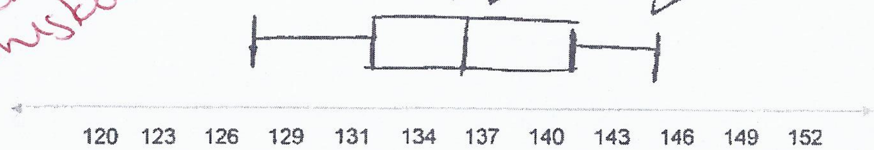
• LQ
• UQ
• IQR

c. Draw a box and whisker diagram to represents the data. (4)

✓ Heading
✓✓ Box 2 marks
✓ Whiskers

Heghts of children ✓

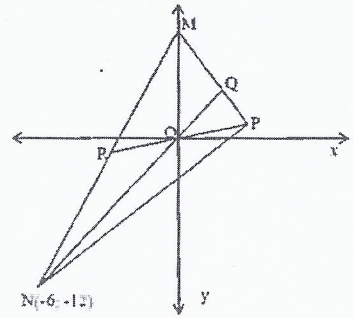
✓✓ ✓



[11]

Question 2

In the diagram, M, N and P are the vertices of ΔMNP, with $N(-6;-12)$.
M is a point on the y-axis. The equation of the line MN is $3x - y + 6 = 0$.
$MR = NR$ and $NQ \perp MP$. PR and NQ intersect at the origin O.



a. Calculate the gradient of NQ. (1)

$\frac{-12-0}{-6-0} = 2$ ✓ Answer

b. Calculate the gradient of MP. (1)

$m_1 \times m_2 = -1$
$m_{MP} = -\frac{1}{2}$ ✓ Answer

mym

c. Calculate the angle of inclination of MP. (3)

$\tan\theta = -\frac{1}{2}$
r.a $= 26,57$ ✓
$\theta = 153,43$ ✓

• $m = \tan\theta$
• RA
• Answer mym

d. Determine the equation of the line MP. (4)

$y - 6 = -\frac{1}{2}(x - 0)$ ✓✓
$y = -\frac{1}{2}x + 6$ ✓✓

(0,6)

• M(0,6)
• gradient
• Subst
• Answer.

If answer only $\frac{4}{4}$

mym M(0,6)
mym gradient according to (b)

e. Determine the coordinates of P. (4)

$m_{NQ} \times m_{mp} = -1$ ✓

$\frac{y-6}{x-0} = \frac{-1}{2}$ ✓

$x = 2 \quad y = 5$ ✓

$P(2;5)$

any method
Answer only
$\frac{4}{4}$
mym from (b)

f. Determine the co-ordinates of R. (3)

$MR = NR$

$R\left(\frac{0-6}{2}; \frac{6-12}{2}\right)$ ✓

$R(-3;-3)$ ✓

*m(0;6)
*N(−6;−12)
• Subst
• x
• y

[16]

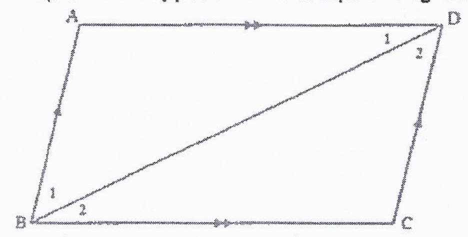
Question 3

Give reasons for your statements.

a. Complete the following statement:
If the opposite angles of a quadrilateral are equal, then the quadrilateral .... (1)

is a parm ✓

b. Use the sketch below to prove that opposite sides of a parallelogram are equal. (6)



In $\triangle ABD$ and $\triangle CDB$
St
Req

$D_1 = B_2$ ✓ alt ∠ AD∥BC ✓

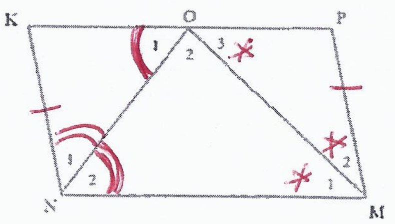
$B_1 = D_2$ alt ∠ AB∥DC ✓ St+R

$BD = BD$ common ✓ St+R

$\therefore \triangle ABD \equiv \triangle CDB$ AAS ✓
conc+R

$\therefore AB = DC$
$AD = BC$ ✓ concl

c. In the sketch below, $KPMN$ is a parallelogram. ON bisects $K\hat{N}M$ and OM bisects $N\hat{M}P$.



1. Show that $N\hat{O}M = 90°$. (3)

Let $N_1 = N_2 = x$ $M_1 + M_2 = y$

$2x + 2y = 180$ co-int KN//PM   sum of ∠s

$\therefore x + y = 90$ $N_2 + O_2 + M_1 = 180$

$O_2 = 90$

2. Prove that O is the midpoint of KP. (6)

$N_2 = O_1$ alt KP//NM

$\therefore O_1 = N_1$ gives $N_1 = N_2$

$\therefore KO = KN$ ∠'s opp = sides

$O_3 = M_1$ alt KP//MN

$\therefore O_3 = M_2$ gives $M_1 = M_2$

$\therefore OP = PM$ ∠'s opp = sides

$KN = PM$ opp sides parm

$\therefore KO = OP$

$\therefore O$ is the midpoint.

Question 4

a. Simplify without the use of a calculator:

1. $\frac{\cos(90°-x).\sin(-x)}{\cos^2(180°+x)}$ (5)

$\frac{\sin x \cdot -\sin x}{\cos^2 x}$

$= -\tan^2 x$

cos(90−x)
sin(−x)
cos² x
answer

2. $\sin 143°.\cos 127° - \sin 53°.\cos 37°$ (5)

$\sin 37 \cdot -\cos 53 - \sin 53 \cos 37$

$-\sin 37 \ \sin 37 - \cos 37 \cos 37$

$-\sin^2 37 - \cos^2 37$

$-(\sin^2 37 + \cos^2 37)$

-1

sin 143
cos 123
cos 53
HCF
Answer

b. Prove the identity: $\left(\tan y + \frac{1}{\tan y}\right)(1-\cos^2 y) = \tan y$. (6)

$\left(\frac{\sin y}{\cos y} + \frac{1}{\tan y}\right)\sin^2 y$

$\left(\frac{\sin y}{\cos y} + \frac{\cos y}{\sin y}\right)\sin^2 y$

$\left(\frac{\sin^2 y + \cos^2 y}{\sin y \cos y}\right)\sin^2 y$ LCD ④

$\frac{\sin y}{\cos y} = \tan y$ ⑥ answer.

⑤ simplify $= RHS$

c. Determine the general solution of: $\cos\theta - \frac{1}{\cos\theta} = \frac{5}{6}$. (6)

$6\cos^2\theta - 6 = 5\cos\theta$

$6\cos^2\theta - 5\cos\theta - 6 = 0$ Std form

$(3\cos\theta + 2)(2\cos\theta - 3) = 0$ Factors

$\cos\theta = -\frac{2}{3}$ $\quad\cos\theta = \frac{3}{2}$

r.a $= 48,19°$ $\quad$ NA

II $\quad$ III

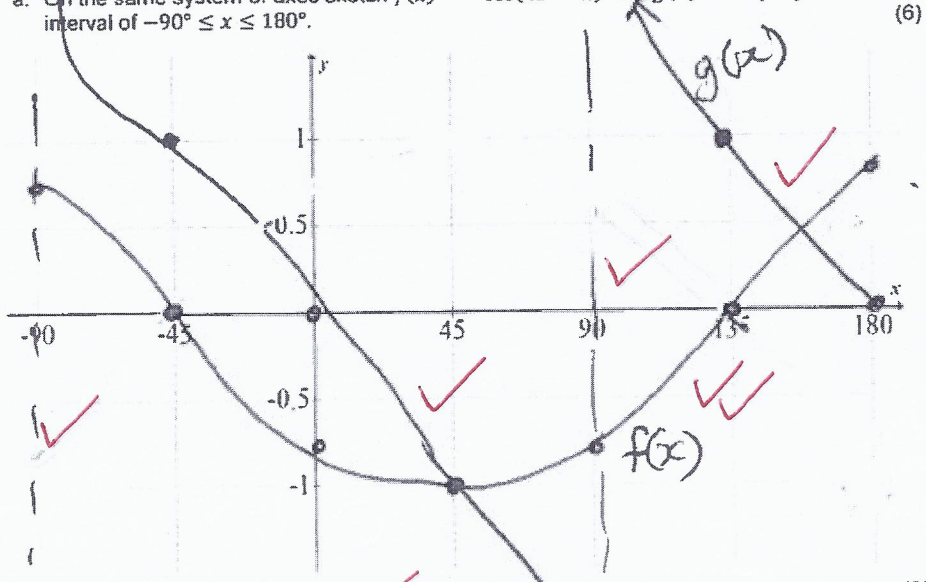
$\theta = 131,81 + k.360$ $\quad \theta = 311,81 + k.360$

$k \in Z$
If left out
-1

[22]

Question 5

a. On the same system of axes sketch $f(x) = -\cos(45° - x)$ and $g(x) = \tan(-x)$ in the interval of $-90° \leq x \leq 180°$. (6)



b. For which values of x is $f(x) - g(x) \leq 0$ for $x \in [-90°; 90°]$ (2)

$-90 \leq x \leq 45$ $\quad$ • values • signs.

c. Write down the equation of $h(x)$ if $h(x) = -f(x - 45°)$. (2)

$-(-\cos(45 - x + 45))$ $\quad$ • subst

$= -(-\cos(90 - x))$ $\quad$ • Answer.

$= \sin x$.

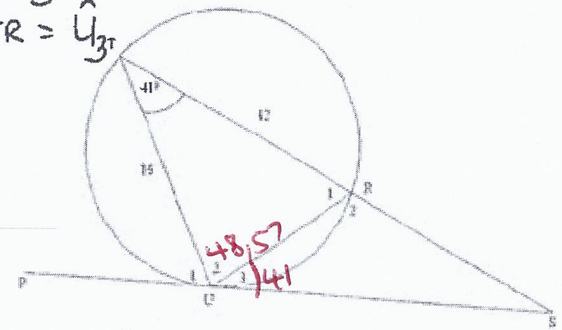
[10]

Question 6

a. Complete: c^2 = $a^2 + b^2 - 2ab\cos C$ ✓ (1) (1)

b. TRS is a secant of the circle, and SU is a tangent at U. $TU = 16$ cm, $TR = 12$ cm and $\hat{T} = 41°$.

$U\hat{T}R = \hat{U}_3$



Calculate:

1. The length of UR, correct to two decimal places. (3)

$UR^2 = 16^2 + 12^2 - 2(16)(12)\cos 41$ ✓ cos Rule ✓ subst

$= 110{,}191$

$UR = 10{,}50$ ✓ answer

2. The size of $\hat{U}_2$. (3)

mym

$\frac{\sin U_2}{12} = \frac{\sin 41}{10{,}50}$ ✓ sin Rule

$\sin U_2 = 0{,}74978\ldots$ ✓ subst

$\hat{U}_2 = 48{,}57°$ ✓ answer

$\cos U_2 = \frac{16^2 + 10{,}50^2 - 12^2}{2(16)(10{,}50)}$

$= 0{,}6614583\dot{3}$

$U_2 = 48{,}59°$

3. The length of secant TRS. (5)

$\hat{U}_3 = 41$

$\hat{S} = 49{,}43$ ✓ sum ∠s △ • $\hat{S}$

$\frac{RS}{\sin 41} = \frac{10{,}5}{\sin 49{,}43}$ ✓ • (sin Rule subst

$RS = 9{,}07$ ✓ • RS

$TRS = 21{,}07$ ✓✓ ✓TRS 2m

[12]

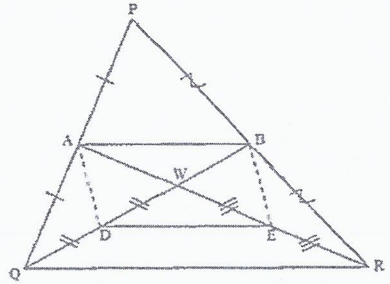
mym from U_2 UR

SECTION B

Question 7

a. Complete the following statement:
The line through the midpoint of two sides in a triangle is parallel to and half ✓ the third side. (1)

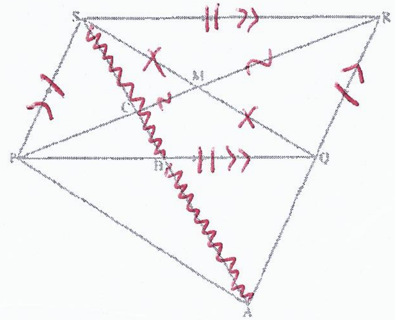
b. In ΔPQR, A and B are the midpoints of sides PQ and PR respectively.
AR and BQ intersect at W.
D and E are the points on WQ and WR respectively such that $WD = DQ$ and $WE = ER$.



Prove that $ADEB$ is a parallelogram. (5)

AB // QR ✓ } midpnt theorem ✓
AB = ½QR }
DE // QR } midpnt theorem ✓
DE = ½QR }
∴ AB // DE ✓
AB = DE
∴ ADEB is a parm
1 pr opp sides = // ✓

c. In the diagram below, $PQRS$ is a parallelogram having diagonals PR and QS intersecting at M.
B is a point on PQ such that SBA and RQA are straight lines and $SB = BA$.
SA cuts PR in C and PA is drawn.



1. Prove that $SP = QA$. (4)

SB = BA ✓ given
QR = QA ✓ converse midpnt ✓
QR = SP ✓ opp sides parm ✓
∴ SP = QA

2. Prove that $SPAQ$ is a parallelogram. (2)

SP = QA proven
SP // QA opp sides parm ✓
∴ SPAQ is a parm
one pr opp sides = // ✓

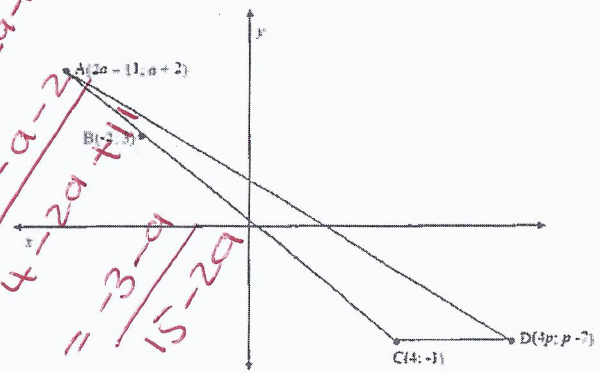
3. Prove that $AR = 4MB$. (4)

M midpnt of PR } diag of ✓
B midpnt of PQ } parm
MB = ½QR ✓ midpnt th ✓
MB = ½(½AR)
4MB = AR ✓

[16]

Question 8

The points $A(2a-11; a+2)$, $C(4;-1)$ and $D(4p; p-7)$ are the vertices of ΔACD with $B(-2;3)$ on AC.



$AC = \frac{-1-(a+2)}{4-(2a-11)} = \frac{-1-a-2}{4-2a+11} = \frac{-3-a}{15-2a}$

a. If points A, B and C are collinear, find the value of a. (4)

$m_{BC} = \frac{-1-3}{4+2} = \frac{-4}{6} = \frac{-2}{3}$ ✓ • m_{BC}

$m_{AC} = \frac{a+2+1}{2a-11-4} = \frac{a+3}{2a-15}$ ✓ • m_{AC}

$\frac{-2}{3} = \frac{a+3}{2a-15}$ ✓ • =

$a = 3$ ✓ • Answer.

$m_{BC} = -\frac{2}{3}$ ✓

$m_{AB} = \frac{3-(a+2)}{-2-(2a-11)} = \frac{1-a}{9-2a}$ ✓

$-\frac{2}{3} = \frac{1-a}{9-2a}$ ✓

$-18+4a = 3-3a$

$-21a = -7a$

$a = 3$ ✓

$\frac{a+2-3}{2a-11+2} = \frac{a-1}{2a-9}$

b. Determine the equation of the line AC. (3)

$y-5 = -\frac{2}{3}(x+5)$ ✓ ✓ • m • Co-ord

$y = -\frac{2}{3}x + \frac{5}{3}$ ✓ • answer.

A(−5; 5) mym

Can also Subst (−2, 3)

$C(4,-1)$

$y+1 = -\frac{2}{3}(x-4)$

$y+1 = -\frac{2}{3}x + \frac{8}{3}$

$y = -\frac{2}{3}x + \frac{5}{3}$

B(−2, 3) C(4, −1)

c. Hence, or otherwise, determine the co-ordinates of M, the midpoint of BC. (3)

$M\left(\frac{-2+4}{2}; \frac{3-1}{2}\right)$ ✓ • Subst

$M(1; 1)$ ✓ • x • y

d. Determine the value of p if CD is parallel to the x-axis. (3)

$m_{CD} = 0$ ✓ • m_{CD}

$\frac{p-7+1}{4p-4} = 0$ ✓ • Subst

$p = 6$ ✓ • Answer.

alt // $p-7 = -1$

$p = 6$

[13]

Question 9

Given: $p.\sin\beta - 4 = 0$ and $p.\cos\beta + 3 = 0$ where $p > 0$

a. Explain why $\beta \in [90°; 180°]$. (3)

$\sin\beta = \frac{4}{p}$ ✓ 2nd Quad

$\cos\beta = -\frac{3}{p}$ ✓ 2nd or 3th Quad ✓

∴ 2nd Quad which is 90 – 180

b. Show that $\tan\beta = -\frac{4}{3}$. (2)

$\tan\beta = \frac{\sin\beta}{\cos\beta}$ ✓ $= \frac{4}{p} \div -\frac{3}{p}$ ✓

$= \frac{4}{3}$

c. Determine the numerical value of p. (2)

$p = \sqrt{4^2 + (-3)^2}$ ✓

$= 5$ ✓

[7]

alt

$\tan\beta = \frac{-4}{3}$

r.a $= 53{,}13....$

$\beta = 126{,}87°$; $306{,}13$

OR $\beta = \cancel{233{,}13}$

$\sin\beta = \frac{-4}{p}$

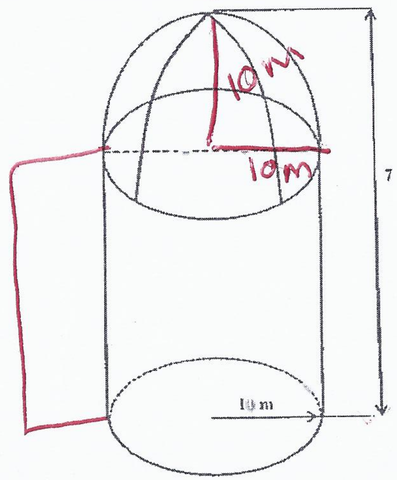
$\sin 126{,}87 = -\frac{4}{p}$

$p = 5$

Question 10

Surface Area = $2\pi rh$	Sphere **Surface Area = $4\pi r^2$**
Volume = $\frac{1}{3}lbh$	**Volume = lbh**

The picture below shows a storage tank in which a farmer stores his grain. The tank is made up of a right cylinder with a hemisphere on top. The perpendicular height of the tank to the top is 7 500 cm and the radius of the tank is 10 m.



Rad = 10
height = 75M
Height of cyl = 65M

a. Calculate the total surface area of the tank (including the base). (5)

SA Hemisphere $= 2\pi r^2$

$= 2(\pi)(10)^2$ ✓

$200\pi = 628{,}32\ m^2$ ✓

$2\pi r^2 + 2\pi rh$

SA cylinder $= 2\pi(10)(65) + \pi(10)^2$ ✓

1400π

$= 4398{,}23\ m^2$ ✓

Total SA $= 1600\pi\ m^2$ ✓

$5026{,}55$

b. Calculate the volume of the tank. (5)

V Hemisphere: $\frac{2}{3}\pi(10)^3$ $\frac{2000}{3}\pi$ $666\frac{2}{3}\pi$

$= 2094,40 \text{ m}^3$ ✓✓

V cylinder $= \pi(10)^2(65)$

6500π

$= 20420,35 \text{ m}^3$ ✓✓

V of tank $= \frac{21500}{3}\pi \text{ m}^3$ ✓

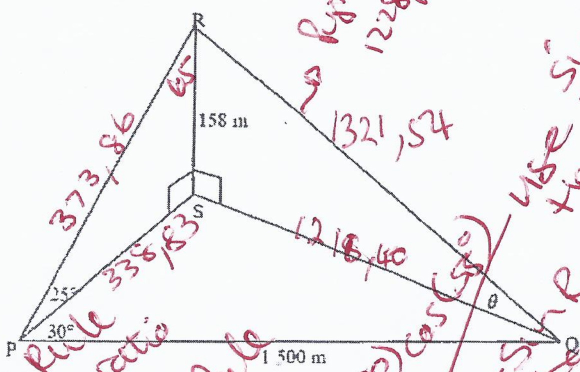
$7166\frac{2}{3}\pi$

$22514,75$

[10]

Question 11

In the diagram below, PQ is a straight line 1 500 m long. RS is a vertical tower 158 m high with P, Q and S points in the same horizontal plane.
The angles of elevation of R from P and Q are 25° and θ.
$S\hat{P}Q = 30°$.



$\frac{PR}{\sin 90} = \frac{158}{\sin 25}$

Pyth
1228,00

use sin ratio
Pyth.
tan ratio

Alt
① Det PR ← Sine Rule or sin ratio
② Det RQ → cosine Rule
$RQ^2 = 373,86^2 + 1500^2 - 2(373,86)(1500)\cos(55)$
$RQ^2 = 1746459,44$
$RQ = 1321,54$
③ Det θ → Sine Rule → Sin ratio
$\frac{1321,54}{\sin 90} = \frac{158}{\sin\theta}$
$\sin\theta = 0,11955...$
$\theta = 6,87°$

① PS — sin Rule or tan ratio
② SQ — cos Rule
③ Q — sin Rule or tan ratio

det RQ with Pyth then Q any ratio

a. Determine the value of θ. (9)

$\frac{PS}{RS} = \tan P\hat{R}S$ ✓

$PS = 158\tan 65$ ✓

$= 338,83$ ✓   (3)

$SQ^2 = PS^2 + PQ^2 - 2PS.PQ\cos 30$

$= 338,83^2 + 1500^2 - 2(338,83)(1500)\cos 30$ ✓

$= 1484499,606$ (3)

$SQ = 1218,40$ ✓

$\frac{RS}{SQ} = \tan\theta$ ✓   ✓   (3)

$\tan\theta = \frac{158}{1218,40}$ ✓ $\theta = 7,39°$

b. Calculate the area of ΔSPQ. (4)

$\frac{1}{2}(SP)(PQ)\sin P$ ✓

✓ $\frac{1}{2}(338,83)(1500)\sin 30$ ✓

$= 127061,25 \text{ m}^2$ ✓

[13]