MEMO

BALLITO

Formula sheet.

Mathematics
Paper 2
FORM 4
July 2018

TIME: 2 hours **TOTAL: 100 marks**

Examiner: Miss M. Eastes **Moderated: Mrs. Gunning, Mrs. Algie**

NAME:

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY BEFORE ANSWERING THE QUESTIONS.

- This question paper consists of 24 pages. This includes an information sheet. Please check that your question paper is complete.
- Answer all questions on this question paper.
- Read and answer all questions carefully.
- It is in your own interest to write legibly and to present your work neatly.
- All necessary working which you have used in determining your answers **must** be clearly shown.
- Approved non-programmable calculators may be used except where otherwise stated. Where necessary give answers correct to 2 decimal places unless otherwise stated.
- Ensure that your calculator is in DEGREE mode.
- Diagrams have not necessarily been drawn to scale.

Questions	**1**	**2**	**3**	**4**	**5**	**6**
Out of	9	11	11	15	6	9
Mark						
Question	**9**	**10**	**11**	**12**	**13**	**TOTAL**
Out of	6	7	9	12	5	100
Mark						

SECTION A
QUESTION 1

The equations of the following lines are given:

AB	$y = {}^{1}/_{2}\,x$
CD	$y = 2$
EF	$3y - 2x - 3 = 0$
GH	$3y + 2x - 6 = 0$
JK	$y = -2x + 4$
LM	$x = \pm 2$

Choose from the above list a line which will satisfy the conditions in each of the following cases. Write only the question number and the letters naming the line.
Each line may only be used once. Show all calculations used to arrive at your answer.

1.1 The line cuts the x-axis at (3; 0). (2)

$3y + 2x - 6 = 0$ GH
$3(0) + 2x - 6 = 0$
$2x = 6$
$x = 3$

1.2 The line is parallel to the line $y = {}^{2}/_{3}\,x + 3$ (2)

$3y = -2x + 6$ $3y = 2x + 3$ EF
$y = \frac{2}{3}x + 1$

1.3 The line has a gradient of zero. (1)

CD

1.4 The line represents all points 2 units from the y-axis. (1)

LM

1.5 The line has an angle of inclination of 116,6° (2)

$\tan\theta = \frac{1}{2}$ $\tan\theta = -2$ JK
$\theta = 63,43^{\circ}$

1.6 The line is perpendicular to line JK. (1)

AB

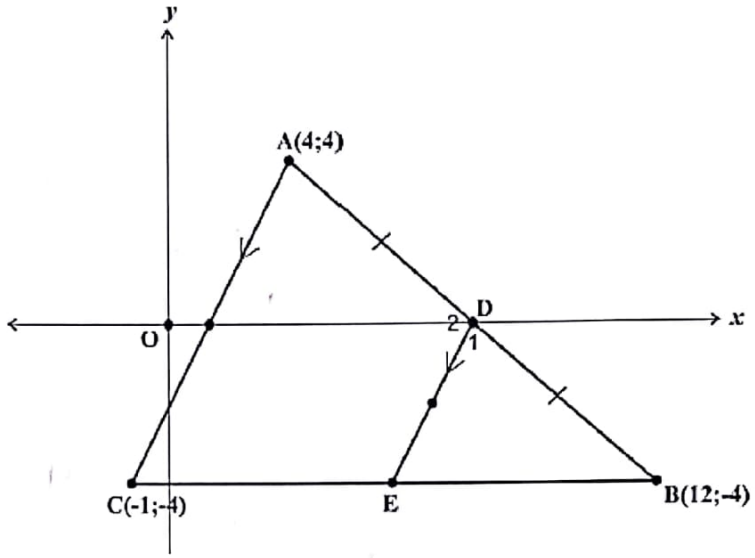
[9]

QUESTION 2

In the diagram below, the coordinates of A(4; 4), B(12; -4) and C(-1; -4) are given.

AC // DE and CEB is a straight line.

D is the midpoint of AB.



Determine:

2.1 the length of AB. Give answer correct to one decimal digit. (2)

$\sqrt{(12-4)^2+(-4-4)^2}$ ✓

$= \sqrt{8^2+(-8)^2}$

$= 11,3$ ✓

2.2 the coordinates of D, the midpoint of AB. (2)

$D\left(\frac{4+12}{2}; \frac{4-4}{2}\right)$ ✓

$D(8; 0)$ ✓

2.3 the equation of the line DE. (4)

$m = \frac{4-(-4)}{4-(-1)}$ ✓m

$= \frac{8}{5}$ ✓a

$y = \frac{8}{5}x + c$

$0 = \frac{8}{5}(8) + c$ ✓ca

$0 = \frac{64}{5} + c$

$-\frac{64}{5} = c$

$y = \frac{8}{5}x - \frac{64}{5}$ ✓ca

2.4 the coordinates of E. (3)

$y = \frac{8}{5}x - \frac{64}{5}$ RP

$-4 = \frac{8}{5}x - \frac{64}{5}$ ✓m

$\frac{11}{2} = x$ ✓a $\quad (\frac{11}{2}, -4)$ ✓ca

[11]

QUESTION 3

3.1 $M(a;2)$ is the midpoint of $A(-2;5)$ and $B(8;-1)$. Find the value of a. (3)

$\frac{-2+8}{2} = a$ ✓m ✓a

$\frac{6}{2} = a$

$3 = a$ ✓ca

$\frac{5-1}{2} = 2$

$\frac{-2+a}{2} = 3$ ✓a

$-2+a = 6$ ✓ca

$a = 8$ ✓ca

RP

3.2 Given the points $P(1;3)$, $Q(3;2)$ and $R(-1;-1)$:
Determine the equation of the straight line through R ~~parallel~~ perpendicular to PQ. (4)

$m_{PQ} = \frac{3-2}{1-3}$

$= \frac{1}{-2}$ ✓a

$m = 2$ ✓ca

$(-1;-1)$

$y = 2x + c$

$-1 = 2(-1) + c$ ✓m

$-1 = -2 + c$

$1 = c$ ✓ca

RP

3.3 Consider the points: $D(-1;1)$, $E(t;t^2)$ *and* $F(1;2t-1)$:
Show that these 3 points are collinear.

CP (4) (5)

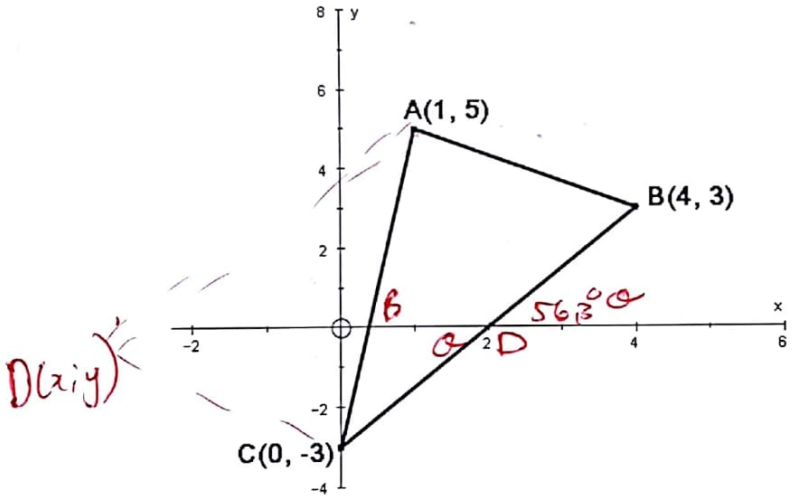
$m_{DE} = \frac{t^2-1}{t+1} = \frac{(t+1)(t-1)}{t+1} = t-1$

$m_{EF} = \frac{t^2-(2t-1)}{t-1} = \frac{(t-1)(t-1)}{t-1} = t-1$

$\therefore m_{EF} = m_{DE}$ (1) (1)
points collinear (2)

QUESTION 4

A(1;5), B(4.3) and C(0;-3) are the vertices of the triangle.



4.1 Determine, using analytical methods, the coordinates of D if ABCD is a parallelogram.

(5) (4) CP

$M_{pt}\ AC = \left(\frac{1+0}{2}; \frac{5-3}{2}\right)$

$= \left(\frac{1}{2}; \frac{2}{2}\right)$ ✓a

$\frac{x+4}{2} = \frac{1}{2}$ ✓CA

$x = 1-4$

$x = -3$

$\frac{y+3}{2} = 1$ ✓CA

$y+3 = 2$

$y = -1$

$(-3;-1)$ ✓a ✓CA

4.2 If the distance between C and F(8;p) is 12 units, find the value(s) of p (to the nearest integer). (5)

CP

$C(0;-3)$

$F(8;p)$ ✓m

$\sqrt{(8-0)^2+(p+3)^2} = 12$ ✓✓cq

$8^2 + p^2 + 6p + 9 = 144$ ✓cq

$p^2 + 6p - 71 = 0$ ✓cq

$p = 6$ or $p = -12$ ✓cq

4.3 Find the size of $A\hat{C}B$ in Δ ABC (7)

CP

$m_{BC} = \frac{3-(-3)}{4-0}$

$= \frac{6}{4}$

$= \frac{3}{2}$ ✓

$m_{AC} = 8$ ✓

$m_{AC} = 8 = 8$ ✓a

$\tan\beta = 8$

$\beta = 82,87^\circ$ ✓

$\tan\theta = \frac{3}{2}$

$\theta = 56,3^\circ$ ✓

$O\hat{D}C = 56,3^\circ$ (vertically opp ∠'s) ✓

$A\hat{C}B = 82,87^\circ - 56,3^\circ$ (ext ∠ of Δ)

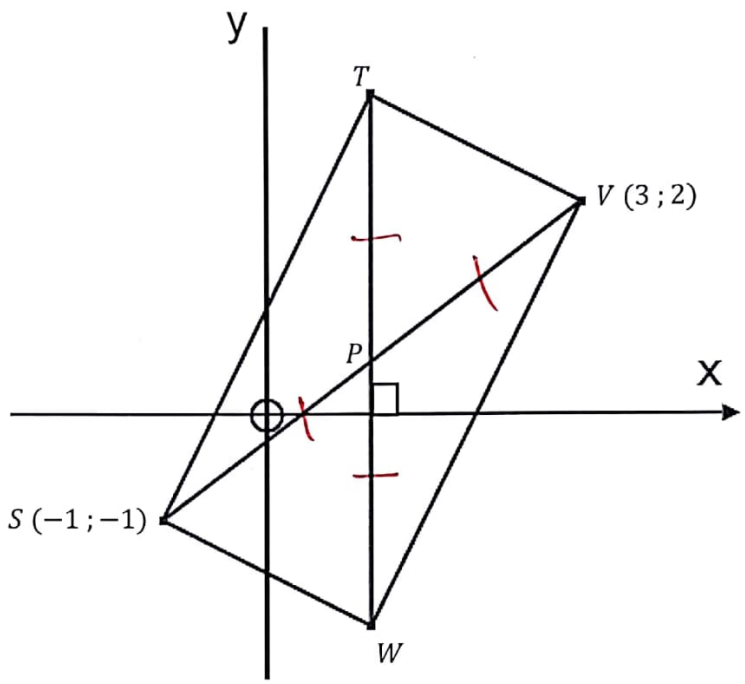
$= 26,57^\circ$ ✓

[15]

QUESTION 5

The diagonals of rectangle $STVW$ are equal in length and bisect each other at P.

Calculate the co-ordinates of T and W.



$P\left(\frac{-1+3}{2}; \frac{-1+2}{2}\right)$ ✓m

$P\left(1; \frac{1}{2}\right)$ ✓a

$SV = \sqrt{(3+1)^2+(2+1)^2}$ ✓m

$= \sqrt{4^2+3^2}$

$= \sqrt{16+9}$

$= \sqrt{25} = 5$ ✓a

$TW = 5$ (diag of rectangles) ✓a

$T(1; 3)$ ✓ca

$W(1; -2)$ ✓ca

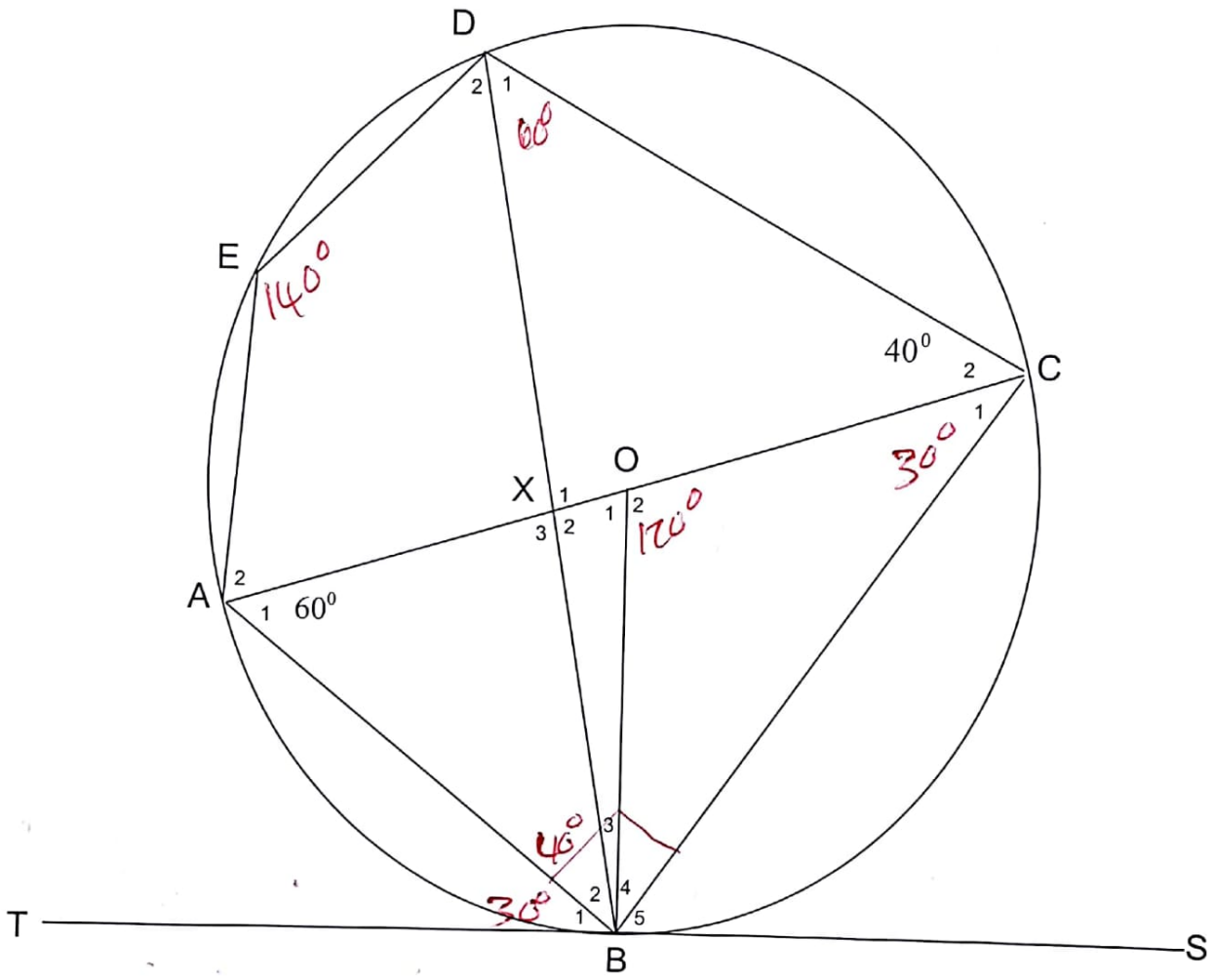
[6]

PS

QUESTION 6

K

A, B, C, D, and E, are points on the circumference of a circle with centre O.
TBS is a tangent to the circle at B. DB and AC intersect at X.
$\hat{A}_1 = 60^0$ *and* $\hat{C}_2 = 40^0$
Give the reasons to the following statements made.



6.1 $\hat{D}_1 = 60^0$

∠'s in same segment ✓ (1)

6.2 $\hat{O}_2 = 120^0$

∠ at centre ✓ (1)

6.3 $\hat{B}_2 + \hat{B}_3 + \hat{B}_4 = 90^0$

∠ in semi ○ ✓ (1)

6.4 $\hat{C}_1 = 30^0$

int ∠'s of △ ✓ (1)

6.5 $\hat{B}_1 = 30^0$ tan chord theorem (1)

6.6 $\hat{B}_4 = 30^0$ isos △ (1)

6.7 $\hat{E} = 140^0$ opp ∠ of cyclic quad (1)

6.8 $\hat{B}_2 = 40^0$ ∠s in same segment / opp ∠s of gm (1)

6.9 $X_3 = 80^0$ int ∠s of △ (1)

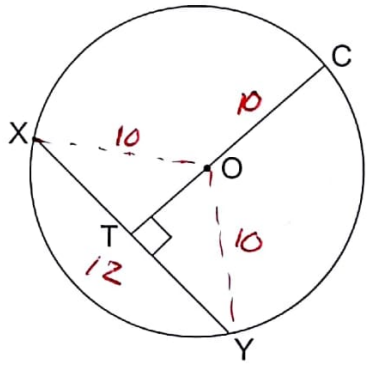
[9]

QUESTION 7

O is the centre of the circle, and COT ⊥ XY.

If OC = 10 and XY = 12,

Find the length of CT.



RT

(6)

OX = OY = 10 (radii) ✓a

XT = 6 ✓a (line ⊥ from centre to chord) ✓a

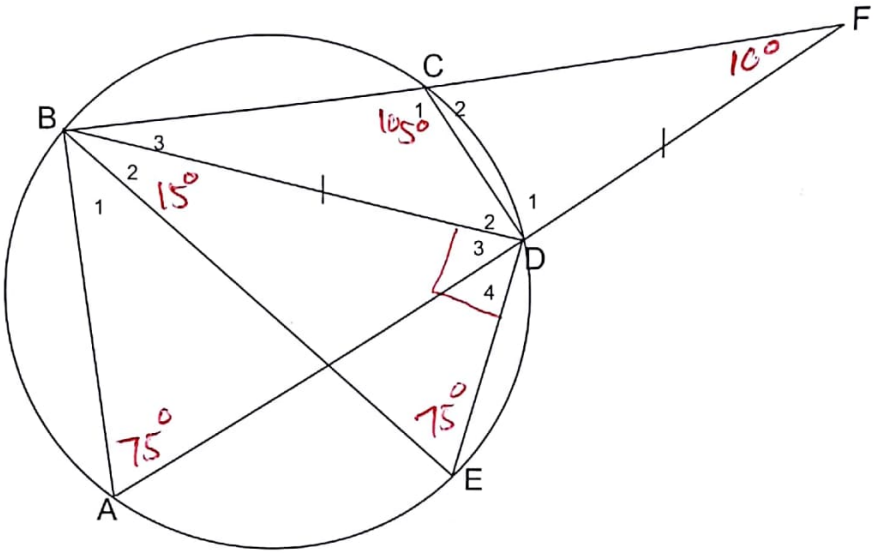
$OT = \sqrt{10^2 - 6^2}$ (pythag) ✓a

$= 8$ ✓m

∴ CT = 10 + 8

= 18 ✓cq

QUESTION 8



BE is the diameter. BD = DF. BCF and ADF are straight lines. $\hat{B}_2 = 15°$ and $\hat{F} = 10°$.
Calculate:

8.1 $\hat{A}$ (3)

$\hat{D}_3 + \hat{D}_4 = 90°$ (∠ in semi ⊙) ✓q
$\hat{E} = 75°$ (ext ∠s of Δ) ✓q
$\hat{A} = 75°$ (∠'s in same segment) ✓q

RP

8.2 $\hat{C}_1$ (1)

$\hat{C}_1 = 105°$ (opp ∠s of cyclic quad) ✓q

RP

8.3 $\hat{D}_4$ (3)

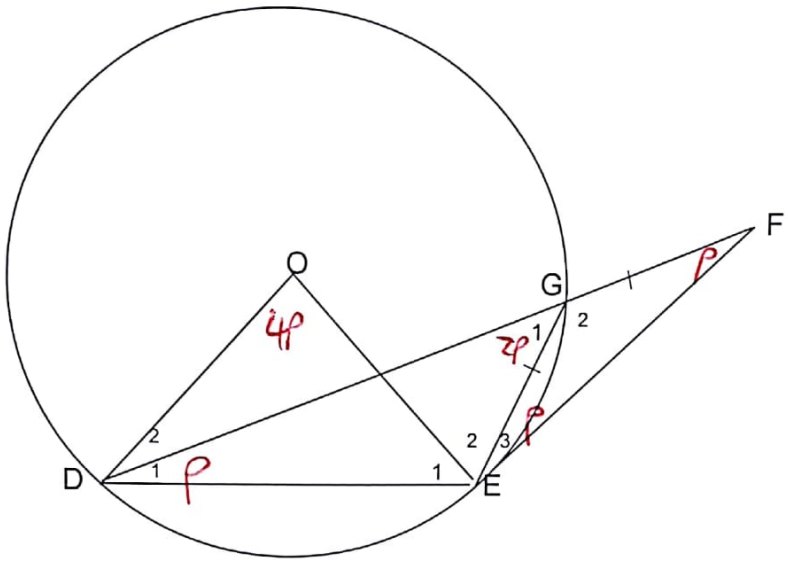
$\hat{B}_3 = 10°$ (∠'s opp = sides) ✓q
$\hat{D}_3 = 20°$ (ext ∠ of Δ) ✓q

$\hat{D}_4 = 90° - 20°$
$= 70°$ ✓cq

RP

[7]

QUESTION 9



FE is a tangent to the circle with centre O. D and F are joined so that EG = GF.

9.1 If $\hat{E}_3$ = p, name, with reasons, two other angles which are equal to p. (2)

$\hat{F} = p$ (isos Δ) ✓a
$\hat{D}_1 = p$ (tan-chord the) ✓a

CP

9.2 Express DÔE in terms of p. (3)

$\hat{G}_1 = 2p$ (ext ∠ of Δ) ✓a
$\hat{O} = 4p$ ✓a (∠ at centre) ✓a

CP

9.3 Express OÊD in terms of p. (4)

$OE = OD$ (radii) ✓a
$\hat{E}_1 = \hat{D}_1 + \hat{D}_2$ (isos Δ) ~~✓a~~
$\hat{E}_1 = \frac{180° - 4p}{2}$ (int ∠'s of Δ) ✓a
$\hat{E}_1 = 90 - 2p$ ✓a

CP

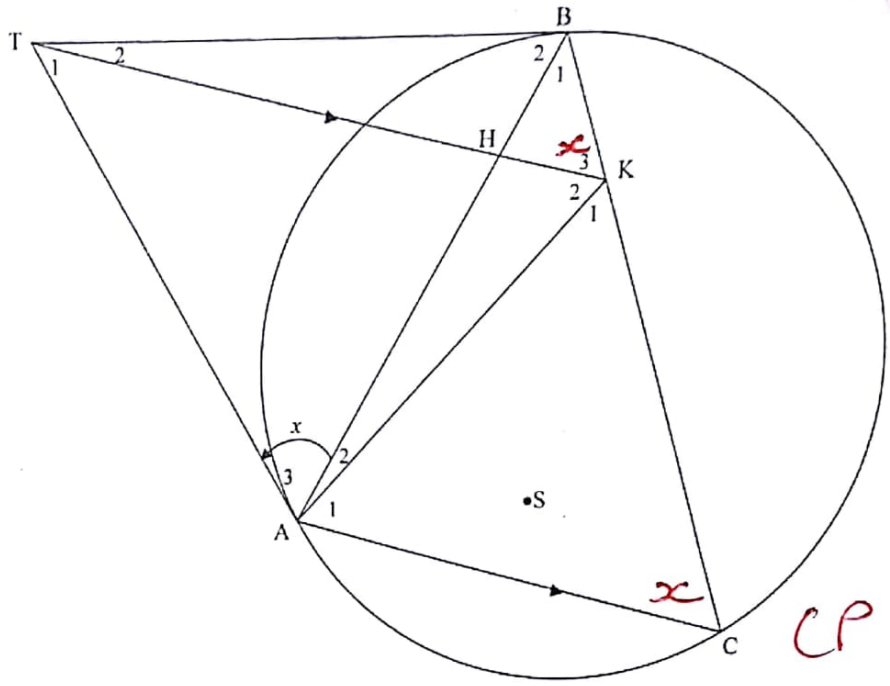
[9]

QUESTION 10

In the diagram below, ΔABC is drawn in the circle.

TA and TB are tangents to the circle.

The straight line THK is parallel to AC with H on BA and K on BC.
AK is drawn.



Let $\hat{A}_3 = x$

10.1 Prove that $\hat{K}_3 = x$. ~~(4)~~ (2)

$\hat{C} = x$ (tan chord th) ✓q
$\hat{K}_3 = x$ (corre ∠'s TK ∥ AC) ✓q

10.2 Prove that AKBT is a cylic quadrilateral. (2)

$\hat{K}_3 = \hat{A}_3$ (proven) ✓q
AKBT is a cyclic quad (converse of ∠'s in same segment) ✓q

10.3 Prove that TK bisects $A\hat{K}B$. ~~(4)~~ (3)

more lines

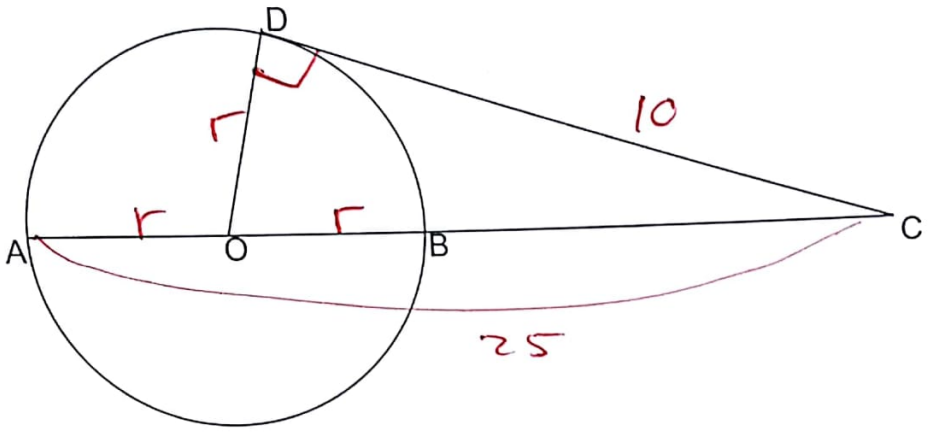
$\hat{K}_2 = \hat{B}_2$ (∠'s in same segment) ✓q
$\hat{B}_2 = \hat{C}$ (tan chord th) ✓q
$\therefore \hat{C} = \hat{K}_3$ (proven)
$\hat{K}_3 = \hat{K}_2$ ✓q

10.4 Prove that TA is a tangent to the circle passing through the points A, K and H. (2)

$\hat{K}_2 = \hat{A}_3$ (proven) ✓q
$\therefore$ TA is a tangent (converse tan chord theorem) ✓q

~~[12]~~ [9]

QUESTION 11



CD is a tangent at D to the circle with centre O. CD = 10 units. AC = 25 units.
If r is the radius, show that r = 10,5 units. (5) PS

$\hat{D} = 90^\circ$ (tan ⊥ rad) ✓a

$r^2 + 10^2 = (25 - r)^2$ ✓m (Pythag) ✓a

$r^2 + 10^2 = 625 - 50r + r^2$ ✓a

$0 = -50r + 525$

$50r = 525$ ✓a

$r = 10,5$