education

Department:
Education
PROVINCE OF KWAZULU-NATAL

NATIONAL SENIOR CERTIFICATE

GRADE 11

MATHEMATICS P2

COMMON TEST

JUNE 2018

MARKS: 100

TIME: 2 hours

This question paper consists of 6 pages.

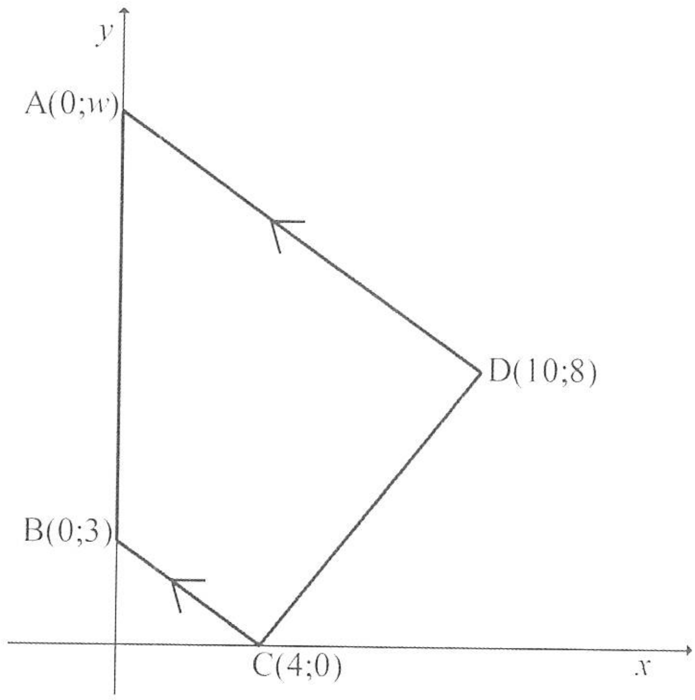
INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions:

1. This question paper consists of 5 questions.
2. Answer ALL the questions.
3. Number the answers correctly according to the numbering system used in this question paper.
4. Clearly show ALL calculations, diagrams, graphs, et cetera, which you have used in determining the answers.
5. Answers only will NOT necessarily be awarded full marks.
6. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
7. If necessary, round off answers to TWO decimal places, unless stated otherwise.
8. Diagrams are NOT necessarily drawn to scale.
9. Write neatly and legibly.

QUESTION 1

A(0 ; w), B(0 ; 3), C(4 ; 0) and D(10 ; 8) are the vertices of a quadrilateral in the Cartesian plane. AD is parallel to BC.



1.1

1.1.1 Calculate the gradient of CD. (2)

1.1.2 Hence, determine the angle of inclination of CD. (2)

1.2 Prove that $B\hat{C}D = 90°$. (3)

1.3

1.3.1 Write down the gradient of AD. (1)

1.3.2 Hence, or otherwise, calculate the value of w. (3)

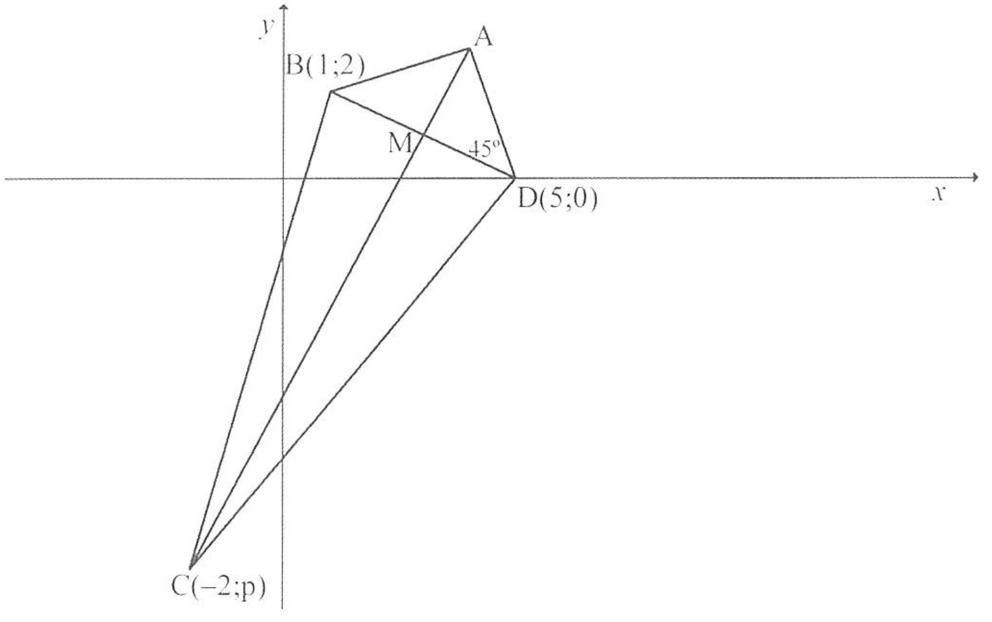
1.4 If it is given that $w = 15\frac{1}{2}$, calculate the length of AD. (3)

1.5 Calculate the area of quadrilateral ABCD. (6)

[20]

QUESTION 2

A, B(1 ; 2), C(–2 ; p) and D(5 ; 0) are the vertices of a KITE.
M is the point of intersection of the diagonals of the kite.
$A\hat{D}B = 45^\circ$.



2.1	Determine the coordinates of M.	(4)
2.2	Calculate the value of p.	(6)
2.3	If $p = -9$, determine the equation of AC.	(5)
2.4	Determine the angle of inclination of AD.	(5)
2.5	Determine the coordinates of A.	(6)
		[26]

QUESTION 3

3.1 If $\tan\theta = \frac{3}{4}$ and $\theta \in [90^\circ; 360^\circ]$, determine the value of $2\sin\theta.\cos\theta$ without the use of a calculator. (4)

3.2 Simplify without the use of a calculator:

3.2.1 $\dfrac{\sin(360^\circ - x) + \cos(90^\circ + x)}{\sin(180^\circ - x) + \tan 540^\circ}$ (5)

3.2.2 $\cos 330^\circ.\tan(-120^\circ) + \sin 73^\circ.\left(\dfrac{1}{\cos 197^\circ}\right)$ (6)

[15]

QUESTION 4

4.1 The identity $\dfrac{(\sin x - \cos x)^2 - 1}{\sin^2 x - 1} = 2\tan x$ is given.

4.1.1 Prove the identity. (5)

4.1.2 For which values of x in the interval $0^\circ \leq x \leq 360^\circ$ will the identity in 4.1.1 not be defined? (3)

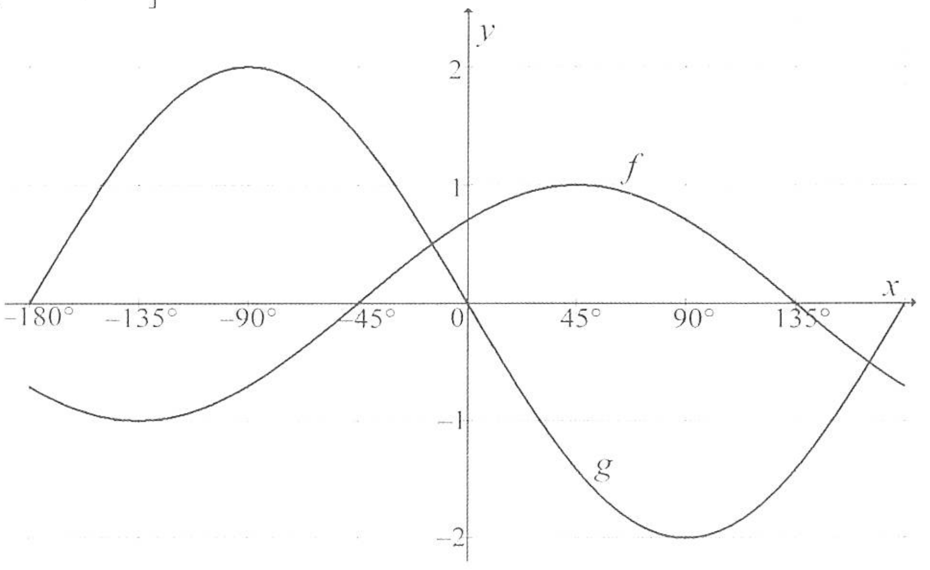
4.2 Solve for x if $\tan(3x + 40^\circ) = -1$ and $x \in [-90^\circ; 90^\circ]$. (5)

4.3 Determine the general solution of $2\sin x = \sqrt{3 + 3\cos x}$. (8)

[21]

QUESTION 5

5.1 The sketch represents the graphs of $f(x) = a\cos(x+b)$ and $g(x) = c\sin x$ for $x \in [-180°; 180°]$.



5.1.1 Write down the values of a, b and c. (3)

5.1.2 If the points of intersection of f and g are $(-14,64°;k)$ and $(m;-0,51)$, write down the values of k and m. (4)

5.1.3 For which values of x in the interval $[-180°;0°]$ will

(a) $f(x) - g(x) < 0$? (2)

(b) $f(x).g(x) \geq 0$? (2)

5.1.4 Determine the minimum value of $h(x)$ if $h(x) = f(x) + 2$. (2)

5.2

5.2.1 Draw a sketch graph of $m(x) = \tan 2x$ for $x \in [0°;90°]$. (3)

5.2.2 Describe how the graph m has to be transformed to form the graph n where $n(x) = \tan(2x + 50°)$. (2)

[18]

TOTAL MARKS: 100

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MARKING GUIDELINE

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QUESTION 1

1.1.1	$m_{CD}=\dfrac{y_2-y_1}{x_2-x_1}$ $=\dfrac{8-0}{10-4}$ $=\dfrac{4}{3}$	✓ substituting in gradient formula ✓ answer (2)
1.1.2	$\tan\theta=m_{CD}$ $=\dfrac{4}{3}$ $\theta=53{,}13^\circ$	✓ $\tan\theta=\dfrac{4}{3}$ ✓ answer (2)
1.2	$m_{BC}=\dfrac{3-0}{0-4}$ $=-\dfrac{3}{4}$ $m_{CD}\times m_{BC}=\dfrac{4}{3}\times-\dfrac{3}{4}$ $=-1$ Therefore $B\hat{C}D=90^\circ$.	✓ gradient of BC ✓ multiplying gradients ✓ answer of -1 and concluding (3)
1.3.1	$m_{AD}=-\dfrac{3}{4}$	✓ answer (1)
1.3.2	$-\dfrac{3}{4}=\dfrac{w-8}{0-10}$ $30=4(w-8)$ $30=4w-32$ $w=\dfrac{31}{2}$ $=15\frac{1}{2}$	✓ substitution in gradient formula ✓ simplification ✓ answer (3)
1.4	$AD=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ $=\sqrt{(0-10)^2+(\frac{31}{2}-8)^2}$ $=\sqrt{\dfrac{625}{4}}$ $=\dfrac{25}{2}$ $=12\frac{1}{2}$ units	✓ substitution into quadratic formula ✓ simplification ✓ answer (3)

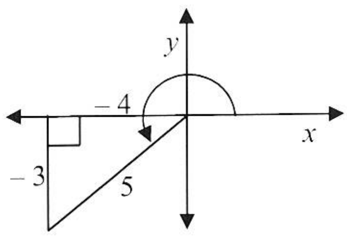
1.5	$BC=\sqrt{(0-4)^2+(3-0)^2}=5$ $CD=\sqrt{(10-4)^2+(8-0)^2}=10$ Area of ABCD = area of trapezium = ½(sum of parallel sides) × height $=\frac{1}{2}(5+12\frac{1}{2})(10)$ $=87\frac{1}{2}$ square units	✓ length of BC ✓ length of CD ✓ formula ✓ substitution of $(5+12\frac{1}{2})$ ✓ substitution of 10 ✓ answer (6)
	OR	**OR**
	$BC=\sqrt{(0-4)^2+(3-0)^2}=5$ $CD=\sqrt{(10-4)^2+(8-0)^2}=10$ Area of ABCD = area of rectangle + area of triangle $=(\ell\times b)+(\frac{1}{2}bh)$ $=(5\times10)+\left[\frac{1}{2}\times10\times(12\frac{1}{2}-5)\right]$ $=87\frac{1}{2}$ square units	✓ length of BC ✓ length of CD ✓ substitution of (5×10) ✓✓ $\left[\frac{1}{2}\times10\times(12\frac{1}{2}-5)\right]$ ✓ answer (6)
	OR	**OR**
	$BC=\sqrt{(0-4)^2+(3-0)^2}=5$ $CD=\sqrt{(10-4)^2+(8-0)^2}=10$ Area of ABCD = area of ΔBCD + area of ΔABD $=(\frac{1}{2}\times5\times10)+\left[\frac{1}{2}\times(15\frac{1}{2}-3)\times10\right]$ $=25+62\frac{1}{2}$ $=87\frac{1}{2}$ square units	✓ length of BC ✓ length of CD ✓ $(\frac{1}{2}\times5\times10)$ ✓ ✓ $\left[\frac{1}{2}\times(15\frac{1}{2}-3)\times10\right]$ ✓ answer (6)
		[18]

QUESTION 2

2.1	$x_M = \frac{x_1 + x_2}{2}$ $= \frac{1+5}{2}$ $= 3$ $y_M = \frac{y_1 + y_2}{2}$ $= \frac{2+0}{2}$ $= 1$ M(3;1)	✓ substitution for x_M ✓ answer for x_M ✓ substitution for y_M ✓ answer for y_M (4)
2.2	BC = CD $\sqrt{(-2-1)^2 + (p-2)^2} = \sqrt{(-2-5)^2 + (p-0)^2}$ $\sqrt{9 + p^2 - 4p + 4} = \sqrt{49 + p^2}$ $\sqrt{p^2 - 4p + 13} = \sqrt{p^2 + 49}$ $p^2 - pk + 13 = p^2 + 49$ $-4p = 36$ $p = -9$	✓ BC = CD ✓ LHS substitution in distance formula ✓ RHS substitution in distance formula ✓ simplifying ✓ squaring both sides ✓ answer (6)
2.3	$m_{CM} = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{1-(-9)}{3-(-2)}$ $= 2$ $y = 2x + c$ Substitute $(-2;-9)$: $-9 = 2(-2) + c$ $c = -5$ $y = 2x - 5$	✓ substitute in gradient formula ✓ value of gradient ✓ substitute in straight line formula ✓ value of c ✓ equation (5)
2.4	$m_{BD} = \frac{0-2}{5-1}$ $= -\frac{1}{2}$ $\tan B\hat{D}X = -\frac{1}{2}$ $B\hat{D}X = 180^\circ - 26,57^\circ$ $= 153,43^\circ$ Angle of inclination of AD $= 153,43^\circ - 45^\circ$ $= 108,43^\circ$	✓ gradient of BD ✓ $\tan B\hat{D}X = m_{BD}$ ✓ angle of inclination of BD ✓ $153,43^\circ - 45^\circ$ ✓ answer (5)

2.5	$m_{AD} = \tan 108,43^\circ$ $= -3$ Equation of AD: $y = -3x + c$ Substitute $(5;0)$: $0 = -3(5) + c$ $c = 15$ $y = -3x + 15$ Solve simultaneous equations for AC and AD: $2x - 5 = -3x + 15$ $5x = 20$ $x = 4$ $y = 3$ A(4;3)	✓ gradient of AD ✓ substituting (5 ; 0) ✓ equation of AD ✓ solving simultaneously ✓ value of x ✓ value of y (6)
	OR DM = AM = BM [sides opposite equal angles] $= \sqrt{5}$ AB = AD $= \sqrt{10}$ [Theorem of Pythagoras] AB = AD $\sqrt{(x-1)^2 + (y-2)^2} = \sqrt{(x-5)^2 + (y-0)^2}$ $x^2 - 2x + 1 + y^2 - 4y + 4 = x^2 - 10x + 25 + y^2$ $8x - 4y = 20$ $y = 2x - 5$...........line 1 $AD = \sqrt{10}$ $\sqrt{(x-5)^2 + (y-0)^2} = \sqrt{10}$ $x^2 - 10x + 25 + y^2 = 10$ ....line 2 $x^2 - 10x + 25 + (2x-5)^2 = 10$ $x^2 - 10x + 25 + 4x^2 - 20x + 25 = 10$ $5x^2 - 30x + 40 = 0$ $x^2 - 6x + 8 = 0$ $(x-4)(x-2) = 0$ $x = 2$ or $x = 4$ N/A $y = 3$	**OR** ✓ DM = AM = BM = $\sqrt{5}$ ✓ $\sqrt{(x-1)^2 + (y-2)^2} = \sqrt{(x-5)^2 + (y-0)^2}$ ✓ simplification to $y = 2x - 5$ ✓ $\sqrt{(x-5)^2 + (y-0)^2} = \sqrt{10}$ ✓ $x = 4$ ✓ $y = 3$ (6)
		[26]

QUESTION 3

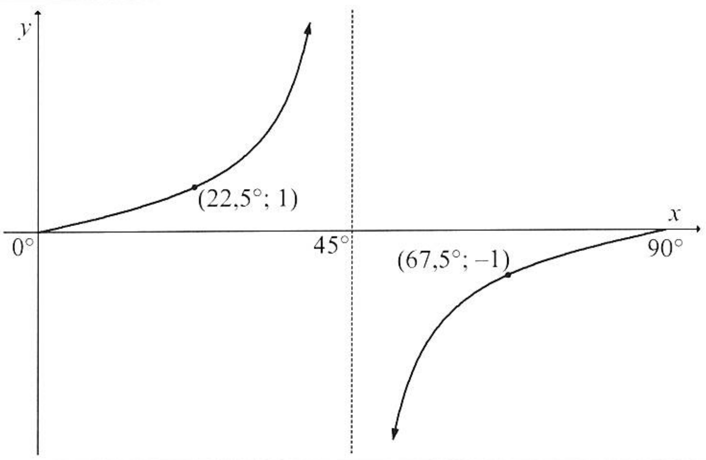
3.1	 $2\sin\theta\cos\theta$ $=2\left(-\frac{3}{5}\right)\left(-\frac{4}{5}\right)$ $=\frac{24}{25}$	✓ sketch ✓ substitution of $-\frac{3}{5}$ ✓ substitution of $-\frac{4}{5}$ ✓ answer (4)
3.2.1	$\frac{\sin(360^\circ-x)+\cos(90^\circ+x)}{\sin(180^\circ-x)+\tan 540^\circ}$ $=\frac{-\sin x+(-\sin x)}{\sin x+\tan 180^\circ}$ $=\frac{-2\sin x}{\sin x+0}$ $=-2$	✓ $-\sin x$ and ✓ $-\sin x$ in numerator ✓ $\sin x$ and ✓ $\tan 180^\circ$ in denominator ✓ answer (5)
3.2.2	$\cos 330^\circ.\tan(-120^\circ)+\sin 73^\circ.\left(\frac{1}{\cos 197^\circ}\right)$ $=\cos 30^\circ.-\tan 120^\circ+\sin 73^\circ.\left(\frac{1}{-\cos 17^\circ}\right)$ $=\cos 30^\circ.\tan 60^\circ-\frac{\sin 73^\circ}{\sin 73^\circ}$ $=\left(\frac{\sqrt{3}}{2}\right).\left(\frac{\sqrt{3}}{1}\right)-1$ $=\frac{3}{2}-1$ $=\frac{1}{2}$	✓ $\cos 30^\circ$ ✓ $-\cos 17^\circ$ ✓ $\tan 60^\circ$ ✓ $-\frac{\sin 73^\circ}{\sin 73^\circ}$ (**OR**: $-\frac{\cos 17^\circ}{\cos 17^\circ}$) ✓ special angle values ✓ answer (6)
		[15]

QUESTION 4

4.1.1	$\dfrac{(\sin x-\cos x)^2-1}{\sin^2 x-1}$ $=\dfrac{\sin^2 x-2\sin x\cos x+\cos^2 x-1}{\sin^2 x-1}$ $=\dfrac{-2\sin x\cos x+\sin^2 x+\cos^2 x-1}{\sin^2 x-1}$ $=\dfrac{-2\sin x\cos x+1-1}{-(1-\sin^2 x)}$ $=\dfrac{-2\sin x\cos x}{-\cos^2 x}$ $=\dfrac{2\sin x}{\cos x}$ $=2\tan x$	✓ multiplying out ✓ applying identity $\sin^2 x+\cos^2 x=1$ ✓ $-(1-\sin^2 x)$ ✓ applying identity $1-\sin^2 x=\cos^2 x$ ✓ simplification (5)
4.1.2	$\sin^2 x-1=0$ $\sin^2 x=1$ $\sin x=-1$ or $\sin x=1$ $x=270^\circ$ or $x=90^\circ$	✓ $\sin^2 x-1=0$ ✓ 270° ✓ 90° (3)
4.2	$\tan(3x+40^\circ)=-1$ reference angle: 45° $3x+40^\circ=180^\circ-45^\circ+n.360^\circ$ or $3x+40^\circ=360^\circ-45^\circ+n.360^\circ$ $3x=95^\circ+n.360^\circ$ $3x=275^\circ+n.360^\circ$ $x=31,67^\circ+n.120^\circ$ $x=91,67^\circ+n.120^\circ$ $x=-88,33^\circ$ or $31,67^\circ$ $x=-28,33^\circ$ where $n\in Z$ **OR** $3x+40^\circ=180^\circ-45^\circ+n.180^\circ$ $3x=95^\circ+n.180^\circ$ $x=31,67^\circ+n.60^\circ$ $x=-88,33^\circ$ or $-28,33^\circ$ or $31,67^\circ$ where $n\in Z$	✓ $3x+40^\circ=180^\circ-45^\circ+n.360^\circ$ ✓ $3x+40^\circ=360^\circ-45^\circ+n.360^\circ$ ✓ $-88,33^\circ$ ✓ $31,67^\circ$ ✓ $-28,33^\circ$ (5) **OR** ✓ $180^\circ-45^\circ$ ✓ $+n.180^\circ$ ✓ $-88,33^\circ$ ✓ $31,67^\circ$ ✓ $-28,33^\circ$ (5)

4.3	$2\sin x = \sqrt{3+3\cos x}$ $4\sin^2 x = 3+3\cos x$ $4(1-\cos^2 x) = 3+3\cos x$ $4-4\cos^2 x = 3+3\cos x$ $4\cos^2 x+3\cos x-1=0$ $(4\cos x-1)(\cos x+1)=0$ $\cos x = \frac{1}{4}$ or $\cos x = -1$ $x = 75{,}52^\circ + n.360^\circ$ or $x = 284{,}48^\circ + n.360^\circ$ or $x = 180^\circ + n.360^\circ$ for $n \in Z$	✓ squaring both sides ✓ $1-\cos^2 x$ ✓ standard form ✓ $\cos x = \frac{1}{4}$ or $\cos x = -1$ ✓✓✓ answers ✓ $n \in Z$ (8)
		[21]

QUESTION 5

5.1.1	$a=1$ $b=-45^\circ$ $c=-2$	✓ $a=1$ ✓ $b=-45^\circ$ ✓ $c=-2$ factors (3)
5.1.2	$k=0{,}51$ $m=165{,}36^\circ$	✓✓ $k=0{,}51$ ✓✓ $m=165{,}36^\circ$ (4)
5.1.3(a)	$-180^\circ < x < -14{,}64^\circ$	✓✓ answer (2)
5.1.3(b)	$-45^\circ \le x \le 0^\circ$	✓✓ answer (2)
5.1.4	$-1+2=1$	✓✓ answer (2)
5.2.1		✓ shape ✓ asymptote at $x=45^\circ$ ✓ indicated $(22{,}5^\circ;1)$ and $(67{,}5^\circ;-1)$ (3)
5.2.2	m has to be translated (shifted) by 25° to the left.	✓ shifted to the left ✓ by 25° (2)
		[18]

TOTAL: 100