Mathematics Paper 2 November 2016

FORM 4

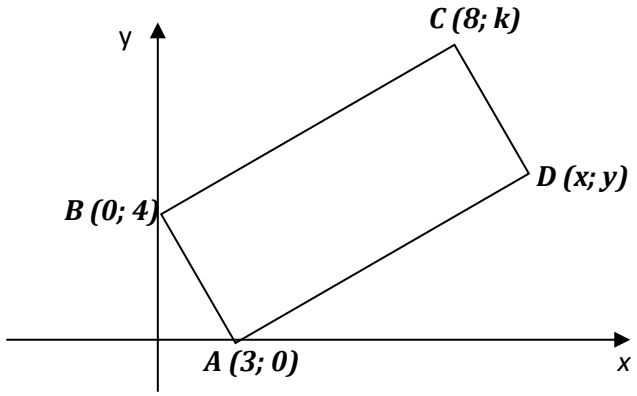
Examiner:	A Gunning	**Moderators:**	P Denissen, C Mundy
Time:	3 hours	**Marks:**	150

NAME:

	SECTION A						SECTION B									
Ques No	1	2	3	4	5	6	7	8	9	10	11	12	13	14		%
Out of	11	14	8	9	10	13	9	7	5	10	8	26	10	10	150	100
Mark																

- All questions are to be answered in this booklet.
- This question paper consists of 24 pages. Included in this, is a list of useful formulae. Please check that your question paper is complete.
- Read and answer all questions carefully.
- It is in your own interest to write legibly and to present your work neatly.
- **All necessary working which you have used in determining your answers must be clearly shown.**
- Approved non-programmable calculators may be used except where otherwise stated. Where necessary give answers **correct to 2 decimal places**.
- Diagrams have not necessarily been drawn to scale.

Question 1



The points *A* (3, 0) and *B* (0, 4) are two vertices of the rectangle *ABCD*, as shown above.

(*a*) Write down the gradient of *AB* and hence the gradient of *BC*. (3)

The point *C* has coordinates (8, *k*), where *k* is a positive constant.

(*b*) Find the length of *BC* in terms of *k*. (2)

Given that the length of *BC* is 10 and using your answer to part (*b*),

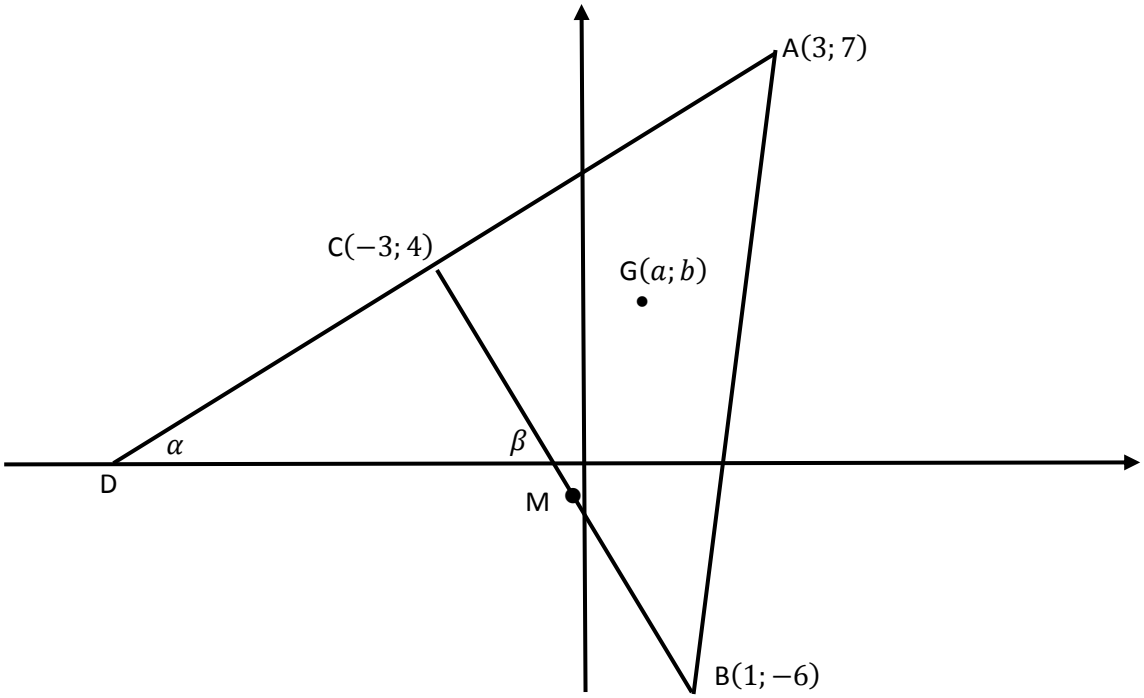
(*c*) find the value of *k*, (4)

(*d*) write down the coordinates of *D.* (2)

_____

_____ [11]

Question 2



In the diagram above, A, B and C are the vertices of a triangle. AC is extended to cut the x-axis at D. (The diagram is not drawn according to scale.)

(a) Calculate the gradient of:

i) AD (2)

_____

_____

ii) BC (2)

_____

_____

_____

_____

(b) Calculate the sizes of α and β and hence write down the size of $D\hat{C}B$. (3)

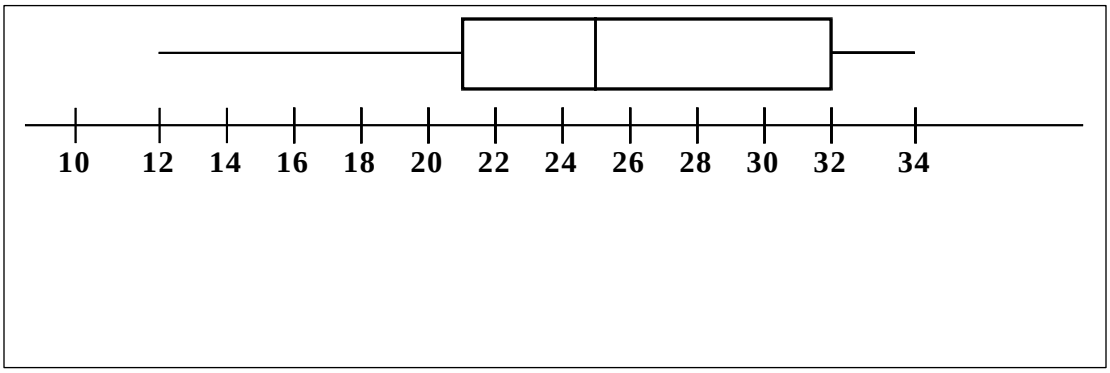
(c) Determine the **coordinates** of M, the midpoint of BC. (2)

(d) If G$(a; b)$ is a point such that A, G and M are collinear, show that $b = 2a + 1$. (5)

[14]

Question 3

A driver, Chris, of a courier motorcycle, recorded the distance (in km) he had travelled on 15 trips. The 5-number summary of his data is: $(12; 21; 25; 32; 34)$ and the box-and-whisker diagram for his travel appears below.



Another driver in the same company, Colin, also recorded the distances (in kilometres) he travelled on 15 trips. The data appear alongside:

24	19	21	27	17	20	32	22
26	18	13	23	30	10	13	

(a) Determine the median for the data. (2)

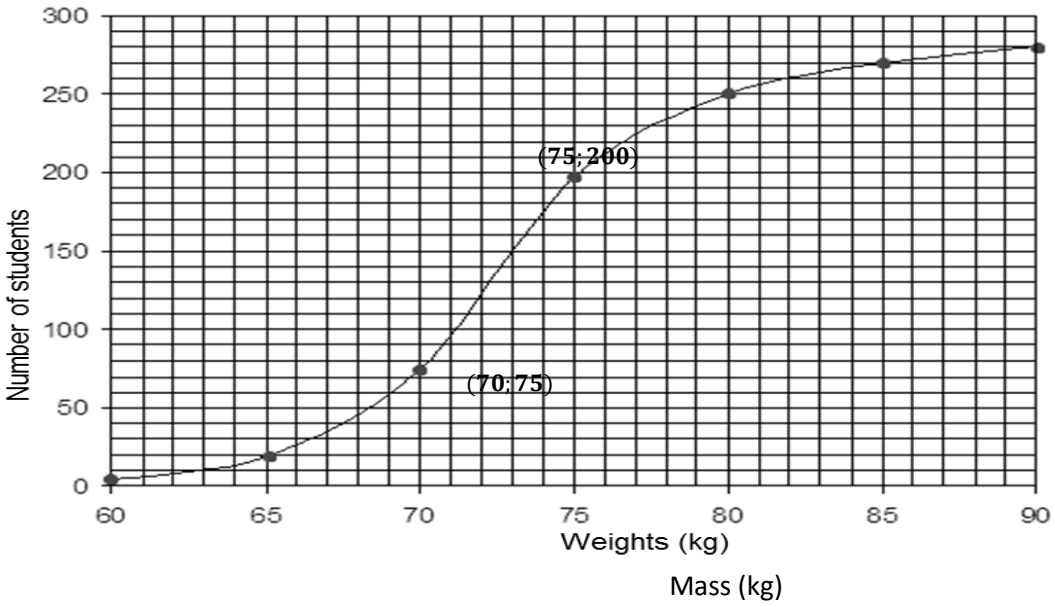
(b) Determine the 5-number summary for the data of Colin's travels and draw a box-and-whisker diagram in the space provided below the box-and-whisker diagram above. (4)

(c) Carefully analyse the box and whisker diagrams for the two drivers, and comment on the differences or similarities, if any, between the distances covered by each on the 15 trips. (2)

[8]

Question 4

The cumulative frequency curve below shows the mass of 280 students.



(a) How many students have a mass of less than 73 kilograms? (1)

_____

_____

(b) Find the number of students that have a mass between 80 and 85 kilogram. (2)

_____

_____

(c) Complete the following frequency table. (2)

Mass (m)	$60 \leq m < 65$	$65 \leq m < 70$	$70 \leq m < 75$	$75 \leq m < 80$	$80 \leq m < 85$	$85 \leq m < 90$
Frequency	20					10

(d) Hence, calculate the estimated mean of the students' mass, using a calculator. (2)

_____

(e) Write down the standard deviation of the mass of the students. (1)

_____

_____

(f) After a relaxing Matric Rage, all the students have gained 2 kilograms of mass each.

Which of the following statements would be correct for the given data set?

(i) The standard deviation of the data set changes and the mean stays the same.

(ii) The standard deviation as well as the mean changes.

(iii) The standard deviation stays the same and the mean changes. (1)

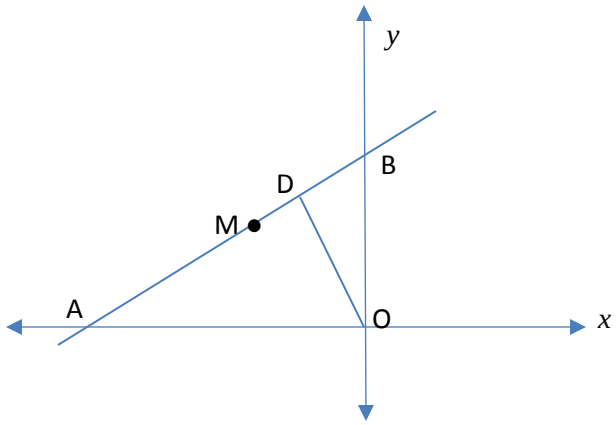
_____

_____

[9]

Question 5

In the figure, the straight line with equation $2y - x - 10 = 0$ cuts the x-axis at A and the y-axis at B. M is the midpoint of AB and $OD \perp AB$.



Determine:

(a) the c**oordinates** of A, B and M. (4)

(b) the **coordinates** of $\widehat{D}$ (6)

[10]

Question 6

(a) Given $sin\alpha = \frac{8}{17}$ and $90° < \alpha < 270°$, with the aid of a sketch and without the use of a calculator, determine the value of:

(i) $\tan \alpha$ (3)

(ii) $\sin(90° - \alpha)$ (2)

(b) If $\sin 40° = p$, determine each of the following in terms of p (without the use of a calculator)

(i) $\sin 140°$ (1)

(ii) $\cos(-40°)$ (3)

(iii) $\cos 50° \sin 220°$ (4)

[13]

Section B

Question 7

Simplify each of the following as far as possible. You may not use a calculator.

(a) $\dfrac{\sin(180°-x)}{\cos(90°+x)+\sin(360°-x)}$ (4)

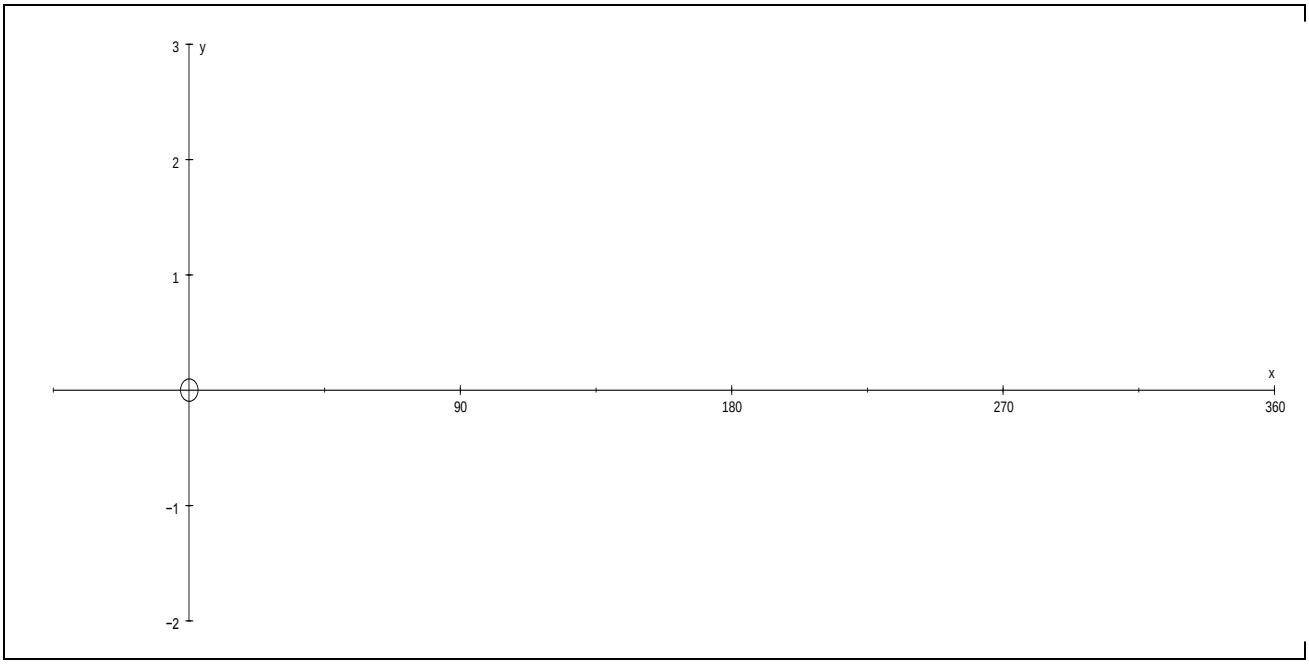
(b) $\dfrac{\cos 115°.\cos 214°}{\cos(-65°).\sin 236°}$ (5)

[9]

Question 8

Given: $f(x) = 1 + \sin x$ and $g(x) = \cos 2x$

(a) Draw sketch graphs of f and g for $x \in [0°; 360°]$ (5)



(b) Using your graphs, read off the values of x for which $f(x) \leq g(x)$ (2)

[7]

Question 9

The straight line passing through $A(-2;-3)$ and $B(7;2)$ is parallel to the straight line with equation $rx - 3y + 5 = 0$. Calculate the value of r. (5)

[5]

Question 10

(a) Given: $\cos\hat{G} = 0{,}726$ and $180^\circ < \hat{G} < 360^\circ$

(i) Use a calculator to solve for $\hat{G}$, correct to one decimal place. (2)

(ii) Hence determine the value of $\tan\left(\frac{2}{3}\hat{G} + 100^\circ\right)$, correct to one decimal place. (1)

(b) You are given $6\,sin^2\theta + cos\theta = 4$

(i) Show that this can be rewritten as $6\,cos^2\theta - \cos\theta - 2 = 0$. (2)

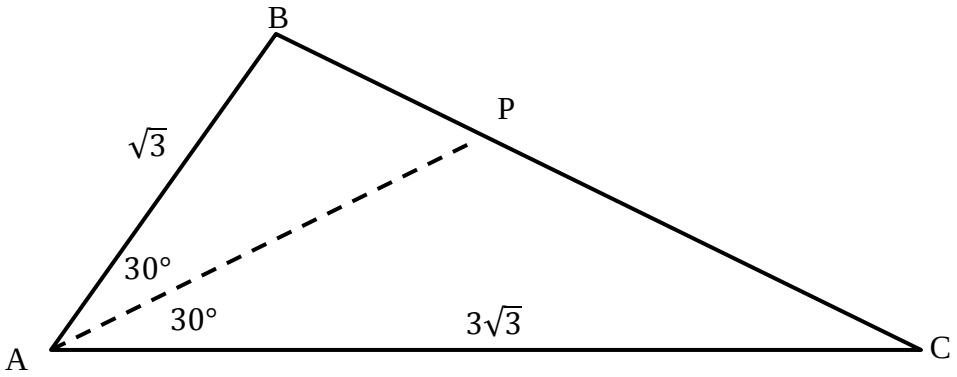
(ii) Hence find the general solution for $6\,sin^2\theta + cos\theta = 4$. (5)

[10]

Question 11

In the diagram below, ABC is a triangular piece of card and AP is the angle bisector of $B\hat{A}C$.

$AB = \sqrt{3}\ cm$, $AC = 3\sqrt{3}\ cm$ and $B\hat{A}P = P\hat{A}C = 30°$



(a) Determine the area of ΔABC leaving your answer in simplest surd form. (2)

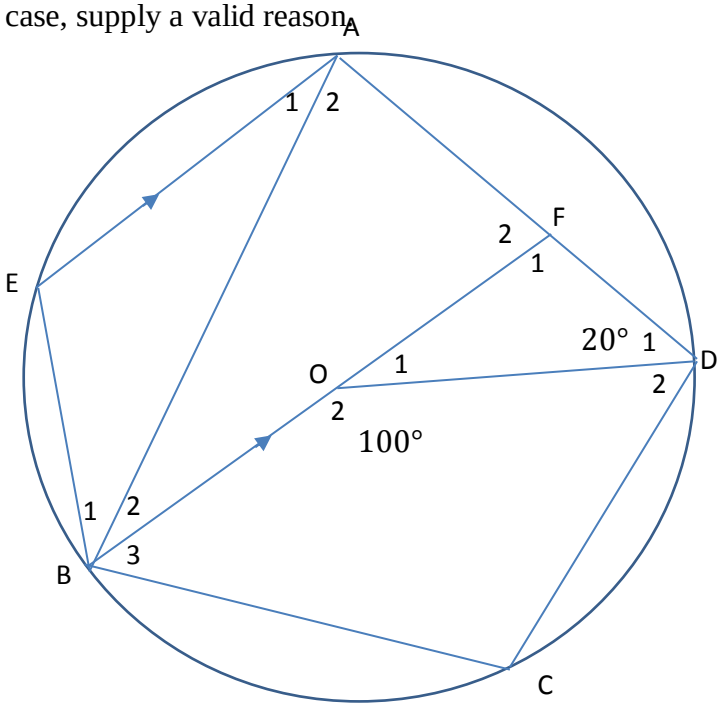
(b) $Area\ \Delta ABC = \ area\ \Delta ABP + area\ \Delta APC$. Use this and the answer you found in (a) to determine the length of AP (3)

(c) Hence determine the length of BP, leaving your answer in the simplest surd form. (3)

[8]

Question 12

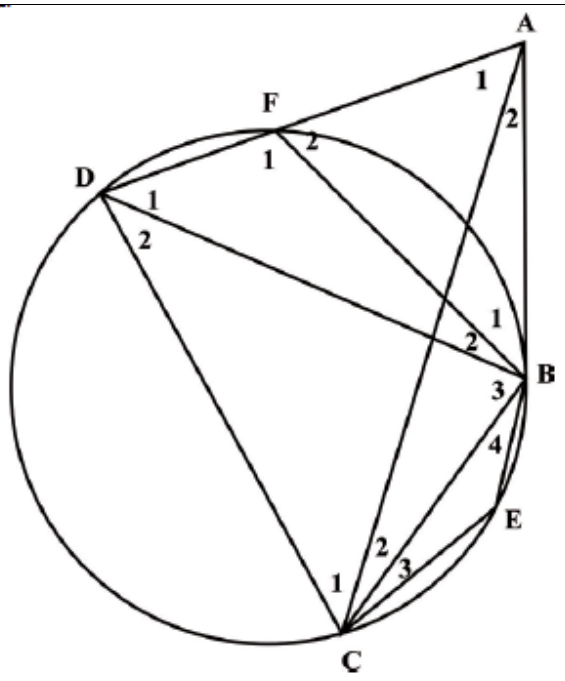
(a) In the figure, O is the centre of the circle $AEBCD$, with line $BOF//EA$, with F on AD. It is given that $B\hat{O}D = 100°$ and $\hat{D}_1 = 20°$. The sizes of the angles are as indicated. In each case, supply a valid reason.



Statement	Reasons
$\hat{A}_2 = 50°$	
$\hat{O}_1 = 80°$	
$\hat{F}_1 = 80°$	
$\hat{A}_1 = 30°$	
$\hat{B}_2 = 30°$	

(5)

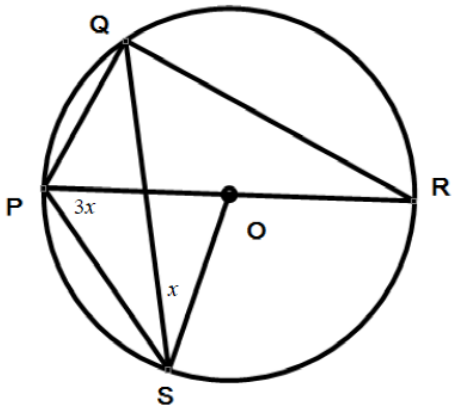
(b) In the diagram below, AB is a tangent to the circle passing through B, E, C and D. AD cuts the circle at F. AC is drawn. A list of statements is given. Give reasons for the statements which are correct. If a statement is not necessarily correct, write "not correct" in the space provided for the reason. (3)



Statement	Reason
$\widehat{C_1} + \widehat{C_2} = \widehat{F_2}$	
$\widehat{D_2} + \hat{E} = 180°$	
$\widehat{B_1} = \widehat{D_1}$	

(c) P, Q and R are points on the circumference of a circle centre O.

PR is a diameter of the circle. $Q\hat{S}O = x$ and $O\hat{P}S = 3x$.



(i) Express each of the following angles in terms of x.

Give a reason for each of your answers.

a. $S\hat{Q}R$ (2)

(ii) $P\hat{Q}S$ (2)

(iii) $P\hat{S}Q$ (3)

(iv) $S\hat{O}P$ (2)

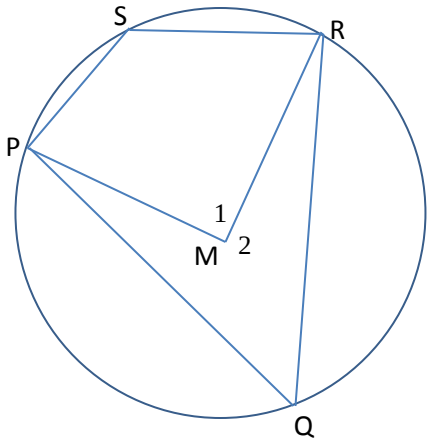
(v) $P\hat{R}Q$ (2)

(vi) $Q\hat{P}R$ (2)

(d) In the figure, P, Q, R and S are points on a circle with centre M.

It is given that $\widehat{M_1} = 4x + 40°$ and $\hat{S} = 5x + 20°$,

Calculate, stating reasons, the size of $\hat{Q}$. (5)

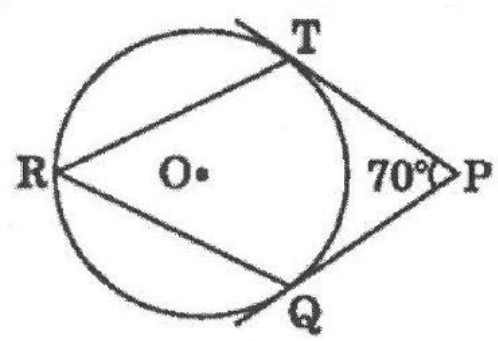


[26]

Question 13

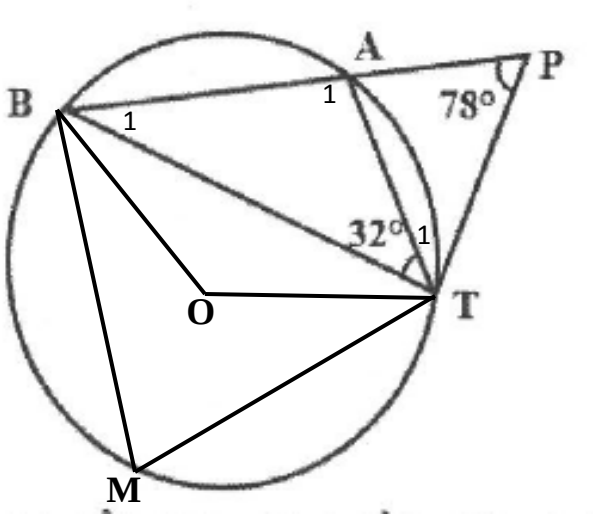
(a) In the diagram below, O is the centre of the circle. PT and PQ are tangents to the circle.

$T\hat{P}Q = 70°$. (Hint: you may need to draw a line.)



Determine the size of $T\hat{R}Q$ (5)

(b) In the diagram below, A, B, M and T are four points on a circle. The tangent at T meets BA produced at P.



Given that $A\hat{T}B = 32°$ and that $A\hat{P}T = 78°$, calculate the angle subtended by BT at the centre of the circle.

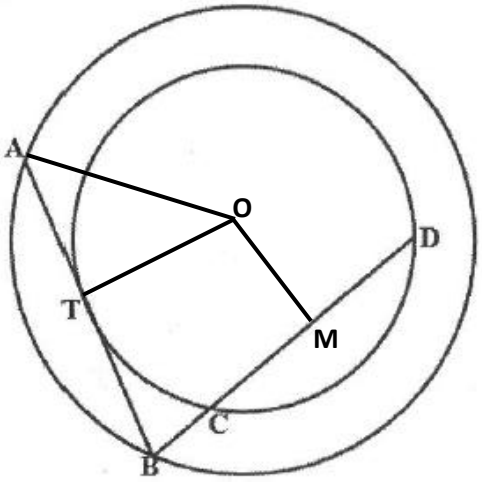
(Hint: let $\widehat{B_1} = x$, and solve for x first.) (5)

[10]

Question 14

In the diagram below, the radii of 2 concentric circles is 9 cm and 15 cm.

Tangent AB touches the smaller circle at T. OT and OM are drawn.



(a) What can you deduce about AB and OT. Give a reason for your answer. (1)

(b) Write down the lengths of OA and OT. (1)

(c) Hence, calculate the length of AT. (2)

(d) Now given that $OM \perp CD \; and \; CD = AT - 2$, write down the length of CD. (1)

(e) Join OC. Hence, or otherwise, calculate the length of OM. (3)

(f) Join OB. Hence, determine the length of BC. (2)

[10]

Useful formulae

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad M\left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c \qquad y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad m = \tan\theta$$

In ΔABC:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc.\cos A$$

$$area\ \Delta ABC = \frac{1}{2}ab.\sin C$$

$$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta \qquad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$$

$$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta \qquad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases} \qquad \sin 2\alpha = 2\sin\alpha.\cos\alpha$$

$$\bar{x} = \frac{\sum fx}{n} \qquad \sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)} \qquad P(A\ or\ B) = P(A) + P(B) - P(A\ and\ B)$$