ASHTON INTERNATIONAL COLLEGE

BALLITO

Mathematics Paper 2 November 2016

FORM 4

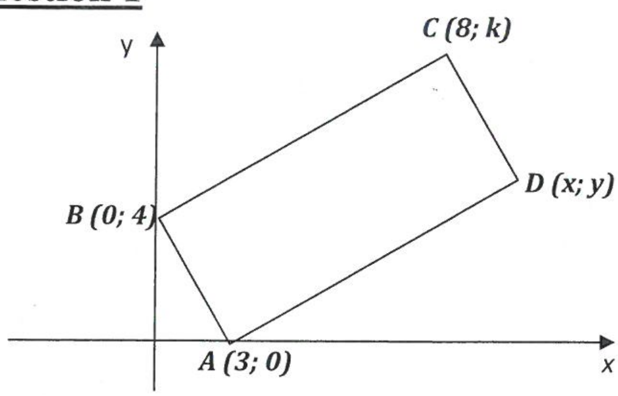
Examiner:	A Gunning	**Moderators:**	P Denissen, C Mundy
Time:	3 hours	**Marks:**	150

NAME: MEMO.

	SECTION A						SECTION B									
Ques No	1	2	3	4	5	6	7	8	9	10	11	12	13	14		%
Out of	11	14	8	9	10	13	9	7	5	10	8	26	10	10	150	100
Mark																

- All questions are to be answered in this booklet.
- This question paper consists of 24 pages. Included in this, is a list of useful formulae. Please check that your question paper is complete.
- Read and answer all questions carefully.
- It is in your own interest to write legibly and to present your work neatly.
- **All necessary working which you have used in determining your answers must be clearly shown.**
- Approved non-programmable calculators may be used except where otherwise stated. Where necessary give answers **correct to 2 decimal places**.
- Diagrams have not necessarily been drawn to scale.

Question 1



The points A (3, 0) and B (0, 4) are two vertices of the rectangle $ABCD$, as shown above.

(a) Write down the gradient of AB and hence the gradient of BC. (3)

$m_{AB} = -\frac{4}{3}$ ✓a $\therefore m_{BC} = \frac{3}{4}$ ✓ca

$AB \perp BC$ ✓m (3)

The point C has coordinates (8, k), where k is a positive constant.

(b) Find the length of BC in terms of k. (2)

$BC = \sqrt{(8-0)^2 + (k-4)^2}$ ✓m

$= \sqrt{64 + k^2 - 8k + 16}$

$= \sqrt{k^2 - 8k + 80}$ ✓a (2)

Given that the length of BC is 10 and using your answer to part (b),

(c) find the value of k, (4)

$10 = \sqrt{k^2 - 8k + 80}$ ✓m. ca.

$100 = k^2 - 8k + 80$ ✓m.

$k^2 - 8k - 20 = 0$ ✓a.

$(k-10)(k+2) = 0$

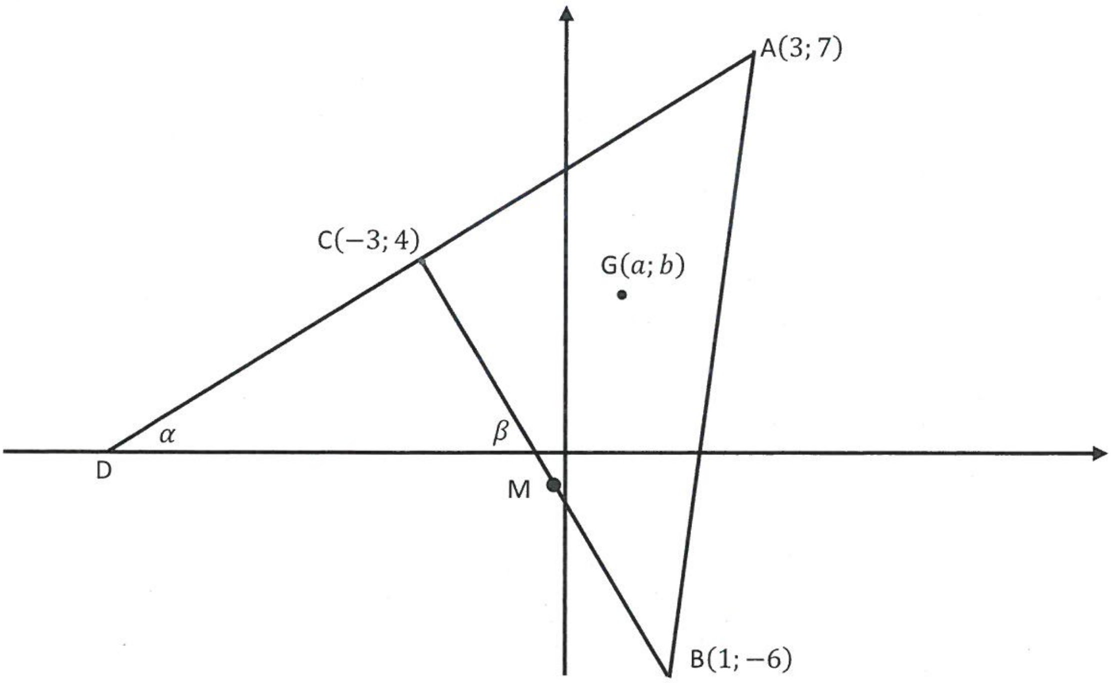
$k = 10$ ✓ or $\cancel{-2}$ ($-\frac{1}{2}$ if -2 not discarded)

(4)

(d) write down the coordinates of D. (2)

$D(x;y)$ $x = 11$ ✓ ca. $y = 6$ ✓ (2) [11]

Question 2



In the diagram above, A, B and C are the vertices of a triangle. AC is extended to cut the x-axis at D. (The diagram is not drawn according to scale.)

(a) Calculate the gradient of:

i) AD (2)

$m_{AD} = \frac{7-4}{3-(-3)}$ ✓m $= \frac{3}{6} = \frac{1}{2}$ ✓a. (2)

ii) BC (2)

$m_{BC} = \frac{4-(-6)}{-3-1}$ ✓m $= \frac{10}{-4} = -\frac{5}{2}$ ✓a (2)

(b) Calculate the sizes of α and β and hence write down the size of $D\hat{C}B$. (3)

$\tan\alpha = \frac{1}{2}$ ✓ m ca. $\tan\beta = \frac{-5}{2}$ ✓ m . ca

$\alpha = 26,565$ ✓ a $\beta = -68,2$ ✓ a

$D\hat{C}B = 180 - (26,565 + 68,2)$ ✓

$= 85,24$ ✓

(3)

(c) Determine the **coordinates** of M, the midpoint of BC. (2)

$M(x;y)$ $x = \frac{-3+1}{2}$ $y = \frac{4-6}{2}$

$= -1$ ✓ $y = -1$ ✓

$M(-1;-1)$ (Do not penalise if not written as coordinates) (2)

(d) If G(a; b) is a point such that A, G and M are collinear, show that $b = 2a + 1$. (5)

$m_{AG} = m_{GM} = m_{AM}$ ✓ m.

$A(3;7)$ $G(a;b)$ $M(-1;-1)$.

$m_{AG} = \frac{b-7}{a-3}$ ✓ $m_{GM} = \frac{b+1}{a+1}$ $m_{AM} = \frac{7-(-1)}{3-(-1)}$

$= \frac{8}{4}$ ✓

$= 2$

$\frac{b-7}{a-3} = 2$ ✓

$b - 7 = 2a - 6$ ✓ (5)

$b = 2a + 1$.

[14]

if used $m_{AG} = m_{GM}$

$\frac{b-7}{a-3} = \frac{b+1}{a+1} \Rightarrow (b-7)(a+1) = (b+1)(a-3)$

$ab - 7a + b - 7 = ab + a - 3b - 3$

$-8a + 4b = 4$

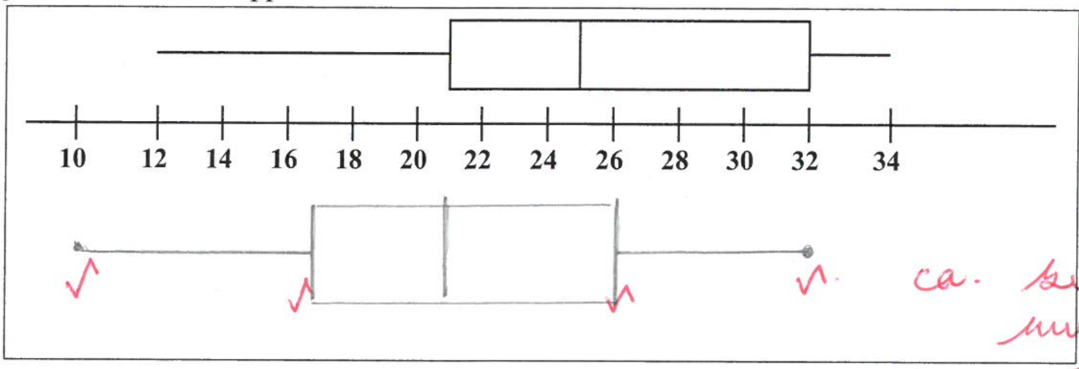
$4b = 8a + 4$

$\therefore b = 2a + 1$

[14]

Question 3

A driver, Chris, of a courier motorcycle, recorded the distance (in km) he had travelled on 15 trips. The 5-number summary of his data is: (12; 21; 25; 32; 34) and the box-and-whisker diagram for his travel appears below.



ca. but must be accurate.

Another driver in the same company, Colin, also recorded the distances (in kilometres) he travelled on 15 trips. The data appear alongside: 10 13 13 17 18 19 20 21 22 23 24 26 27 30 32

24	19	21	27	17	20	32	22
26	18	13	23	30	10	13	

(a) Determine the median for the data. (2)

median = 21. ✓✓ (2)

(b) Determine the 5-number summary for the data of Colin's travels and draw a box-and-whisker diagram in the space provided below the box-and-whisker diagram above. (4)

min = 10 ✓

Q_1 = 17 ✓

med = 21

Q_3 = 26 ✓

Max = 32. ✓ (4)

(c) Carefully analyse the box and whisker diagrams for the two drivers, and comment on the differences or similarities, if any, between the distances covered by each on the 15 trips. (2)

Colin – on whole shorter distances than Chris.
– normal distribution

[8]

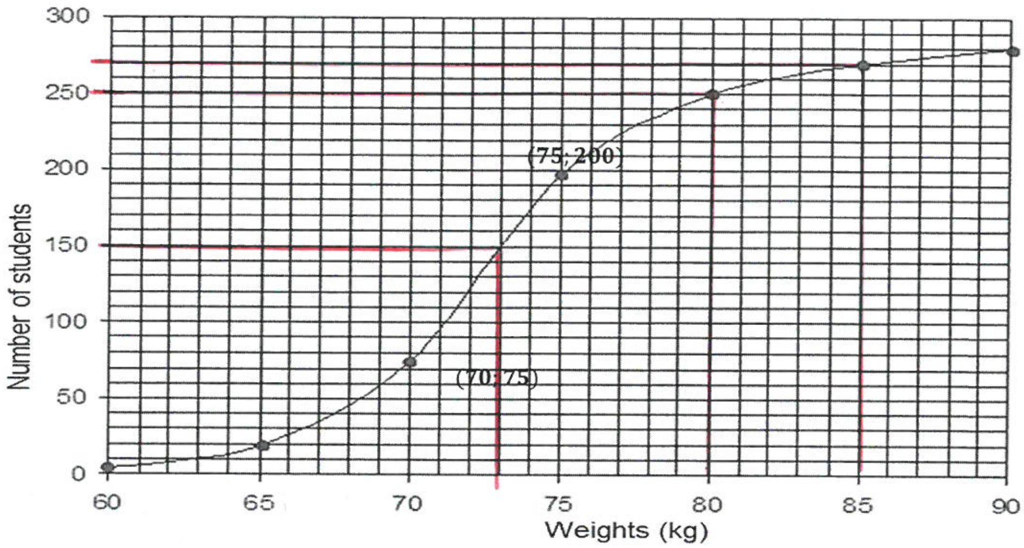
Chris had 25% of his trips between 32 and 34 km.

compared to Colin with 25% between 10 and 17 km. ✓✓ (2)

[8]

Question 4

The cumulative frequency curve below shows the mass of 280 students.



Mass (kg)

(a) How many students have a mass of less than 73 kilograms? (1)

150 students. (1)

(b) Find the number of students that have a mass between 80 and 85 kilogram. (2)

$270 - 250 = 20$ (2)

(c) Complete the following frequency table. (2)

Mass (m)	$60 \leq m < 65$	$65 \leq m < 70$	$70 \leq m < 75$	$75 \leq m < 80$	$80 \leq m < 85$	$85 \leq m < 90$
Frequency	20	55	125	50	20	10

(2)

(d) Hence, calculate the estimated mean of the students' mass, using a calculator. (2)

$\bar{x} = 72,95$ (2)

(e) Write down the standard deviation of the mass of the students. (1)

Std deviation = 5,61. (1)

(f) After a relaxing Matric Rage, all the students have gained 2 kilograms of mass each.

Which of the following statements would be correct for the given data set?

(i) The standard deviation of the data set changes and the mean stays the same.

(ii) The standard deviation as well as the mean changes.

(iii)The standard deviation stays the same and the mean changes. (1)

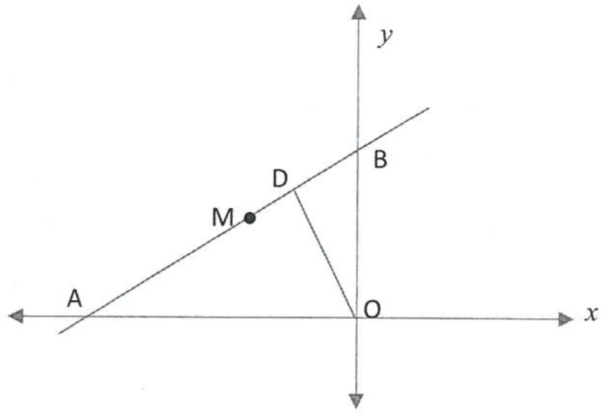
(iii) Std deviation stays the same.
mean changes. (1)

[9]

[9]

Question 5

In the figure, the straight line with equation $2y - x - 10 = 0$ cuts the x-axis at A and the y-axis at B. M is the midpoint of AB and $OD \perp AB$.



Determine:

(a) the **coordinates** of A, B and M. (4)

$2y - x - 10 = 0$

Xint $x = -10$ Yint $y = 5$

$A(-10; 0)$ $B(0; 5)$ $M\left(-5; \frac{5}{2}\right)$

1 mark given for coordinates (4)

(b) the **coordinates** of $\widehat{D}$ (6)

$m_{AB} = \frac{1}{2}$ $\therefore m_{OD} = -2$

eqn of OD

$y = -2x$

Solve simultaneously.

$2y - x - 10 = 0$

$2(-2x) - x - 10 = 0$

$-5x = 10$

$x = -2$

$y = 4$

$D(-2; 4)$

final mark for working as coordinates. (6)

[10]

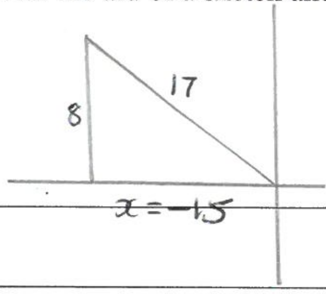
[10]

Question 6

(a) Given $sin\alpha = \frac{8}{17}$ and $90° < \alpha < 270°$, with the aid of a sketch and without the use of a calculator, determine the value of:

(i) $\tan\alpha$ (3)

$\tan\alpha = -\frac{8}{15}$ ✓ca.



• 1 mark correct quad
• 1 mark correct x (write −).

(3)

(ii) $\sin(90° - \alpha)$ (2)

$= \cos\alpha$ ✓ m.

$= -\frac{15}{17}$ ✓ ca. (from diagram)

(2)

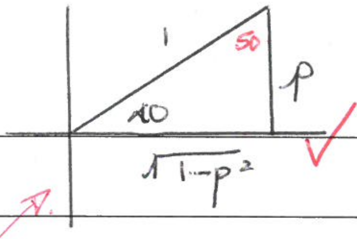
(b) If $\sin 40° = p$, determine each of the following in terms of p (without the use of a calculator)

(i) $\sin 140°$ (1)

Must show "working"

$= \sin 40°$

$= p.$



(1)

(ii) $\cos(-40°)$ (3)

$= \cos 40.$ ✓

$= \sqrt{1-p^2}.$ ✓

(3)

(iii) $\cos 50° \sin 220°$ (4)

$= \sin 40. - \sin 40$ ✓ ✓

$= p. \quad -p \quad \Rightarrow \quad -p^2.$ ✓

(4)

[13]

[13]

Section B

Question 7

Simplify each of the following as far as possible. You may not use a calculator.

(a) $\dfrac{\sin(180°-x)}{\cos(90°+x)+\sin(360°-x)}$ (4)

$= \dfrac{\sin x}{-\sin x - \sin x} = \dfrac{\sin x}{-2\sin x} = \dfrac{-1}{2}$

ca.

(4)

(b) $\dfrac{\cos 115°.\cos 214°}{\cos(-65°).\sin 236°}$ (5)

$\dfrac{-\cancel{\cos 65}.(-\cancel{\cos 34})}{\cancel{\cos 65}.(-\cancel{\sin 56})}$ $\cos 34 = \sin 56.$

$= -1.$

(5)

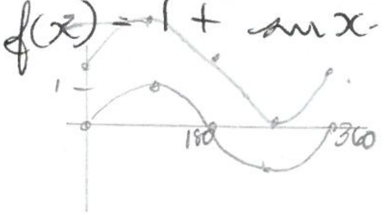
[9]

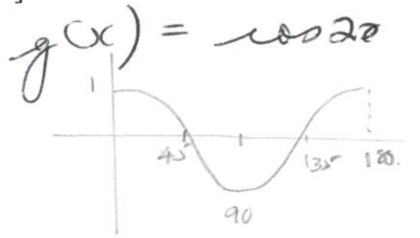
[9]

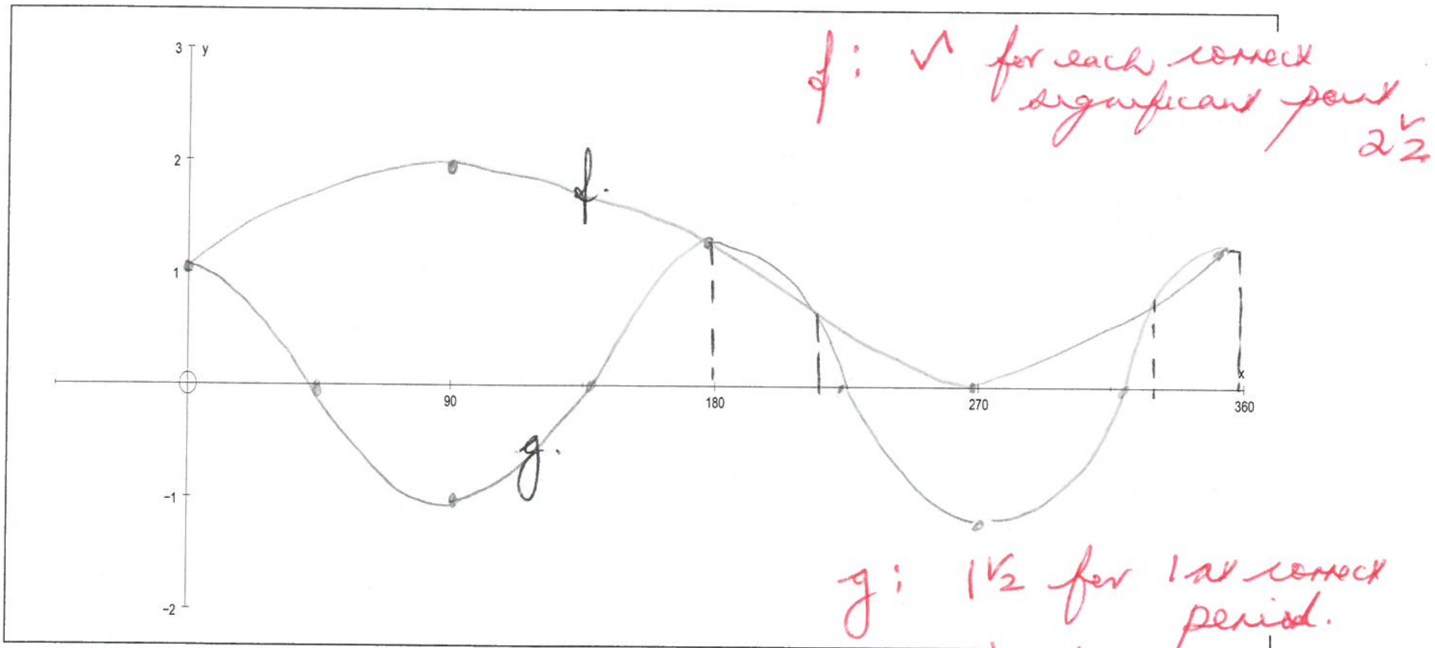
Question 8

Given: $f(x) = 1 + \sin x$ and $g(x) = \cos 2x$

(a) Draw sketch graphs of f and g for $x \in [0°; 360°]$ (5)







(b) Using your graphs, read off the values of x for which $f(x) \le g(x)$ (2)

✓ ✓ ✓ ✓

$x \in [180; 210]$ or $x \in [330; 360]$

all ca from their graphs.

$-\frac{1}{2}$ if not square brackets.

(2)

[7]

[7]

Question 9

The straight line passing through $A(-2;-3)$ and $B(7;2)$ is parallel to the straight line with equation $rx - 3y + 5 = 0$. Calculate the value of r. (5)

$AB \parallel rx - 3y + 5 = 0$ $\quad +3y = +rx + 5$

$m_{AB} = m_{line}$ ✓ $\quad y = \frac{r}{3}x + \frac{5}{3}$ ✓

$\frac{5}{9} = \frac{r}{3}$ ✓ $\quad m = \frac{r}{3}$ ✓

$r = \frac{5}{3}$ ✓ $\quad m_{AB} = \frac{-3-2}{-2+-7} = \frac{5}{9}$ ✓

(5)

[5] [5]

Question 10

(a) Given: $\cos\hat{G} = 0{,}726$ and $180^\circ < \hat{G} < 360^\circ$

(i) Use a calculator to solve for $\hat{G}$, correct to one decimal place. (2)

$\hat{G} = 316{,}6^\circ$ ✓

$43{,}45^\circ$ ✓

(2)

(ii) Hence determine the value of $\tan\left(\frac{2}{3}\hat{G} + 100^\circ\right)$, correct to one decimal place. (1)

$-1{,}15$

$\approx -1{,}2$ ✓

Penalty 1 for whole paper if rounding incorrect

(b) You are given $6\,sin^2\theta + cos\theta = 4$

(i) Show that this can be rewritten as $6\,cos^2\theta - \cos\theta - 2 = 0$. (2)

$6 \sin^2\theta + \cos\theta = 4$

$6(1 - \cos^2\theta) + \cos\theta = 4$

$6 - 6\cos^2\theta + \cos\theta - 4 = 0$

$-6\cos^2\theta + \cos\theta + 2 = 0$

$\therefore 6\cos^2\theta - \cos\theta - 2 = 0$

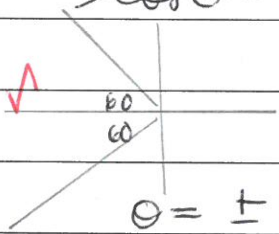
(2)

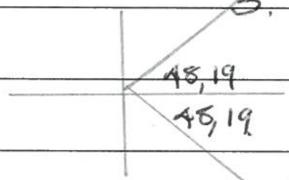
(ii) Hence find the general solution for $6\,sin^2\theta + cos\theta = 4$. (5)

$6\cos^2\theta - \cos\theta - 2 = 0$

$(2\cos\theta + 1)(3\cos\theta - 2) = 0$

$\cos\theta = -\frac{1}{2}$ or $\cos\theta = \frac{2}{3}$





$\theta = \pm 120^\circ + n360$ or $\theta = \pm 48{,}19 + n360$

$n \in \mathbb{Z}$

($-\frac{1}{2}$ if don't put $n \in \mathbb{Z}$.)

[10]

can write solutions as.

$\theta = 120 + n360.$

or $\theta = 240 + n360.$

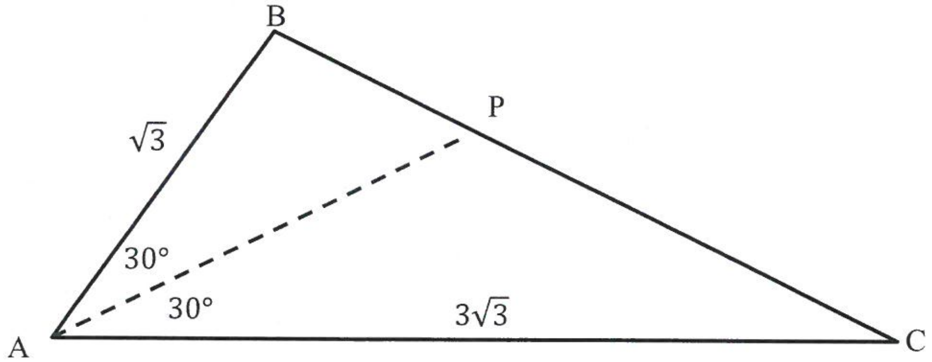
or $\theta = 48{,}19 + n360$

or $\theta = 311{,}81 + n360$ $n \in \mathbb{Z}$.

Question 11

In the diagram below, ABC is a triangular piece of card and AP is the angle bisector of $B\hat{A}C$.

$AB = \sqrt{3}\ cm$, $AC = 3\sqrt{3}\ cm$ and $B\hat{A}P = P\hat{A}C = 30°$



(a) Determine the area of ΔABC leaving your answer in simplest surd form. (2)

$\text{area } \Delta ABC = \frac{1}{2} AB . AC . \sin A$ ✓ m

$= \frac{1}{2} \sqrt{3} . 3\sqrt{3} . \sin 60$ ✓ a

$= \frac{9}{2} . \frac{\sqrt{3}}{2} = \frac{9\sqrt{3}}{4}$ ✓ ca. (2)

(b) $Area\ \Delta ABC = area\ \Delta ABP + area\ \Delta APC$. Use this and the answer you found in (a) to determine the length of AP (3)

$\text{area } \Delta ABP + \text{area } \Delta APC$ m

$= \frac{1}{2} \sqrt{3} . AP . \sin 30 + \frac{1}{2} 3\sqrt{3} . AP . \sin 30 = \frac{9\sqrt{3}}{4}$ ✓ ca.

$\frac{\sqrt{3}}{4} AP$ ✓ a $+ \frac{3\sqrt{3}}{4} AP = \frac{9\sqrt{3}}{4}$ ✓ a

$AP + 3AP = 9$

$4AP = 9$

$AP = \frac{9}{4}$ ✓ a. (3)

(c) Hence determine the length of BP, leaving your answer in the simplest surd form. (3)

In $\triangle$ BAP.

$BP^2 = AB^2 + AP^2 - 2\,AB.\,AP.\cos A$ ✓m

$= (\sqrt{3})^2 + \left(\frac{9}{4}\right)^2 - 2.\sqrt{3}.\frac{9}{4}.\frac{\sqrt{3}}{2}$ ✓a $\hat{A} = 30.$

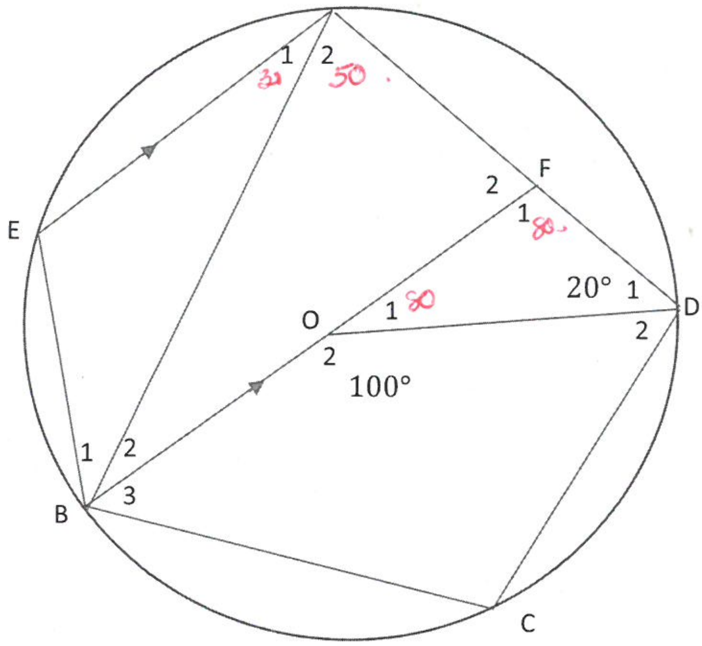
$BP^2 = \frac{21}{16}$

$BP = \frac{\sqrt{21}}{4}$ ✓ ca. (3) [8]

[8]

Question 12

(a) In the figure, O is the centre of the circle $AEBCD$, with line $BOF//EA$, with F on AD. It is given that $B\hat{O}D = 100°$ and $\hat{D}_1 = 20°$. The sizes of the angles are as indicated. In each case, supply a valid reason.



Statement	Reasons
$\hat{A}_2 = 50°$	∠ at centre = 2∠ on circumf. ✓
$\hat{O}_1 = 80°$	∠'s on str line. ✓
$\hat{F}_1 = 80°$	ext ∠ △FOD or ∠'s in △ FOD. ✓
$\hat{A}_1 = 30°$	$\hat{A} = \hat{F}_1$ corresp ∠'s =; EA // BF ✓ *
$\hat{B}_2 = 30°$	alt ∠'s = ; AE // BF ✓

(5)

* or $\hat{F}_2 = 100°$ ∠'s on str line

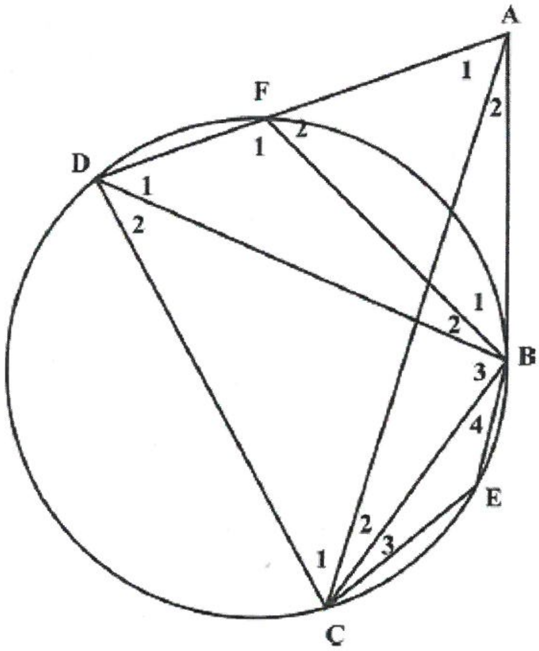
$\hat{B}_2 = 30°$ ∠'s in △ ABF

$\hat{A}_1 = 30$ alt ∠'s =; EA // BF

but must give all of it!

(5)

(b) In the diagram below, AB is a tangent to the circle passing through B, E, C and D. AD cuts the circle at F. AC is drawn. A list of statements is given. Give reasons for the statements which are correct. If a statement is not necessarily correct, write "not correct" in the space provided for the reason. (3)

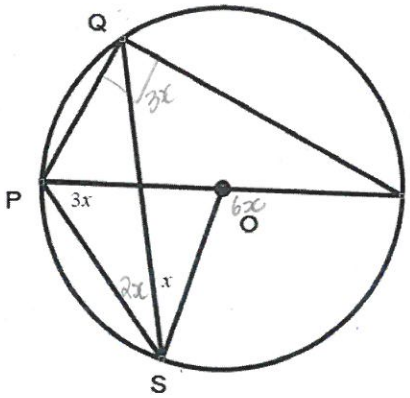


Statement	Reason
$\widehat{C_1} + \widehat{C_2} = \widehat{F_2}$	ext ∠ cyclic quad (FDCB) ✓ no need to name the quad.
$\widehat{D_2} + \hat{E} = 180°$	opp ∠'s cyclic quad suppl. (DBEC) ✓
$\widehat{B_1} = \widehat{D_1}$	tan chord Th. ✓

(3)

(c) P, Q and R are points on the circumference of a circle centre O.

PR is a diameter of the circle. $Q\hat{S}O = x$ and $O\hat{P}S = 3x$.



(i) Express each of the following angles in terms of x.

Give a reason for each of your answers.

a. $S\hat{Q}R$ (2)

$S\hat{Q}R = 3x$ ∠'s in same segment chord SR (2)

(ii) $P\hat{Q}S$ (2)

$\hat{Q} = 90°$ ∠ in semi circle.

$\therefore P\hat{Q}S = 90 - 3x$ (2)

(iii) $P\hat{S}Q$ (3)

△ POS is isosceles

$\therefore P\hat{S}O = 3x$

$Q\hat{S}O = x$ given $\therefore P\hat{S}Q = 2x$. (3)

(iv) $S\hat{O}P$ (2)

$S\hat{O}P = 180 - 6x$ ∠'s in △ SOP

OR ∠'s on str line (2)

(v) $P\hat{R}Q$ (2)

$PRQ = 2x$ ∠'s in same segment chord PQ (2)

(vi) $Q\hat{P}R$ (2)

$Q\hat{P}R = 180 - (2x + 90)$

$= 90 - 2x$ ∠'s in △ PQR (2)

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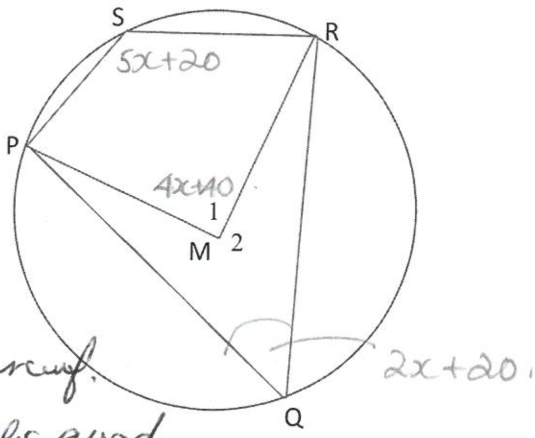
(d) In the figure, P, Q, R and S are points on a circle with centre M.

It is given that $\widehat{M_1} = 4x + 40°$ and $\hat{S} = 5x + 20°$,

Calculate, stating reasons, the size of $\hat{Q}$. (5)



$\hat{M}_1 = 4x + 40°$ given

$\hat{Q} = 2x + 20$ ∠ at centre = 2 ∠ on circumf.

$\hat{S} + \hat{Q} = 180°$ opp ∠'s cyclic quad

$5x + 20 + 2x + 20 = 180$

$7x = 140$

$x = 20$

$\therefore \hat{Q} = 60°$

(5)

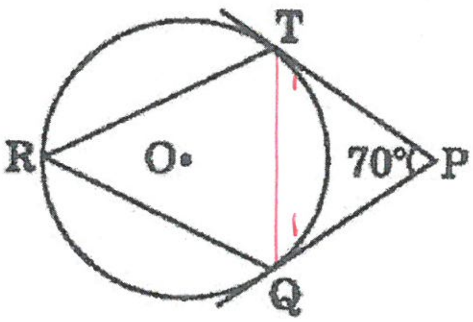
[26]

[26]

Question 13

(a) In the diagram below, O is the centre of the circle. PT and PQ are tangents to the circle.

$T\hat{P}Q = 70°$. (Hint: you may need to draw a line.)



Determine the size of $T\hat{R}Q$ (5)

Join TQ.

$\hat{T}_1 = \hat{Q}_1$ isos △ 2 tangents drawn from same point.

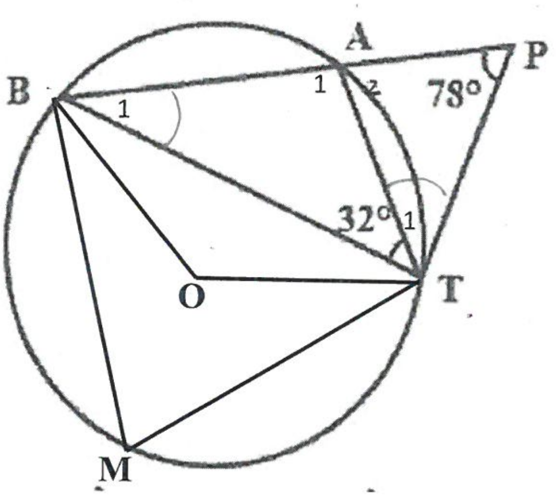
$\therefore \hat{T}_1 = \hat{Q}_1 = 55°$ ∠'s of △.

$\hat{T}_1 = \hat{R}$

$= 55°$ tan chord Thr

5

(b) In the diagram below, A, B, M and T are four points on a circle. The tangent at T meets BA produced at P.



Given that $A\hat{T}B = 32°$ and that $A\hat{P}T = 78°$, calculate the angle subtended by BT at the centre of the circle.

(Hint: let $\widehat{B_1} = x$, and solve for x first.) (5)

$\hat{B}_1 = x$

$\hat{T}_1 = x$ tan chord Th

In $\triangle$ PTB

$78° + x + 32 + x = 180°$ ∠s in $\triangle$.

$2x + 110 = 180$

$x = 35°$.

$\hat{A}_2 = 180 - (78 + 35)$

$= 67°$ ∠s in $\triangle$ APT

$\hat{A}_2 = \hat{M}$ ext ∠ cyclic quad

$= 67°$.

$\therefore B\hat{O}T = 134°$ ∠ at centre = 2∠ at circum

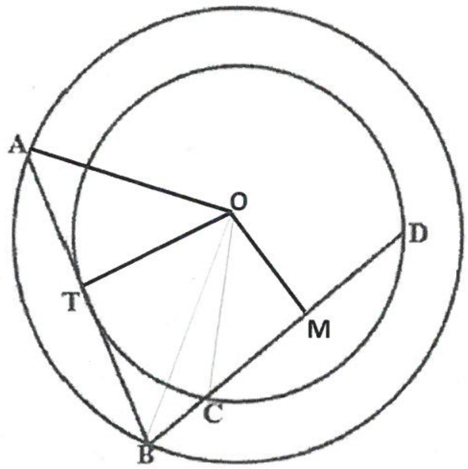
5 [10]

[10]

Question 14

In the diagram below, the radii of 2 concentric circles is 9 cm and 15 cm.

Tangent AB touches the smaller circle at T. OT and OM are drawn.



(a) What can you deduce about AB and OT. Give a reason for your answer. (1)

$AB \perp OT$. radius $\perp$ to tangent. (1)

(b) Write down the lengths of OA and OT. (1)

$OA = 15$

$OT = 9$ (1)

(c) Hence, calculate the length of AT. (2)

In $\triangle AOT$ $AO^2 = AT^2 + TO^2$

$15^2 = AT^2 + 9^2$

$225 - 81 = AT^2$ $\therefore AT = 12$. (2)

(d) Now given that $OM \perp CD$ and $CD = AT - 2$, write down the length of CD. (1)

$CD = AT - 2 = 12 - 2 = 10$. (1)

ca a

(e) Join OC. Hence, or otherwise, calculate the length of OM. (3)

In $\triangle$ OCM. $OC^2 = CM^2 + OM^2$.

$CM = 5$ line from centre $\perp$ to chord

$81 = 5^2 + OM^2$

$OM = 2\sqrt{14}$

(3)

(f) Join OB. Hence, determine the length of BC. (2)

In $\triangle$ OBM

$OB^2 = BM^2 + OM^2$

$15^2 = BM^2 + 56.$

$13^2 = BM^2$ $BM = 13$

$CM = 5$ (proved above)

$\therefore BC = 8.$

(2)

[10]

[10]

Useful formulae

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad M\left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c \qquad y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad m = \tan\theta$$

In ΔABC:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc.\cos A$$

$$area\ \Delta ABC = \frac{1}{2}ab.\sin C$$

$$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta \qquad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$$

$$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta \qquad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases} \qquad \sin 2\alpha = 2\sin\alpha.\cos\alpha$$

$$\overline{x} = \frac{\sum fx}{n} \qquad \sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \overline{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)} \qquad P(A\ or\ B) = P(A) + P(B) - P(A\ and\ B)$$