BALLITO

Mathematics Paper 2 23rd November 2017
FORM 4

Examiner	A Gunning	**Moderator**	C Mundy
Time	3 hours	**Marks**	150

NAME: MEMO

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY BEFORE ANSWERING THE QUESTIONS.

- This question paper consists of 21 pages. Please check that your question paper is complete.
- You have also been given an information sheet.
- All answers are to be written in this booklet.
- Read and answer all questions carefully.
- Number your answers exactly as the questions are numbered.
- It is in your own interest to write legibly and to present your work neatly.
- **All necessary working, which you have used in determining your answers, must be clearly shown.**
- Approved non-programmable calculators may be used except where otherwise stated. Where necessary give answers correct to 2 decimal places.
- Diagrams have not necessarily been drawn to scale.

Ques No	1	2	3	4	5	6	7	8	9	10	11	12	Total	%
Out of	17	10	16	13	32	10	13	5	10	6	5	13	150	
Mark														

QUESTION 1 – a calculator may be used in this question.

(a) The results of 10 students' grade 11 examination results are given below:

43 70 55 60 85 92 65 62 75 58

(i) Calculate the mean test mark.

$\bar{x} = 66,5$ ✓

(1) (1)

(ii) Calculate the standard deviation.

13,78 ✓ (1) (1)

(iii) How many grade 12 students obtained marks within one standard deviation from the mean?

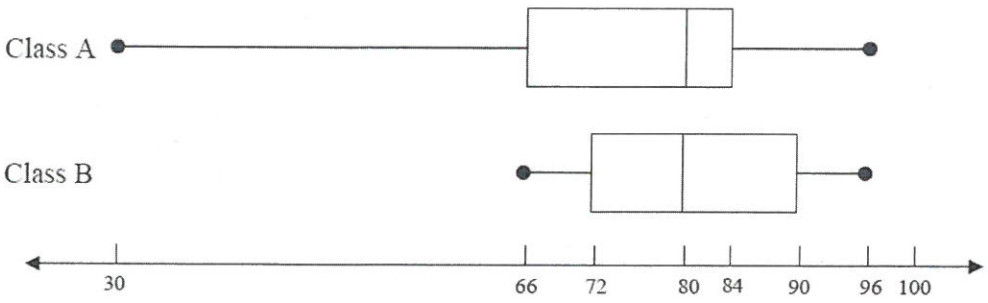
$66,5 + 13,78 = 80,28$ ✓ ca

$66,5 - 13,78 = 52,72$ ✓ ca

$[52,72 ; 80,28]$ HH II

7 students ✓ ca (2) (2)

(b) The box and whisker plots below summarise the final test scores for two of Mrs Smith's Mathematics classes from Grade 11.



(i) Write down two features in the scores that are the same for both classes?

max = 96 ✓

median = 80 ✓ (2) (2)

(ii) Calculate the range for Class A.

Range = 96 − 30 = 66 ✓ (1)

(1)

(iii) Calculate the inter quartile range for Class B?

IQR = 90 − 72 = 18 ✓ (1)

(1)

(iv) Comment on the distribution of the data set for Class A.

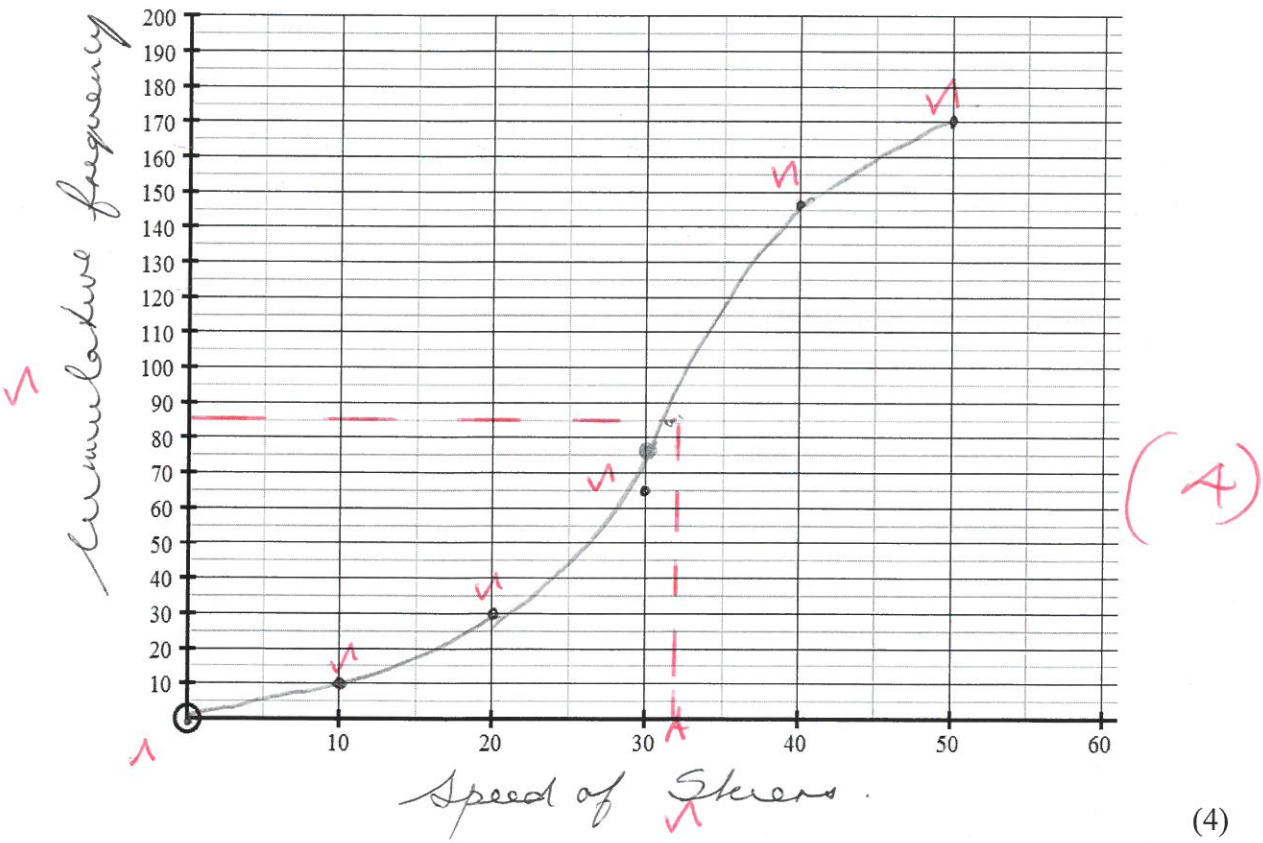
Negatively skewed ✓ (1) (1)

(c) The speed in km/hr of 170 snow skiers passing a certain point on a ski slope was recorded and summarised in the table below.

SPEED	FREQUENCY	CUMULATIVE FREQUENCY
$0 \le x < 10$	10	10
$10 \le x < 20$	20	30 ✓
$20 \le x < 30$	45	75 ✓
$30 \le x < 40$	71	146 ✓
$40 \le x < 50$	24	170 ✓

(i) Complete the table above. (2) (2)

(ii) Draw an ogive curve of the cumulative frequency vs speed of skiers.



(4) (4)

(iii) From your graph, determine the value of the median. Show clearly on your graph where this answer was obtained.

$Q_2 = 32$ ✓ (from A) ✓ Must show where this was obtained (1)

(iv) Use your graph to estimate the number of skiers that passed the point with speed greater than 35. (1)

no. greater than 35 = 170 − 120 ✓

= 60 ✓ (1) (1)

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QUESTION 2

(a) A learner claims that in general, the higher the range of a set of data, the higher the standard deviation. Determine whether the following pair of data sets prove or disapprove the learner's claim. Justify fully.

Data set 1: 60; 80. 80. 80, 80, 100
Data set 2: 62; 62; 62; 98; 98; 98 (2)

Range Set 1 = 40 ✓ Std dev. = 11,55 ✓

Range Set 2 = 36 ✓ Std dev. = 18 ✓

(2)

Not correct.

(b) Let a, b, c and d be integers such that $a < b < c$ and $c = d$.

The mode of these four numbers is 11.

The range of these four numbers is 8.

The mean of these four numbers is 8.

Calculate the values of a, b, c and d. (4)

$c = d = 11$ ✓

Range $= 11 - a = 8$

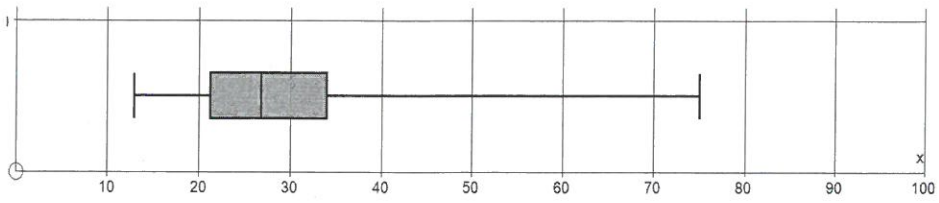
$a = 3$ ✓ ca

$\bar{x} = \frac{3 + b + 22}{4} = 8$ ✓ ca.

$b + 25 = 32$

$b = 7$ ✓ ca. (4)

(c)



The above diagram represents the age demographics of Facebook users in South Africa.

Use this diagram to answer the following questions:

State whether the statement is true or false. If false, correct the statement.

(i) This data is skewed to the left.

F; skewed right

(ii) The median age is between 25 and 30.

T.

(iii) 50% of the users are over 34.

F 25% over 34.

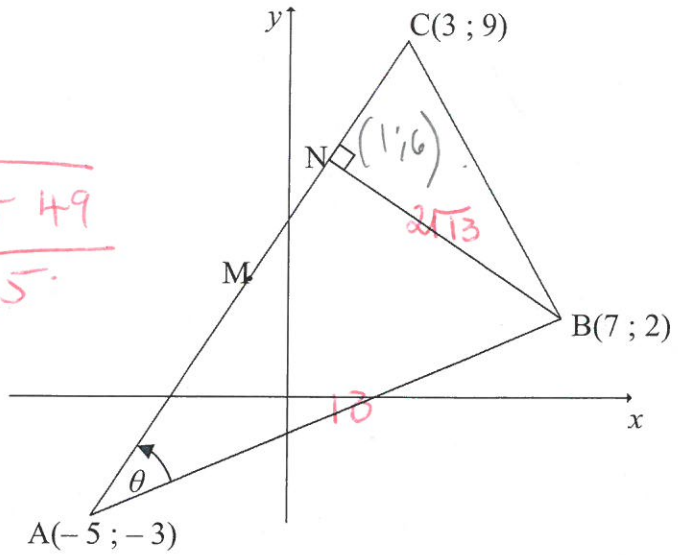
(iv) The Interquartile range is approximately 15 years of age.

F approx 13 (4)

(4)

[10]

QUESTION 3



(a) Calculate the length of AC. (Leave your answer in surd form.) (3)

$AC = \sqrt{(3+5)^2 + (9+3)^2}$ ✓ ✓ no ca.

$= \sqrt{64 + 144}$

$= \sqrt{208}$ ✓

$= 4\sqrt{13}$ ✓ (3)

(b) Determine the coordinates of M, the midpoint of AC. (2)

$M\left(\frac{-5+3}{2} ; \frac{-3+9}{2}\right)$

$= (-1 ; 3)$ ✓ ✓ (2)

(c) Calculate the gradient of AC. (2)

$m_{AC} = \frac{9-(-3)}{3-(-5)}$ ✓ $= \frac{12}{8} = \frac{3}{2}$ ✓ (2)

(d) Hence, determine the equation of BN. (3)

$m = -\frac{2}{3}$ ✓

1 pt (7; 2) $y - 2 = -\frac{2}{3}(x-7)$

$y = -\frac{2}{3}x + \frac{20}{3}$ ✓ ✓ (3)

(f) $AN = \sqrt{6^2 + 9^2} = \sqrt{36+81} = \sqrt{117} = 3\sqrt{13}$.

In $\triangle BAN$: $\tan\theta = \frac{BN}{AN} = \frac{2\sqrt{13}}{3\sqrt{13}} = \frac{2}{3}$

$\theta = \tan^{-1}\left(\frac{2}{3}\right) = 33.69$.

BC $= \sqrt{16+49} = \sqrt{65}$.

$AB = \sqrt{12^2 + 5^2} = \sqrt{144+25} = \sqrt{169} = 13$

(e) Calculate the area of ΔABC if N is the point (1 ; 6). (3)

area ΔABC
$= \frac{1}{2} AC \cdot BN$
$= \frac{1}{2} \cdot 4\sqrt{13} \cdot 2\sqrt{13}$
$= 52u^2$ ✓

$BN = \sqrt{(7-1)^2 + (2-6)^2}$
$= \sqrt{36+16}$
$= \sqrt{52} = 2\sqrt{13}$ ✓

(3)

(f) Calculate θ, correct to 1 decimal place. (3)

$\theta = \tan^{-1}(m_{AC}) - \tan^{-1}(m_{AB})$
$= \tan^{-1}\left(\frac{3}{2}\right) - \tan^{-1}\left(\frac{5}{12}\right)$ ✓
$= 56,31° - 22,62°$
$= 33,69°$ ✓

$m_{AB} = \frac{2-(-3)}{7-(-5)}$
$= \frac{5}{12}$

(3)

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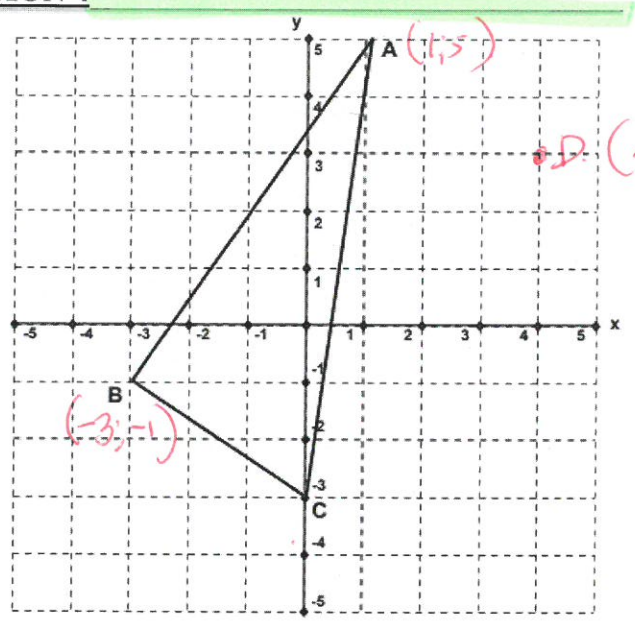
QUESTION 4



$AC = \sqrt{1+64} = \sqrt{65}$
$BD = \sqrt{49+16} = \sqrt{65}$
$AC = BD$
diag =
∴ rect.

$A(1;5)$, $B(-3;-1)$ and $C(0;-3)$ are the vertices of a triangle.

(a) Write down the coordinates of D if ABCD is a parallelogram. (2)

$M_{AC}\left(\frac{1+0}{2}; \frac{5-3}{2}\right) = \left(\frac{1}{2}; 1\right)$

$M_{BD} = M_{AC}$

$\frac{-3+x}{2} = \frac{1}{2}$ ✓ $\quad \frac{-1+y}{2} = 1$ ✓

$x = 4$ $\quad y = 3$

D(4;3)

(2)

can just be read off graph

(b) Show that ABCD is in fact a rectangle. (3)

$m_{AB} = \frac{5-(-1)}{1-(-3)} = \frac{6}{4} = \frac{3}{2}$ ✓

$m_{BC} = \frac{-3-(-1)}{0-(-3)} = \frac{-2}{3}$ ✓

$m_{AB} \cdot m_{BC} = \frac{3}{2} \cdot \frac{-2}{3} = -1$ ✓

$\therefore \hat{B} = 90°$ $\therefore$ ABCD is a rect, ($1 \angle = 90°$). (3)

(c) If A, B and $E(5; y)$ are three collinear points, find the value of y. (3)

$A(1; 5)$ $B(-3; -1)$ $E(5; y)$

$m_{AB} = \frac{3}{2}$ $m_{BE} = \frac{y+1}{8}$ ✓

Can get 1 mark if use meaning lines

$\frac{3}{2} = \frac{y+1}{8}$ ✓ ca.

$24 = 2y + 2$

$22 = 2y$

$y = 11$ ✓ (3)

(d) If the distance between C and $F(8; p)$ is 10 units, find the possible values of p. (5)

$C(0; -3)$ $F(8; p)$

$CF = 10 = \sqrt{8^2 + (p+3)^2}$ ✓

$100 = 64 + (p+3)^2$ ✓

$36 = (p+3)^2$ ✓

$p + 3 = \pm 6$

$p = 3$ or -9 ✓

OR

$p^2 + 6p + 9 - 36 = 0$

$p^2 + 6p - 27 = 0$ ✓

$(p+9)(p-3) = 0$ ✓

(5)

[13]

QUESTION 5

SHOW ALL STEPS OF WORKING.

(a) Simplify the following to a single trigonometric ratio of x: (6)

$$\frac{\cos(180^\circ - x)\sin(180^\circ + x)\tan(-x - 180^\circ)}{\cos(90^\circ + x)}$$

$$\frac{-\cos x \cdot -\sin x \cdot -\tan x}{-\sin x}$$

$$= \cos x \cdot \frac{\sin x}{\cos x} = \sin x$$

✓ for args ✓ for ratios of each. Must see this.

(b) Prove that $\frac{1}{1+\sin x} + \frac{1}{1-\sin x} = \frac{2}{\cos^2(180^\circ - x)}$ (4)

Lhs. $\frac{1-\sin x + 1+\sin x}{(1+\sin x)(1-\sin x)}$ ✓

$= \frac{2}{1-\sin^2 x}$ ✓

$= \frac{2}{\cos^2 x}$.

rhs $\frac{2}{(-\cos x)^2}$ ✓

$= \frac{2}{\cos^2 x}$.

Lhs = rhs (4)

(c) For which values of x in the interval $0^\circ \le x \le 360^\circ$ is the identity in Question 5(b) undefined? (2)

$1 + \sin x = 0$ | $1 - \sin x = 0$

$\sin x = -1$ ✓ | $\sin x = 1$ ✓

$x = 270^\circ$ ✓ or $x = 90^\circ$ ✓ (2)

(d) Using a sketch, drawn in the relevant quadrant, given that $13\sin x + 5 = 0$

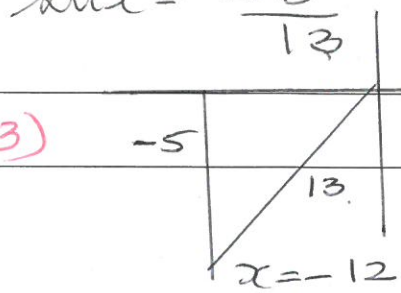
and $x \in [90°; 270°]$, without using a calculator, find the value of each of the following:

(i) $\sin(90° - x)$ (3)

$= \cos x$ ✓

$= -\frac{12}{13}$ ✓

$\sin x = -\frac{5}{13}$



quadrant and ; negative x ✓

(3)

(ii) $\tan(180° - x)$ (2)

$= -\tan x$ ✓

$= -\frac{5}{12}$ ✓ ca

(2)

$= 81,55$ gr.

$73,398 = 1,28°$

(e) Solve for x in each of the following (4)

(i) $7\cos 2x + 2 = 0$, giving solutions for $x \in [0°; 360°]$

$\cos 2x = -\frac{2}{7}$ ✓

73,398 ✓ ca.

73,398

$2x = 180 - 73,398 + k360$ or $2x = 180 + 73,398 + k360$ ✓

$2x = 106,6 + k360$ or $2x = 253,398 + k360$

$x = 53,3° + k180°$ ✓ $x = 126,7° + k180°$ ✓

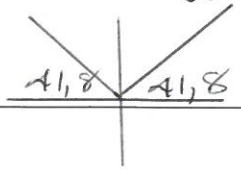
$x \in \{53,3°; 126,7°; 233,3°; 306,7°\}$ ✓

(4)

max 3/4 if did not give ∈ ℤ

(ii) $(\cos x - 2)(3\sin 2x - 2) = 0$ giving a general solution. (5)

$\cos x = 2$ ✓ $\sin 2x = \frac{2}{3}$



no solution

$2x = 180 - 41,8 + k360$ ✓ OR $2x = 41,8 + k360$ ✓

$2x = 138,189 + k360$ ✓

$x = 69,1 + k180°$ ✓ $x = 20,9 + k180°$ ✓

$k \in \mathbb{Z}$ ✓

(5)

(f) Given that $\sin 34° = t$, write each of the following in terms of t.

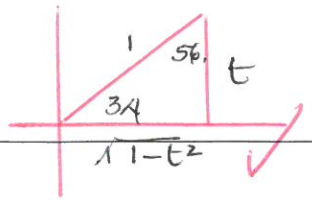
(i) $\sin 214°$ (2)

$= -\sin 34$ ✓

$= -t$ ✓ (2)

(ii) $\sin 56°$ (2)

$= \sqrt{1-t^2}$ ✓



(can work.

$\sin 56 = \cos 34$ (2)

$= \sqrt{1-t^2}$

(iii) $\tan(-34°)$ (2)

$= -\tan 34$ ✓

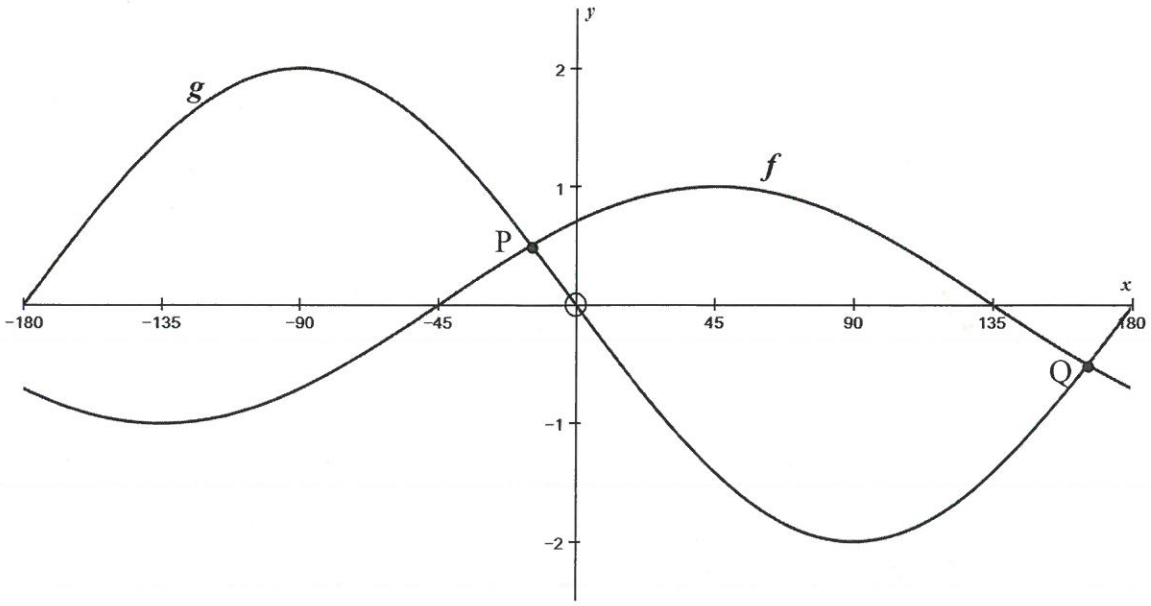
$= \dfrac{-t}{\sqrt{1-t^2}}$ ✓ (2)

[32]

QUESTION 6

The sketch graph below shows the curves of f and g, defined by:

$f(x) = \cos(x+b)$ and $g(x) = c\sin x$ where $x \in [-180° ; 180°]$



$g(x) = -2\sin x$

OR $f(x) = \cos(x - 45^\circ)$

$0,51 = \cos(x - 45^\circ)$

$x - 45 = 59,34$ or

$x = 104,34$ or $-14,34$ ✓

(a) Determine the values of b and c. (2)

$b = -45^\circ$ ✓

$c = -2$ ✓ (2)

(b) If P(d; 0,51) and Q(165,36°; e) are the points of intersection of the two graphs, write down the values of d and e. (3)

can be read off the graph.

$g(x) = -2\sin x$

sub $y = 0,51$ ✓ $\quad 0,51 = -2\sin d$ $\quad$ sub $x = 165,36^\circ$ ✓

$-0,255 = \sin d$ $\quad e = -2\sin 165,36$

$d = -14,77$ ✓ $\quad = -0,51$ ✓ (3)

(c) For which values of x is:

(i) $f(x) > 0$ (1)

$x \in (-45^\circ; 135^\circ)$ ✓ ✓ brackets ✓ (1)

(ii) $f(x).g(x) > 0$ (2)

$x \in (-45^\circ; 0)$ ✓ ✓ brackets ✓

or $x \in (135^\circ; 180^\circ)$ ✓ ✓ brackets ✓ (2)

(d) If the y-axis is shifted 60° to the right, what will the new equation of f be? (1)

old $y = \cos(x - 45)$ ✓ Accept $\cos(x - 105)$.

$\therefore y = \cos(x + 15)^\circ$ (1)

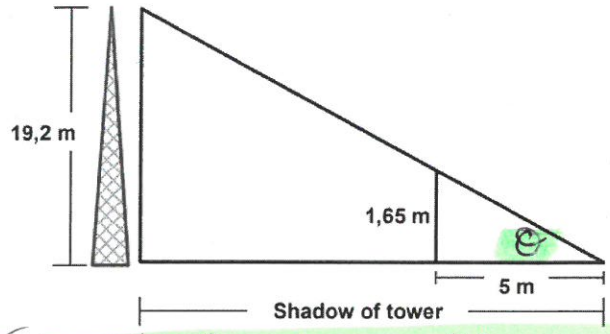
(e) Write down the co-ordinates of the maximum point of $y = -c\sin x$ (1)

$(+90^\circ; 2)$ ✓ ✓ (1)

[10]

QUESTION 7

(a) At a particular time during the day, a tower of height 19,2 m casts a shadow. At the same time, a person who is 1,65 m tall casts a shadow which is 5 m long.



(a) Calculate the value of θ. (2)

(b) What is the length of the shadow cast by the tower at that time? (4) (6)

(a) $\tan\theta = \frac{1,65}{5}$ ✓

$\theta = \tan^{-1}\left(\frac{1,65}{1,5}\right) = 18,26°$ ✓ (2)

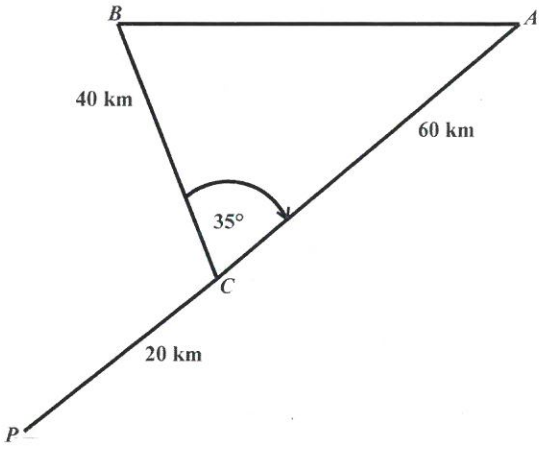
(b) $\frac{\text{tower shadow}}{19,2}$ ✓ $= \tan(90 - 18,26)$ ✓ or $\frac{1}{\tan 18,26°}$

Shadow $= 19,2 \times 3,03$

$= 58,19$ ✓✓ (4)

(b) A cyclist at training is on his way from P to A. When he reaches point C, the road forks. The road to the right leads directly to A, which is 60 km from C. P and C are 20 km apart. The road to the left leads to A via B. C and B are 40 km apart.

The angle between the roads (BC and AC), is $35°$.



(i) Calculate the difference between the distances of the two routes from P to A, correct to the nearest kilometre. (4)

Direct distance = 80 km. ✓

In $\triangle ABC$ $AB^2 = 40^2 + 60^2 - 2.40.60.\cos 35$ ✓

$AB^2 = 1268,07$

$AB = 35,61. = 36$ ✓ (36 + 60)

alt route $= 20 + 40 + 35,61.$ (4)

$= 95,61$

$= 96$ ✓ Difference = 16. km.

(ii) Calculate $\hat{B}$ (3)

$\frac{\sin B}{60} = \frac{\sin 35}{36}$ ✓ OR. $\frac{\sin B}{60} = \frac{\sin 35}{35,61}$

$\sin B = 0,96$ ✓ or. $\hat{B} = 75,11°$

$\hat{B} = 73°$ ✓

(3)

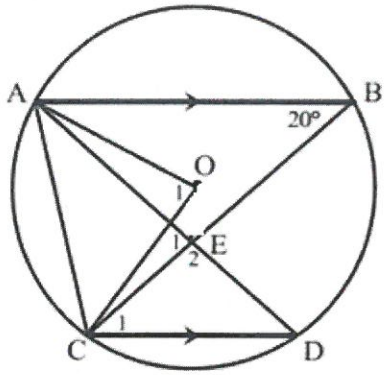
[13]

$\sin B = 0,96$ 

$B = 73°$ or $104,89°$.

QUESTION 8

In the diagram, O is the centre of the circle passing through A, B, D and C. AB // CD and $\hat{B} = 20°$.



Complete the following statements and reasons to prove that AOEC is a cyclic quadrilateral. No extra steps or calculations can be added.

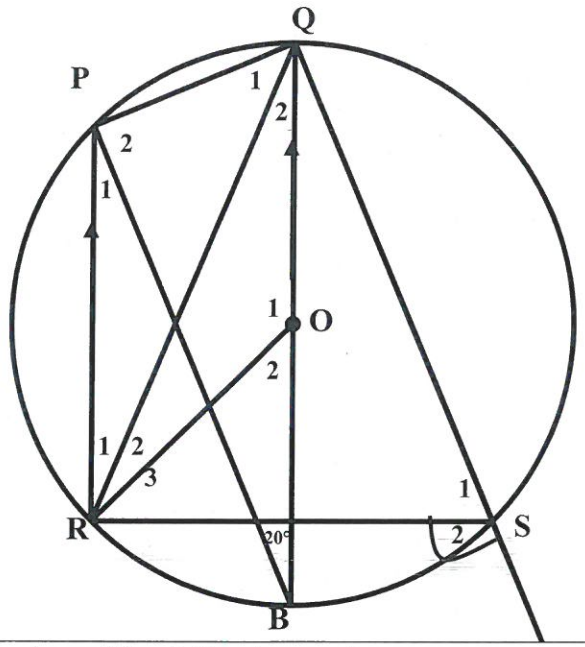
Statements	Reasons
$\hat{C}_1 =$ ……20°…… ✓	Alt angles equal; AB //CD
$\hat{O}_1 = 40°$	…∠ at centre… = 2 ∠ at circumference. ✓
$\hat{D} = 20°$	∠s in same segment ✓
$\hat{E}_1 =$ ……40°…… ✓	Ext angle Δ
∴ AOEC is cyclic	…converse ∠s in same segment ✓

[5]

(5)

QUESTION 9

In the diagram below, O is the centre of the circle. PR is parallel to QB. QOB is a diameter.
$P\hat{B}Q = 20^\circ$



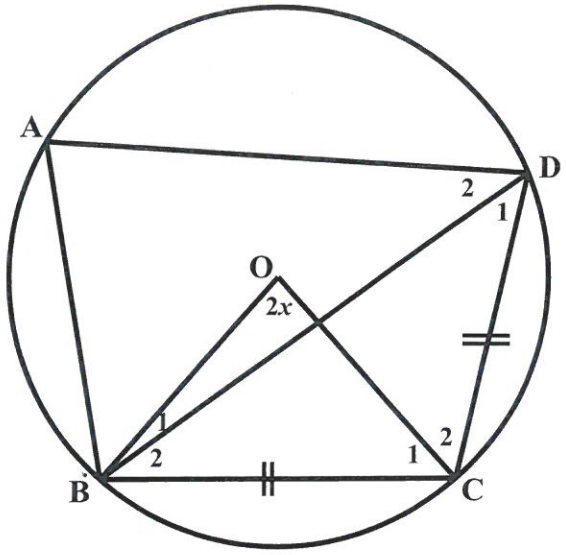
Find, with reasons, the sizes of the following angles. (2 marks each)

Statement			Reason
(i)	$\hat{P}_2$	= 90° ✓	∠ in semi circle, ✓ (2)
(ii)	$\hat{P}_1$	= 20° ✓	alt ∠'s ✓ = ; PR // ✓ BQ (2)
(iii)	$\hat{Q}_2$	= 20° ✓	∠'s in same segment ✓ (2)
(iv)	$\hat{O}_2$	= 40°	∠ at centre = 2 ∠ on circumf (2)
(v)	$\hat{S}_2$	= 110°	ext ∠ cyclic quad (2)

[10]

QUESTION 10

In the diagram alongside, the circle with centre O has BC = CD and $B\hat{O}C = 2x$.

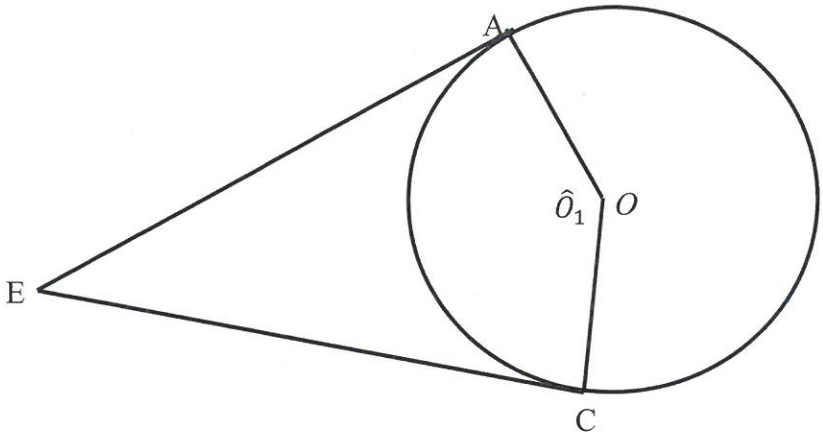


Determine, with reasons, in terms of x.	Reasons
(i) $\hat{D}_1$ (2) $= x$ ✓	∠ at centre = 2 ∠ on circumf ✓ (2)
(ii) $B\hat{A}D$ (4) In △ BCD. $\hat{D}_1 = \hat{B}_2 = x$ ✓ $\therefore \hat{C} = 180 - 2x$ ✓ $\therefore \hat{A} = 2x$ ✓	given radii. ∠'s in △. opp ∠'s cyclic quad ✓ (4) **[6]**

QUSTION 11

Refer to the sketch below.

In the diagram O is the centre of the circle with AE and CE tangents to the circle centre O. $\hat{O}_1 = 127°$



Calculate, with reasons, the size of $\hat{E}$. (5)

STATEMENT	REASONS
AO ⊥ AE ✓	radius ⊥ tan
$\therefore \hat{A} = 90°$	
Similarly $\hat{C} = 90$ ✓	rad ⊥ tang
Either AOCE cyclic ✓	Conv. opp ∠'s suppl.
$\therefore \hat{E} + \hat{O}_1 = 180$ ✓	
$\hat{E} = 53°$ ✓	

OR

∠'s in quadrilateral ✓

$180 + 127 + \hat{E} = 360$ ✓

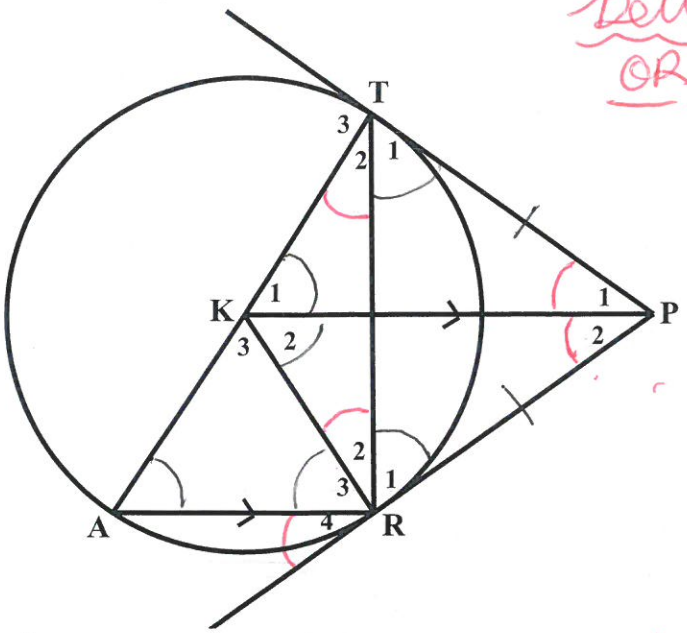
$\therefore \hat{E} = 53°$ ✓

(5)

[5]

QUESTION 12

In the figure below PT and PR are tangents to the circle at T and R respectively. Also PK ∥ RA.
$\hat{P}_1 = \hat{P}_2$. **Do not assume** that K is the centre of the circle.



Better
OR $P_2 = R_4$ ✓ proved
$R_4 = T_2$ ✓ tan chord Th
$\therefore P_2 = T_2$ ✓
$\therefore$ KTPR cyclic ✓ converse ∠s in same segment.

(i) Why is $\hat{R}_4 = \hat{P}_2$? (1)

corresp ∠s = ; KP // AR (1)

(ii) Prove, with reasons, that KTPR is a cyclic quadrilateral. (4) (5) (6)

$\hat{R}_1 = \hat{T}_1$ ✓ equal tangents from same point
$\hat{T}_1 = \hat{A}$ ✓ ∴ ∠s in alt segment Th. ✓
$\hat{A} = \hat{K}_1$ ✓ corresp ∠s = ; KP // AR ✓
$\therefore \hat{R}_1 = \hat{K}_1$ ✓
$\therefore$ KTPR is cyclic ✓ Converse ∠s in same segment

(iii) Prove that PK bisects $T\hat{K}R$. ie $\hat{K}_1 = \hat{K}_2$ (5) (3 4)

Proof.
$\triangle TKP \equiv \triangle RKP$ KP common.
$TP = RP$ ✓ equal tangents.
$\hat{P}_1 = P_2$ given
SAS. ✓

$\therefore \hat{K}_1 = K_2$ ✓ (3 4)

(iv) Prove $\hat{A} = \hat{R}_3$ (3)

$\hat{K}_1 = K_2$ proved above

$\hat{K}_1 = A$ ✓ corresp $\angle$'s ; proved above.

$\hat{K}_2 = \hat{R}_3$ ✓ alt $\angle$'s =; KP // AR ✓

(3)

$\therefore \hat{A} = R_3$

[13]