basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL
SENIOR CERTIFICATE
NASIONALE
SENIOR SERTIFIKAAT

GRADE/*GRAAD* 11

MATHEMATICS P2/*WISKUNDE V2*

NOVEMBER 2019

MARKING GUIDELINES/*NASIENRIGLYNE*

MARKS/*PUNTE*: 150

These marking guidelines consist of 21 pages./
Hierdie nasienriglyne bestaan uit 21 bladsye.

NOTE:
- If a candidate answered a question TWICE, mark only the FIRST attempt.
- If a candidate crossed out an answer and did not redo it, mark the crossed-out answer.
- Consistent accuracy applies to ALL aspects of the marking memorandum.
- Assuming values/answers in order to solve a problem is unacceptable.

LET WEL:
- *As 'n kandidaat 'n vraag TWEE keer beantwoord het, sien slegs die EERSTE poging na.*
- *As 'n kandidaat 'n antwoord deurgehaal en nie oorgedoen het nie, sien die deurgehaalde antwoord na.*
- *Volgehoue akkuraatheid is op ALLE aspekte van die memorandum van toepassing.*
- *Dit is onaanvaarbaar om waardes/antwoorde te veronderstel om 'n probleem op te los.*

QUESTION/*VRAAG* 1

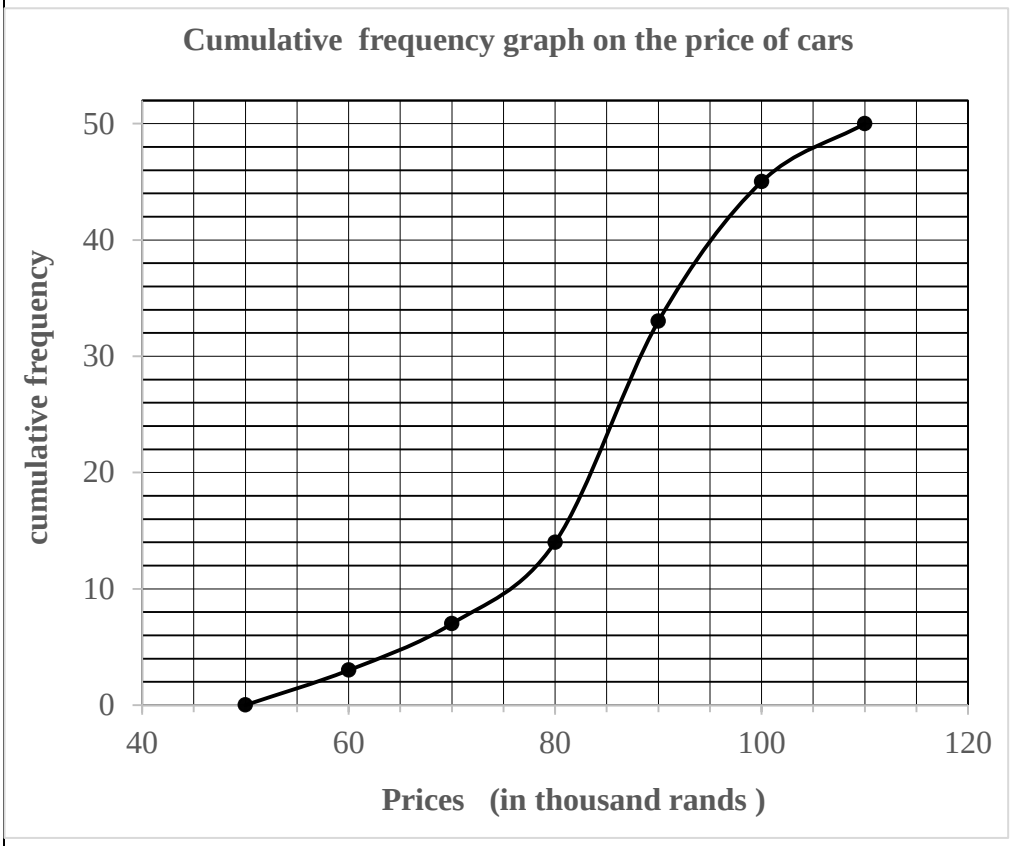
1.1

SELLING PRICE/ *VERKOOPSPRYS* (IN THOUSANDS OF RANDS/ *IN DUISENDE RAND*)	**FREQUENCY/ *FREKWENSIE***	**CUMULATIVE FREQUENCY/ *KUMULATIEWE FREKWENSIE***
$50 \leq x < 60$	3	3
$60 \leq x < 70$	4	7
$70 \leq x < 80$	**7**	14
$80 \leq x < 90$	19	33
$90 \leq x < 100$	12	**45**
$100 \leq x < 110$	5	50

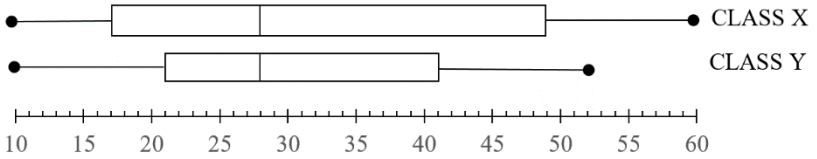
✓$a = 7$

✓$b = 45$

(2)

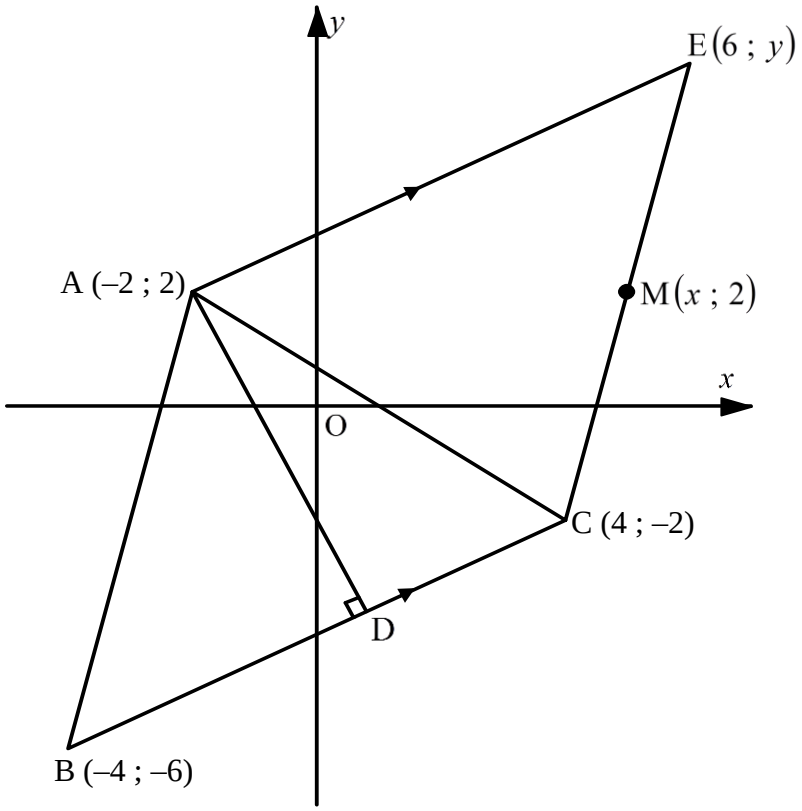
1.2		✓grounding/ *geanker* (50:0) ✓upper boundry/ *boonste grens* ✓smooth curve/ *gladde kurwe* (3)
1.3	(95 000 ; 40) 40 cars to choose from/*40 motors om vanuit te kies* (Accept any answer between 37 and 43/ *Aanvaar enige antwoord tussen 37 en 43*)	(1)
		[6]

QUESTION/*VRAAG* 2



2.1.1	Positively skewed/*Positief skeef* or/*of* skewed to the right/*skeef na regs*	✓answer/*antwoord* (1)
2.1.2	Class/*Klas* X	✓answer/*antwoord* (1)
2.1.3	The average performance is the same./*Die gemiddelde prestasie is dieselfde.* Both classes have the same median mark./*Albei klasse het dieselfde mediaanpunt.*	✓average performance is the same/*gemiddelde prestasie dieselfde* ✓same median mark/ *dieselfde mediaan-punt* (2)
2.2.1	$a = 5$ $g = 5 + 48$ $= 53$ $d = 22$ $b = 5 + 7$ $= 12$ $f = 12 + 28$ $= 40$ $\frac{c + 2c + 132}{7} = 27$ $3c = 57$ $c = 19$ $e = 38$	✓ $a = 5$ ✓ $b = 12$ ✓ $\frac{c + 2c + 132}{7} = 27$ ✓ $c = 19$ ✓ $d = 22$ ✓ $e = 38$ ✓ $f = 40$ ✓ $g = 53$ (8)
2.2.2	$(\bar{x} - \sigma; \bar{x} + \sigma)$ $(27 - 15{,}87\,;\, 27 + 15{,}87)$ $(11{,}13\,;\, 42{,}87)$ 5 goals were scored within one standard deviation of the mean/ *5 doele is binne een standaardafwyking van die gemiddeld aangeteken*	✓✓ $(11{,}13\,;\, 42{,}87)$ ✓answer/*antwoord* (3) **[15]**

QUESTION/*VRAAG* 3

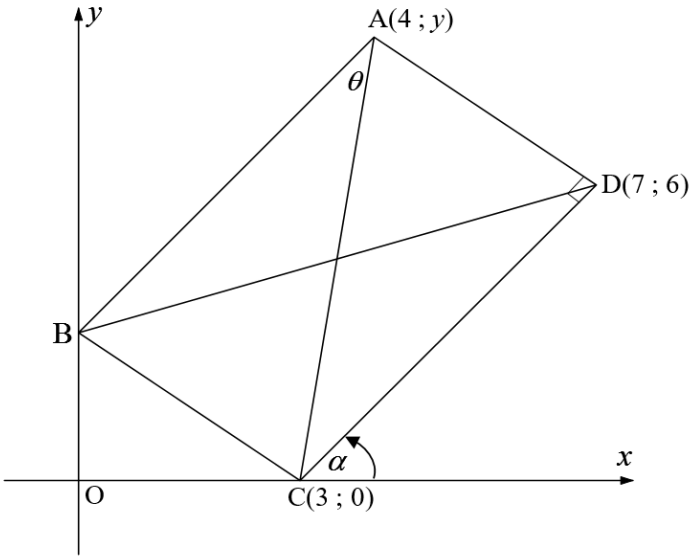


3.1	B(–4;–6) C(4;–2) $m_{BC} = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{-2-(-6)}{4-(-4)}$ $= \frac{4}{8}$ $= \frac{1}{2}$ OR/*OF* $m_{BC} = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{-6-(-2)}{-4-4}$ $= \frac{-4}{-8}$ $= \frac{1}{2}$	✓gradient formula/ *gradiëntformule* ✓ subst into/ *vervanging in* gradient form./ *gradiëntform.* ✓answer/*antwoord* (3)
3.2	$x = \frac{6+4}{2}$ $x = 5$ $\frac{y+(-2)}{2} = 2$ $y = 6$	✓ $x = 5$ ✓ $\frac{y+(-2)}{2} = 2$ ✓ $y = 6$ (3)

3.3	$\text{BC} = \sqrt{\left(x_{2-} - x_1\right)^2 + \left(\text{y}_2 - y_1\right)^2}$ $= \sqrt{\left((-4)-4\right)^2 + \left((-6)-(-2)\right)^2}$ $= \sqrt{80}$	✓subst. in distance formula/*verv. in afstandformule* ✓answer/*antwoord* (2)
3.4	Parallelogram	✓answer/*antwoord* (1)
3.5	$m_{\text{AD}} = -2$ (AD ⊥ BC) $y = \frac{4}{3}x + c$ $2 = -2(-2) + c$ $-2 = c$ $\therefore y = -2x - 2$ **OR/*OF*** $m_{\text{AD}} = -2$ (AD ⊥ BC) $y - 2 = -2\left(x - (-2)\right)$ $y - 2 = -2x - 4$ $\therefore y = -2x - 2$	✓ $m_{\text{AD}} = -2$ ✓subst. of/*verv. m* and point/*en punt* $(-2;2)$/ ✓ answer/*antwoord* ✓ $m_{\text{AD}} = -2$ ✓subst. of/*verv. m* and point/*en punt* $(-2;2)$ ✓answer/*antwoord* (3)

3.6	$m_{BC} = \frac{1}{2}$ Equation of/*Verg. van* BC: $y = \frac{1}{2}x + c$ $-2 = \frac{1}{2}(4) + c$ $-4 = c$ $\therefore y = \frac{1}{2}x - 4$ **OR/*OF*** $y-(-2) = \frac{1}{2}(x-4)$ $y+2 = \frac{1}{2}x-2$ $\therefore y = \frac{1}{2}x-4$ $-2x-2 = \frac{1}{2}x-4$ $-4x-4 = x-8$ $4 = 5x$ $x = \frac{4}{5}$ $y = (-2)\frac{4}{5}-2$ $= \frac{-18}{5}$ **OR/*OF*** $y = \frac{1}{2}\left(\frac{4}{5}\right)-4$ $= \frac{-18}{5}$ $D\left(\frac{4}{5};\frac{-18}{5}\right)$	✓subst. of/*verv. m* and point/*en punt* $(4;-2)$/ ✓ $y = \frac{1}{2}x-4$ ✓equating both equations/*vergelyking van beide vergelykings* ✓*x*-value/-*waarde* ✓*y*-value/-*waarde* (5)
3.7	$AD = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ $= \sqrt{\left(\left(\frac{4}{5}\right)-(-2)\right)^2+\left(\left(\frac{-18}{5}\right)-2\right)^2}$ $= \frac{14\sqrt{5}}{5}$ Area of/*Oppervlakte van* $\Delta AEC = \frac{1}{2}AE \times AD$ $= \frac{1}{2}\times\sqrt{80}\times\frac{14\sqrt{5}}{5}$ $= 28$ units^2 / *eenhede*^{2}	✓length of AD/ *lengte van AD* ✓subst into area formula ✓answer/*antwoord* (3)
		[20]

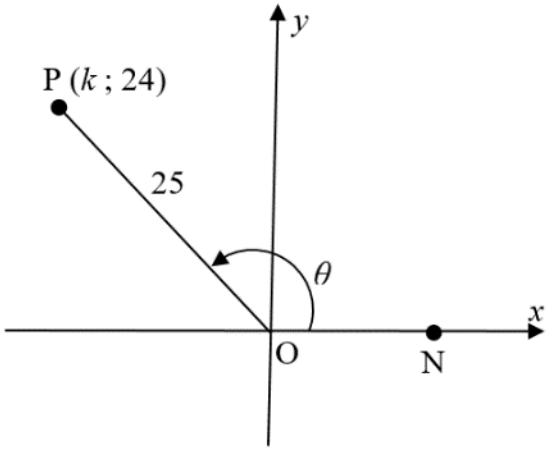
QUESTION/*VRAAG* 4



4.1	$m_{DC} = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{6-0}{7-3}$ $= \frac{3}{2}$	✓substitution in gradient formula/ *vervanging in gradiëntformule* ✓ $m_{DC} = \frac{3}{2}$ (2)
4.2	$\tan\alpha = m_{CD}$ $\tan\alpha = \frac{3}{2}$ $\alpha = 56{,}31°$	✓ $\tan\alpha = \frac{3}{2}$ ✓ $\alpha = 56{,}31°$ (2)
4.3	$m_{CD} \times m_{AD} = -1$ $\frac{3}{2} \times \frac{y-6}{4-7} = -1$ $\frac{y-6}{-3} = -\frac{2}{3}$ $3y - 18 = 6$ $y - 6 = 2$ $y = 8$	✓✓ $\frac{y-6}{-3} = -\frac{2}{3}$ ✓ $3y - 18 = 6$ ✓ $y = 8$ (4)

4.4	$m_{AC} = \frac{8-0}{4-3} = 8$ $\therefore \text{Inclination of AC} = \tan^{-1}(8)$ $= 82{,}87°$ $A\hat{C}D = 82{,}87° - 56{,}31°$ $= 26{,}56°$ $\therefore \theta = 26{,}56°$	✓ $m_{AC} = 8$ ✓ $82{,}87°$ ✓ $A\hat{C}D = 82{,}87° - 56{,}31°$ ✓ $A\hat{C}D = 26{,}56°$ ✓ $\theta = 26{,}56°$ (5)
		[13]

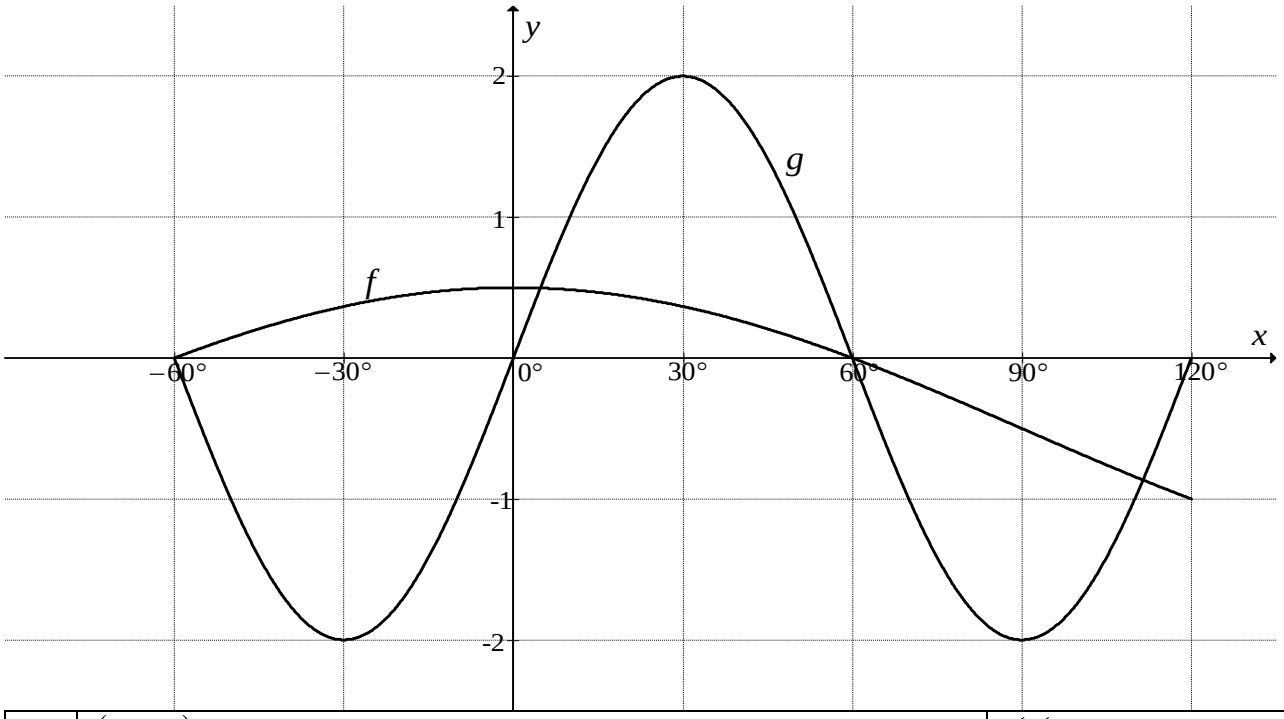
QUESTION/*VRAAG* 5



5.1.1	$x^2 + y^2 = r^2$ $(k)^2 + (24)^2 = 25^2$ $k^2 = 49$ $k = -7$	✓subst./ *vervanging* ✓answer/*antwoord* (2)
5.1.2	$\tan\theta = -\frac{24}{7}$	✓answer/*antwoord* (1)
5.1.3	$\theta + \alpha = 360°$ $\alpha = 360° - \theta$ $\sin\alpha = \sin(360° - \theta)$ $= -\sin\theta$ $= -\frac{24}{25}$	✓ $\sin\alpha = \sin(360° - \theta)$ ✓ $-\sin\theta$ ✓answer/*antwoord* (3)
5.1.4	$\cos^2\theta - \sin^2\alpha$ $= \left(\frac{-7}{25}\right)^2 - \left(-\frac{24}{25}\right)^2$ $= \frac{-527}{625}$	✓✓substitution/ *vervanging* ✓answer/*antwoord* (3)

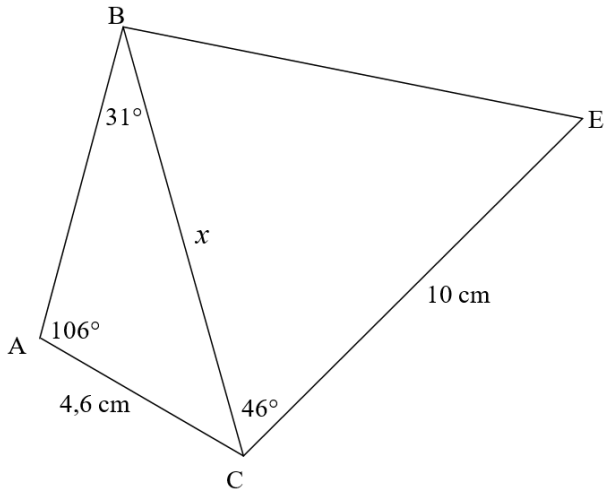
5.2	$\dfrac{\cos 210°.\tan 135°}{\sin(-60°).\cos 420°}$ $=\dfrac{-\cos 30°.-\tan 45°}{-\sin 60°.\cos 60°}$ $=\dfrac{\left(-\dfrac{\sqrt{3}}{2}\right)(-1)}{\left(-\dfrac{\sqrt{3}}{2}\right).\dfrac{1}{2}}$ $=-2$	✓$-\cos 30°.-\tan 45°$ ✓$-\sin 60°.\cos 60°$ ✓$\left(-\dfrac{\sqrt{3}}{2}\right)(-1)$ ✓$\left(-\dfrac{\sqrt{3}}{2}\right).\dfrac{1}{2}$ ✓answer/*antwoord* (5)
5.3	$\text{LHS}=\dfrac{1}{\tan^2 x}-\cos^2 x$ $==\dfrac{1}{\dfrac{\sin^2 x}{\cos^2 x}}-\cos^2 x$ $=\dfrac{\cos^2 x}{\sin^2 x}-\cos^2 x$ $=\dfrac{\cos^2 x-\cos^2 x\sin^2 x}{\sin^2 x}$ $=\dfrac{\cos^2 x(1-\sin^2 x)}{\sin^2 x}$ $\dfrac{\cos^2 x(\cos^2 x)}{\sin^2 x}$ $=\dfrac{\cos^4 x}{\sin^2 x}$ $=\text{RHS}$	✓$\dfrac{\sin^2 x}{\cos^2 x}$ ✓common denominator/*gemene noemer* ✓factors/*faktore* ✓$1-\sin^2 x=\cos^2 x$ (4)
5.4	$\sqrt{2}\sin x\cos x=\cos x$ $\sqrt{2}\sin x\cos x-\cos x=0$ $\cos x(\sqrt{2}\sin x-1)=0$ $\cos x=0$ $x=90°+360°k\ ,\quad k\in Z$ or $x=270°+360°k,\quad k\in Z$ *OR* $x=90°+180°k\ ,\quad k\in Z$ **or/*of*** $\sin x=\dfrac{1}{\sqrt{2}}$ $x=45°+360°.k,\ k\in Z$ or $x=135°+360°.k,\ k\in Z$	✓standard form/*stand.vorm* ✓factors/*faktore* ✓both equations/*beide vergelykings* ✓ $x=90°+360°k\ ,\ k\in Z$ ✓ $x=45°+360°.k,\ k\in Z$ ✓ $x=135°+360°.k,\ k\in Z$ (6) **[24]**

QUESTION/*VRAAG* 6

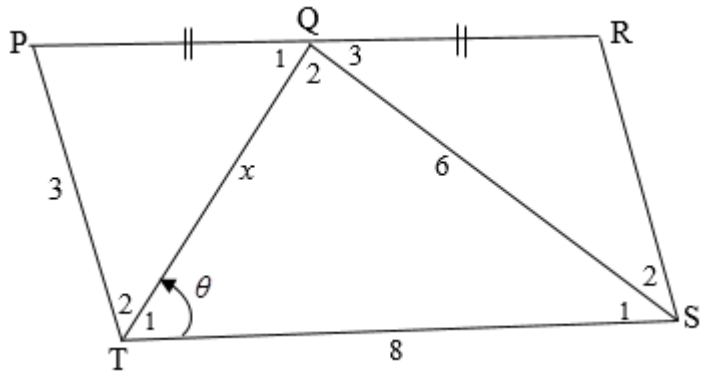


6.1	$(30°;2)$	✓✓answer/*antwoord* (1)
6.2	$-60° < x < 0°$	✓endpoints/*eindpunte* ✓notation/*notasie* (2)
6.3	$q = -\frac{1}{2}$ $b = 3$	✓ $q = -\frac{1}{2}$ ✓ $b = 3$ (2)
6.4	$2\cos x\sin 3x - \sin 3x \geq 0$ $2\sin 3x\left(\cos x - \frac{1}{2}\right) \geq 0$ $g(x).f(x) \geq 0$ $0° \leq x \leq 120°$	✓✓ $2\sin 3x\left(\cos x - \frac{1}{2}\right) \geq 0$ ✓endpoints/*eindpunte* ✓notation/*notasie* (4)
		[9]

QUESTION/*VRAAG* 7

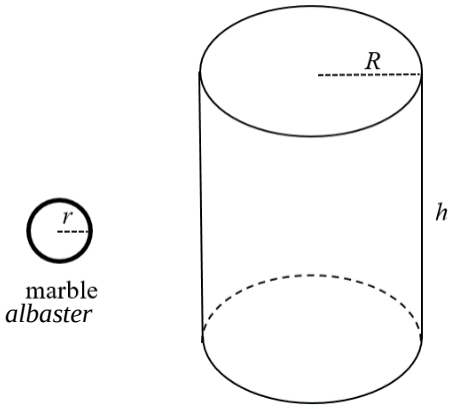


7.1.1	$\frac{BC}{\sin\hat{A}} = \frac{AC}{\sin\hat{B}}$ $\frac{x}{\sin 106°} = \frac{4,6}{\sin 31°}$ $x = \frac{4,6\sin 106°}{\sin 31°}$ $x = 8,59\text{ cm}$	✓substitution into sine rule/*vervanging in sinusreël* ✓ $x = \frac{4,6\sin 106°}{\sin 31°}$ ✓answer/*antwoord* (3)
7.1.2	$A\hat{C}B = 43°$ $\text{Area ACEB} = \left(\frac{1}{2}\times AC\times BC\times\sin A\hat{C}B\right) + \left(\frac{1}{2}\times BC\times CE\times\sin B\hat{C}E\right)$ $= \left(\frac{1}{2}\times 4,6\times 8,59\times\sin 43°\right) + \left(\frac{1}{2}\times 8,59\times 10\times\sin 46°\right)$ $= 44,37\text{ cm}^2$	✓ $A\hat{C}B = 43°$ ✓✓substitution/ *vervanging* ✓answer/*antwoord* (4)



7.2.1	In ΔQTS $6^2 = 8^2 + x^2 - 2(8)(x)\cos\theta$ $16x\cos\theta = 8^2 + x^2 - 6^2$ $16x\cos\theta = x^2 + 28$ $\cos\theta = \frac{28 + x^2}{16x}$	✓✓substitution into cosine rule/ *vervanging in cosinusreël* ✓$16x\cos\theta = x^2 + 28$ (3)
7.2.2	$\hat{Q}_1 = \theta$ and PQ= 4 In ΔQTP $3^2 = 4^2 + x^2 - 2(4)(x)\cos\theta$ $8x\cos\theta = 4^2 + x^2 - 3^2$ $8x\cos\theta = x^2 + 7$ $\cos\theta = \frac{7 + x^2}{8x}$ $\frac{7 + x^2}{8x} = \frac{28 + x^2}{16x}$ $112x + 16x^3 = 224x + 8x^3$ $8x^3 = 112x$ $8x^3 - 112x = 0$ $8x(x^2 - 14) = 0$ $x \neq 0 \quad x = \sqrt{14}$	✓ $\hat{Q}_1 = \theta$ and PQ= 4 ✓ $3^2 = 4^2 + x^2 - 2(4)(x)\cos\theta$ ✓ $\cos\theta = \frac{7 + x^2}{8x}$ ✓equating/ *vergelyking* ✓factors/*faktore* ✓ $x \neq 0 \quad x = \sqrt{14}$ (6)
		[16]

QUESTION/VRAAG 8



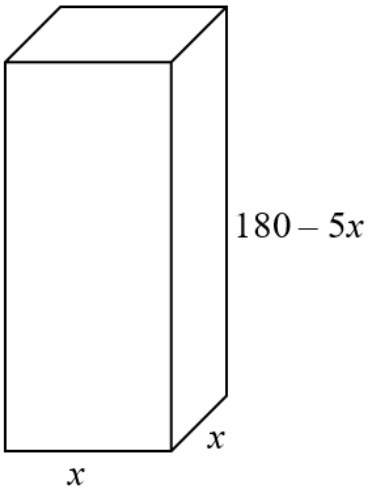
Volume of sphere = $\frac{4}{3}\pi r^3$

Surface area of a sphere = $4\pi r^2$

Volume of sfeer = $\frac{4}{3}\pi r^3$

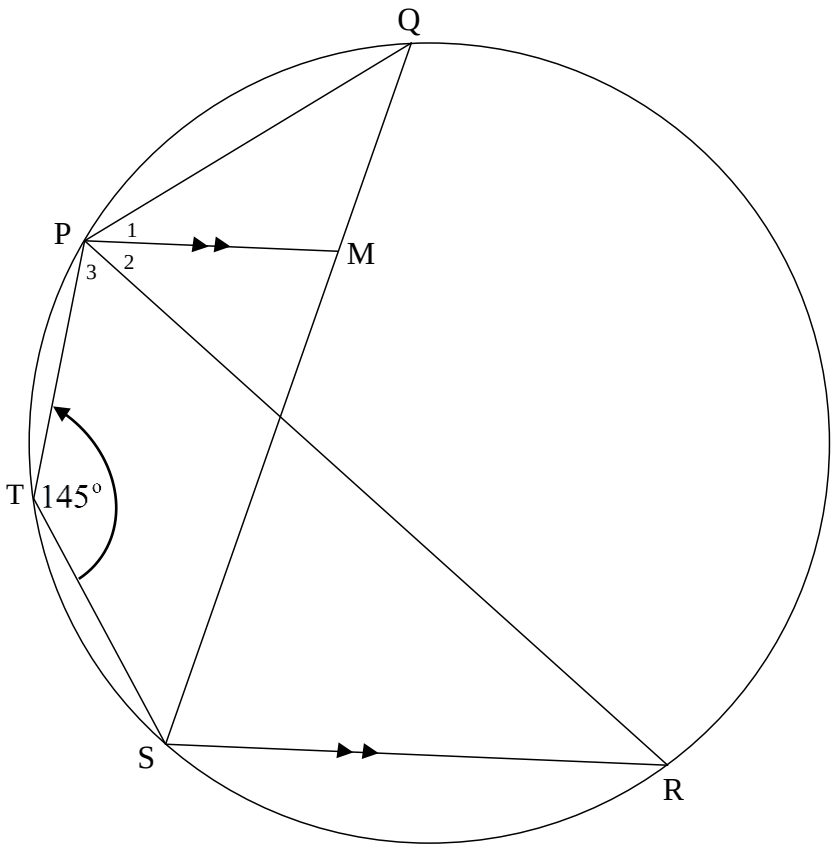
Oppervlakte van 'n sfeer = $4\pi r^2$

8.1.1	$V = \pi r^2 h$ $300 = \pi R^2 h$ $h = \frac{300}{\pi R^2}$	✓substitution/ *vervanging* ✓isolating *h*/ *isoleer h* (2)
8.1.2	$\text{Volume of marbles} = 100\left(\frac{4}{3}\pi r^3\right)$ $= 100 \times \frac{4}{3}\pi(0.75)^3$ $= \frac{225\pi}{4}$ $= 176,71 \text{cm}^3$ $\text{amount of water} = 300 - 176,71$ $= 123,29 \text{ cm}^3$	✓subst. into formula/*vervanging in formule* ✓176,71 ✓123,29 cm^3 (3)



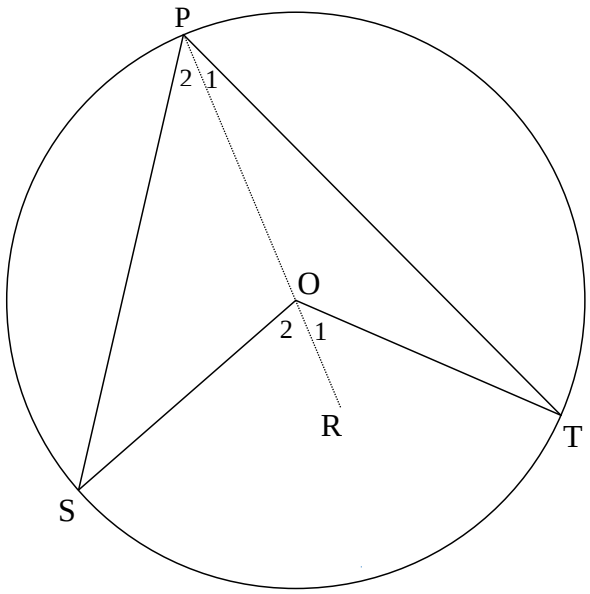
8.2	TSA $= 2\times$ area of the base $+ (\text{perimeter}\times\text{height})$ *TSA* $= 2\times$ *oppervlakte van basis* $\pm$ (*omtrek* $\times$ *hoogte*) $= 2(x^2)+4x(180-5x)$ $= 2x^2+720x-20x^2$ $= -18x^2+720x$ Surface area will be maximum when *Oppervlakte sal op maksimum wees wanneer* $x=\frac{-b}{2a}=\frac{-720}{2(-18)}=20$ max. surface area/maks. oppervlakte $= -18(20)^2+720(20)$ $= 7200\ \text{cm}^2$	✓subst. into formula/*vervanging in formule* ✓simplification/ *vereenvoudiging* ✓value of/*waarde van x* ✓subst./*verv* $x=20$ ✓answer/*antwoord* (5) **[10]**

QUESTION/*VRAAG* 9

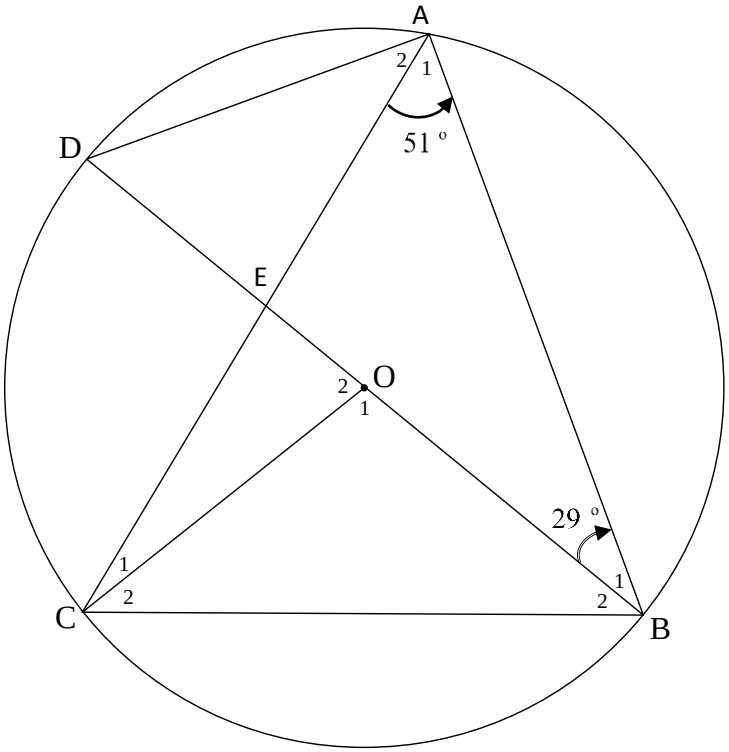


9.1.1	$\hat{Q}=180°-145°$ [opp $\angle^s$ of cyclic quad] $=35°$	✓S ✓ R (2)
9.1.2	$\hat{R}=180°-145°$ [opp $\angle^s$ of cyclic quad] or [$\angle^s$ in the same segment] $=35°$	✓S ✓ R (2)
9.1.3	$\hat{P}_2=\hat{R}=35°$ [alt $\angle^s$, PM ‖ SR]	✓S / R (1)
9.2	$\therefore \hat{P}_2=\hat{Q}$ $\therefore$ PR is a tangent to circle PMQ [converse tan-chord theorem]	✓S / R (1)
		[6]

QUESTION/*VRAAG* 10

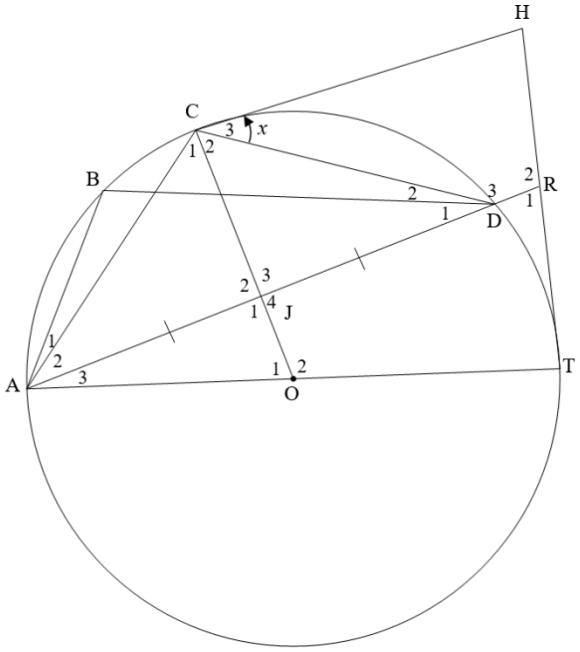


10.1	Construction/*Konstruksie*: Draw/*Trek* POR Let $\hat{P}_2 = x$ $\hat{P}_2 = \hat{S}$ $[angles\ opp = sides]$ $\hat{O}_2 = 2x$ $[exterior\ \angle of\ \Delta]$ Let $\hat{P}_1 = y$ $\hat{P}_1 = \hat{T}$ $[angles\ opp = sides]$ $\hat{O}_1 = 2y$ $[exterior\ \angle of\ \Delta]$ $\hat{O}_1 + \hat{O}_2 = 2x + 2y = 2(x + y)$ $\hat{P}_1 + \hat{P}_2 = x + y$ $\therefore S\hat{O}T = 2 \times S\hat{P}T$	✓Construction/ *Konstruksie* ✓ S/R ✓ S/R ✓ S ✓S (5)



10.2.1	$\hat{O}_1 = 102°$ $\left[\begin{array}{l}\text{angle at centre} = 2 \text{ times angle at circumference}/\\ \textit{middelpuntshoek} = 2\,\textit{keer omtrekshoek}\end{array}\right]$	✓ S ✓ R (2)
10.2.2	$\hat{A} = 90°$ [$\angle$ in a semi – circle] $\hat{A}_2 = 39°$	✓ S ✓ R (2)
10.2.3.	$\hat{D} = 61°$ [sum of int $\angle^s$ *of* Δ]	✓ S (1)
10.2.4	$A\hat{C}B = \hat{D} = 61°$ [$\angle^s$ in the same segment] $\hat{C}_2 = \frac{180° - 102°}{2}$ [sum of int $\angle^s$ *of* Δ] $= 39°$ $A\hat{C}O = 61° - 39°$ $= 22°$	✓ S /R ✓ S ✓S (3)
		[13]

QUESTION/*VRAAG* 11



11.1	$\hat{C}_3 = \hat{A}_2 = x$ [tan - chord]	✓ S ✓ R (2)
11.2	$\hat{J}_3 = 90°$ [line from centre to midpt of chord] $O\hat{C}H = 90°$ [tan ⊥ rad] CH ∥ JR [co - interior $\angle = 180°$] ∴ CHRJ is a trapezuim [one prop sides ∥] **OR/*OF*** $\hat{J}_4 = 90°$ [line from centre to midpt of chord] $O\hat{C}H = 90°$ [tan ⊥ rad] CH ∥ JR [corresp $\angle$ =] ∴ CHRJ is a trapezuim [one prop sides ∥]	✓ S ✓ R ✓ S ✓ R ✓ S **OR/*OF*** ✓ S ✓ R ✓ S ✓ R ✓ S (5)
11.3	In ΔCJA and ΔCJD $\hat{J}_2 = \hat{J}_3$ [line from centre to midpt of chord] AJ = JD [given] CJ = CJ [common side] ∴ ΔCJA ≡ ΔCJD [SAS] $\hat{C}_1 = \hat{C}_2$ [ΔCJA ≡ ΔCJD] OC bisects $A\hat{C}D$	✓ S ✓ S ✓ S (3)

11.4	$\hat{C}_2 = 90° - x$ [tan ⊥ rad] $\hat{B} = \hat{C}_1 + \hat{C}_2$ [$\angle^s$ in the same segment] $= 90° - x + 90° - x$ [$\hat{C}_1 = \hat{C}_2$] $= 180° - 2x$ **OR/*OF*** $A\hat{D}C = x$ [alt $\angle$ = , CH ∥ JR] $\hat{C}_1 + \hat{C}_2 = 180° - 2x$ [sum of int $\angle^s$ *of* Δ] $A\hat{B}D = 180° - 2x$ [$\angle^s$ in the same segment]	✓ S ✓ S (2) ✓ S ✓ S (2)
11.5	$\hat{T} = 90°$ [tan ⊥ rad] $C\hat{A}O = 90° - x$ [$\angle^s$ opp = sides] $x + \hat{A}_3 = 90° - x$ $\hat{A}_3 = 90° - 2x$ $\therefore \hat{R}_2 = 90° + 90° - 2x$ [ext $\angle^s$ of Δ] $= 180° - 2x$	✓ S ✓R ✓ S/ R ✓ S ✓ S ✓ S (6) **[18]**

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