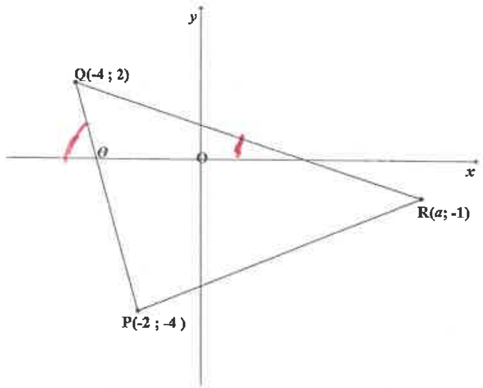
① Q1 - 5 (52 marks)

SECTION A

QUESTION 1

Refer to the sketch below:



In the diagram, P(−2 ; −4); Q(−4 ; 2); and R(a; −1) are the vertices of Δ QPR.

(a) Determine the gradient of the line PQ. (2)

$m_{PQ} = \frac{2-(-4)}{-4-(-2)}$ ✓m/a

$= \frac{6}{-2}$

$= -3$ ✓ca

RP

(b) Determine the gradient of the line PR, if PQ ⊥ PR. (1)

$m_{PR} = \frac{1}{3}$ ✓

RP.

(c) Hence, determine the value of *a*. (3)

(ca) $\frac{1}{3} = \frac{-4+1}{-2-a}$ ✓a.

$-2-a = -12+3$ ✓ca

$7 = a$ ✓ ca.

RP

(d) Calculate the area of Δ QPR. (4)

area Δ QPR $= \frac{1}{2}.PQ.PR$ RP.

$PQ = \sqrt{(-4+2)^2+(2+4)^2} = \sqrt{4+36} = \sqrt{40} = 2\sqrt{10}.$ ✓a

$PR = \sqrt{(-2-a)^2+(-4+1)^2} = \sqrt{81+9} = 3\sqrt{10}.$ ✓a

area $= \frac{1}{2}.2\sqrt{10}.3\sqrt{10}$ ✓ca.

$= 30\ u^2$ ✓ ca.

(e) Determine the coordinates of the midpoint M of QR. (1)

$M\left(\frac{-4+7}{2} ; \frac{2-1}{2}\right)$

$= \left(\frac{3}{2} ; \frac{1}{2}\right)$ ✓a

RP

(f) Hence, determine the equation of the line MN passing through M and parallel to PR. (3)

// to PR ∴ $m = \frac{1}{3}$

1 pt $\left(\frac{3}{2}; \frac{1}{2}\right)$ ✓ca

$y - \frac{1}{2} = \frac{1}{3}\left(x - \frac{3}{2}\right)$ ✓ca from (e)

$y = \frac{1}{3}x$. ✓ ca.

RP

#check.

$\tan \hat{Q} = \frac{PR}{PQ}$ ✓✓

$\hat{Q} = 56{,}31°$

(g) Find the size of $\hat{Q}$. (5)

$\tan \theta = m_{PQ} = -3$ ✓a

$\theta = 180 - 71{,}565$ ✓ca

$= 108{,}4°$

$m_{QR} = \frac{2+1}{-4-7} = \frac{3}{-11} =$

$\tan \alpha = \frac{-3}{11}$ ✓ a

$\alpha = 180 - 15{,}255$

$= 164{,}74$ ✓ ca

ext ∠ Δ.

$\therefore \theta = 164{,}74 - 108{,}4$

$= 56{,}3°$ ✓ ca.

CP.

[19]

$m_{QP} = 3$

$\tan \beta = 3$

$\beta = -71{,}57°$

QUESTION 2

The frequency table below represents the marks out of a maximum of 150 marks, obtained by a group of Grade 11 students in a Mathematics examination.

Marks Obtained	Frequency f	Cumulative Frequency
$0 < x \leq 30$	6	6
$30 < x \leq 60$	12	18
$60 < x \leq 90$	46	64
$90 < x \leq 120$	42	106
$120 < x \leq 150$	14	120

✓ ✓ a

RP

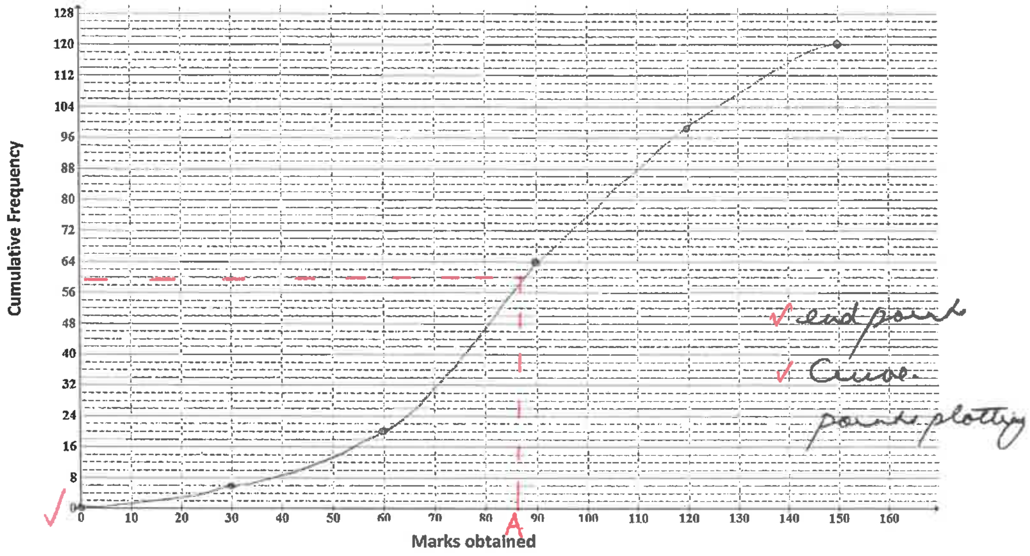
(a) Use the table to complete the cumulative frequency column. (2)

$\frac{\sin Q}{9{,}49} = \frac{\sin 90°}{11{,}40}$

$Q = 56{,}35°$

RP

(b) On the grid below, draw an Ogive, using the information from the table above. (3)



(c) Use the Ogive

(i) to determine the median. (show where you took your reading) (1)

87 (reading at A) ✓ need both parts of this answer for the mark

R.P.

(ii) Comment on the skewness of the data, showing all necessary calculations to prove your reasoning. (3)

$\max - Q_2 = 150 - 87 = 63$ ✓ ca.

$Q_2 - \min = 87 - 0 = 87$ ✓ ca.

negatively skewed ✓ a

$Q_2 - \min > \max - Q_2$

CP

#check

(d) Complete the table below: (3) RP.

Marks Obtained	Midpoint x_i	Frequency f	$f \times x_i$
$0 < x \leq 30$	15	6	90.
$30 < x \leq 60$	45	12	540
$60 < x \leq 90$	75	46	3450
$90 < x \leq 120$	105	42	4410.
$120 < x \leq 150$	135 ✓	14	1890 ✓a
		120.	$\Sigma(f \times x_i)=$ 10380 ✓ca

Using the information from the table above, determine the estimated mean. (2) RP.

approx mean $= \dfrac{10380}{120}$

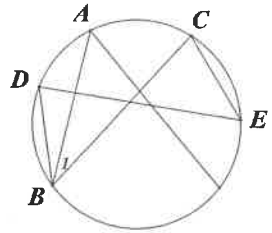
$= 86,5$

[14]

QUESTION 3

Circle the correct solution only.

(a)



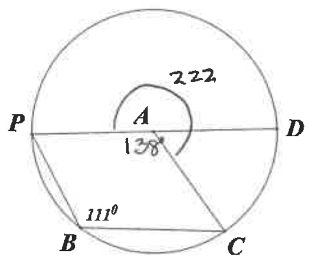
A, B, C, D, and E are points on the circumference of the circle.

Which statement is true? (1) K

A. $\hat{A} = \hat{C}$ B. $\hat{D} = \hat{C}$

C. $\hat{A} = \hat{D}$ D. $\hat{B}_1 = \hat{E}$

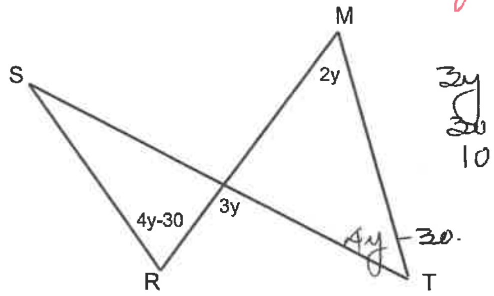
(b)



A is the centre of the circle, PAD is a straight line, and $\hat{B} = 111°$. Determine the magnitude of $C\hat{A}D$. (2) RP

A. 69° B. 62°

C. 59° D. 42°



if working showed

$3y = 2y + 4y - 30$ ✓

$30 = 3y$

$10 = y$

S, R, T and M lie on the circumference of a circle.

Determine the numerical value of y. (2)

A. 35°   B. 30°

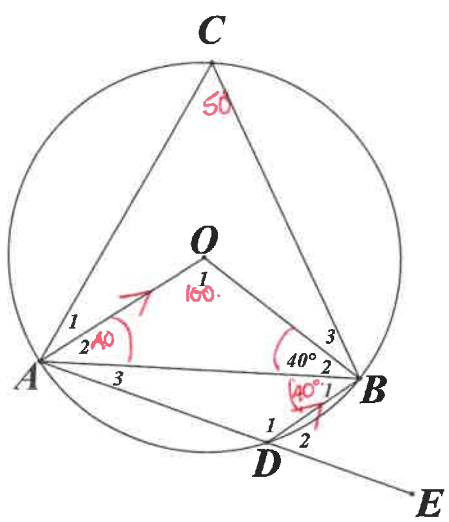
C. 20°   D. 10° ✓

CP.

[5]

QUESTION 4

Refer to the figure below:



O is the centre of the circle CADB. ADE is a straight line. $\hat{B}_2 = 40°$.

Determine the following, **stating all necessary reasons**:

(a) $\hat{C}$. (4)

$\hat{O} = 100°$ ✓a ∠'s in isos △ ; equal radii ✓

∴ $\hat{C} = 50°$ ✓a ∠ at centre = 2∠ on circumf ✓

RP

(b) $\hat{D}_2$ (1)

$\hat{D}_2 = 50°$ ext ∠ cyclic quad ✓

Must have the reason to get the mark.

RP

(c) $\hat{A}_3$ if AO // DB. (2)

$\hat{B}_1 = 40°$ alt ∠'s = ✓ AO // DB.

$\hat{D}_2 = 50 = \hat{A}_3 + \hat{B}_1$ ext ∠ △ADB

$50 = A_3 + 40$

$\hat{A}_3 = 10°$ ✓

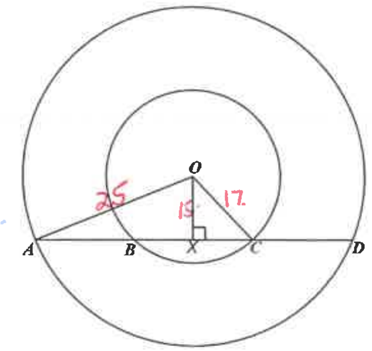
RP

[7]

② 10 Q6 – 9. (47 marks) 11

QUESTION 5

Refer to the diagram below:



In the diagram, O is the centre of two concentric circles.

ABCD is a straight line that intersects the circle as shown.

OX ⊥ AD; OA = 25 cm; OC = 17 cm; OX = 15 cm.

(a) Determine, with reasons, the length of AC. (4) RP

In $\triangle OXC$ $XC^2 = 17^2 - 15^2$ ✓m

$XC = 8$ ✓ca

In $\triangle OAX$ $25^2 - 15^2 = AX^2$ $625 - 225 = 400$

$AX = 20$ ✓a

$\therefore AC = 28$ ✓ca

(b) Prove, with reasons, that AB = CD. (3) RP

$BX = XC$ } line from centre ⊥ to chord ✓

and $AX = XD$ } ✓ line from centre ⊥ to chord ✓

$\therefore AB = CD$

[7]

QUESTION 6

Simplify, without the aid of a calculator. Show **all** calculations.

(a) $\dfrac{3.\cos 150°.\sin 270°}{\tan(-45°)+\cos 600°}$ (6) RP

$\dfrac{3.(-\cos 30).(-1)}{-\tan 45 - \cos 60}$ ✓✓✓✓

$= \dfrac{3.\frac{\sqrt{3}}{2}}{-1-\frac{1}{2}}$ ✓ca $= -\dfrac{\cancel{3}\sqrt{3}}{\cancel{2}} \times \dfrac{\cancel{2}}{\cancel{3}} = -\sqrt{3}$ ✓

(b) $\dfrac{\sin(180°-\theta).\sin(90°+\theta).\sin 310°}{\cos(-\theta).\sin(360°-\theta).\cos 140°}$ (7) RP

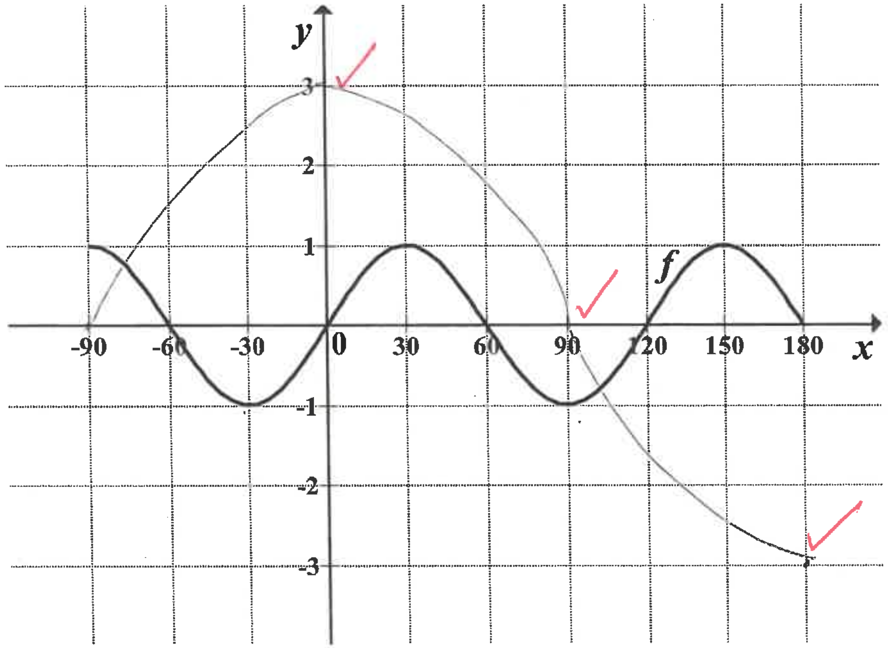
$= \dfrac{\cancel{\sin\theta}.\cancel{\cos\theta}.-\cancel{\sin 50}}{\cancel{\cos\theta}.-\cancel{\sin\theta}.-\cancel{\cos 40}}$ ✓✓✓✓✓✓

$= -1$ ✓ca

[13]

QUESTION 7

The graph $f(x) = \sin 3x;\ x \in [-90^0\ ;180^0\ ]$, is drawn below.



(a) Write down the value(s) of x, which satisfy the equation $\sin 3x = -1$,

in the interval $x \in [-90^0\ ;180^0\ ]$ (2)

$x = -30°$ or $90°$

RP

(b) Given $h(x) = f(x) - 2$, determine the maximum value of h. (1)

-1

CP.

(c) Draw the graph of $g(x) = 3\cos x$ for $x \in [-90^0\ ;180^0\ ]$ on the same system of axes, as f (3)

RP

(e) Use the graphs to determine the **number of solutions** that exist for the equation

$\frac{\sin 3x}{3} - \cos x = 0$ in the interval $x \in [-90^0\ ;180^0\ ]$. (2)

$\sin 3x = 3\cos x$ OR show on graph. CP.

2 solutions. ($x = -80°$ or $102°$)

ca. from graph.

(f) **Use the graphs** to solve for x if $x \in [-90^0\ ;180^0\ ]$:

$f(x).g(x) \le 0$ (3)

$x \in [-60°;\ 0°]$

or $x \in [60°;\ 90°]$

or $x \in [120°;\ 180°]$ Accept any 3.

or $x = -90$

CP

(i) If the graph of $f(x)$ is shifted 30° to the left, give the new equation. (1)

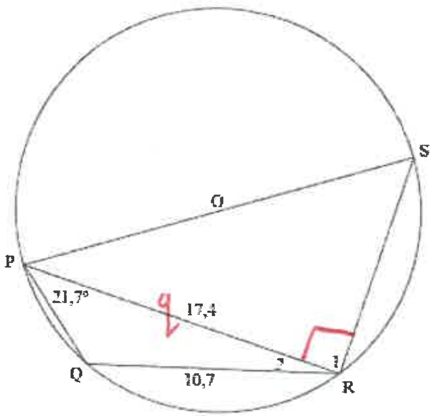
$g(x) = \sin 3(x + 30)$

CP

[12]

QUESTION 8

The accompanying diagram shows a cyclic quadrilateral PQRS with $\hat{Q} > 90°$. PS is a straight line. O is the centre of the circle. PR = 17,4 units, QR = 10,7 units and $Q\hat{P}R = 21,7°$.



Determine, giving reasons where relevant:

(a) The size of $\hat{Q}$ (3) RP

In △PQR

$$\frac{\sin Q}{17,4} = \frac{\sin 21,7°}{10,7}$$

$$\sin Q = \frac{17,4 \times \sin 21,7}{10,7}$$ ambiguous case.

$\hat{Q} = \cancel{36,96°}$ or 143,04 ca.

(b) The size of $\hat{S}$ (2) RP

36,96° ca (opp ∠'s cyclic quad)

(c) The length of the diameter of the circle. (4) RP

$\hat{R}_1 = 90°$ ∠ in semi circle.

In △PRS

$$\frac{PS}{17,4} = \frac{1}{\sin 36,96}$$

$$PS = \frac{17,4}{\sin 36,96}$$

$= 28,94$

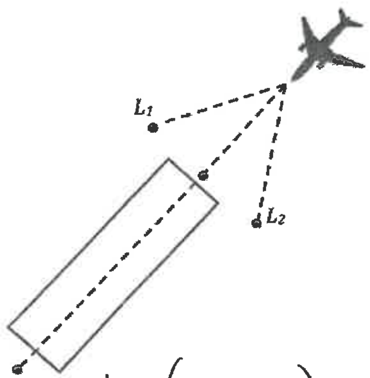
[9]

SECTION B

QUESTION 9

(a) The pilot of a plane coming in to land has to make sure that his plane is constantly equidistant from the two outer landing lights L_1 and L_2. The line of landing lights is at a right angle to the runway. The coordinates of L_1 and L_2 are $(16;30)$ and $(20;25)$ respectively. Find the equation of his flight path in the form $ax+by+c=0$. (6)

CP



$L_1(16;30)$ $L_2(20;25)$

$m_{L_1L_2} = \frac{30-25}{16-20} = \frac{5}{-4}$ ✓a

$\therefore m_{\text{flight path}} = \frac{4}{5}$ ✓ca

$M_{L_1L_2} = \left(\frac{16+20}{2};\frac{30+25}{2}\right)$

$= (18; 27\tfrac{1}{2})$ ✓a

$y - 27\tfrac{1}{2} = \frac{4}{5}(x-18)$ ✓ca

$y = \frac{4}{5}x - \frac{72}{5} + \frac{55}{2}$

generous.

or rearrange.

$10y = 8x - 144 + 275$ ✓ca

$-8x + 10y - 131 = 0$

$8x - 10y + 131 = 0$ ✓ca

$y = \frac{4}{5}x + c$

$\frac{55}{2} = \frac{4}{5}(18) + c$ ✓ca

$\frac{131}{10} = c$

$y = \frac{4}{5}x + \frac{131}{10}$ ✓ca

$0 = \frac{4}{5}x - y + \frac{131}{10}$ ✓ca

(b) The equation of a straight line AB is given by $y = -\frac{2}{3}x + 2$

The equation of the straight line CD is given by $3x + ry = -2;\ r \neq 0$

Determine the value(s) of r such that:

$y = -\frac{3x}{r} - \frac{2}{r}$ ✓

(i) CD // AB (3)

CP

$m_{CD} = m_{AB}$

$-\frac{3}{r} = -\frac{2}{3}$ ✓m

$9 = 2r$

$r = \frac{9}{2}$ ✓ 4,5

(ii) If the angle of inclination of the line AB is the same as that as the line CD, solve for r. (4)

CP

incl. AB = incl CD

AB // CD ✓ $m \rightarrow \tan\theta = \tan\theta$

$\theta = -33,6\ldots$ etc.

$-\frac{2}{3} = -\frac{3}{r}$ ✓ ✓a

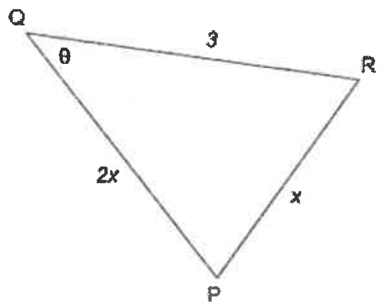
$r = \frac{9}{2}$ ✓ca 4,5.

[13]

QUESTION 10

[7]

In ΔPQR, $QR = 3$ units, $PR = x$ units, $PQ = 2x$ units and $\hat{Q} = \theta$



(a) Show that $\cos\theta = \frac{x^2+3}{4x}$ (3)

using cosine Rule. CP

$\cos\theta = \frac{(2x)^2 + 3^2 - x^2}{2 \cdot 2x \cdot 3} = \frac{3x^2 + 9}{12x}$

$= \frac{x}{4} + \frac{3}{4x}$

$= \frac{4x^2+12}{4x}$

(b) Hence, or otherwise, calculate the value of x for which a solution to the equation $\cos\theta = \frac{x^2+3}{4x}$ exists. (4)

Marked very generously

CP/PS.

$\frac{x^2+3}{4x} < 1$

$x^2 + 3 < 4x$

$x^2 - 4x + 3 < 0$

$(x-3)(x-1) < 0$ 1 3

$x \in (1; 3)$

if got 1 and 3 only give 1/4

or $\frac{x^2+3}{4x} > -1$ $x^2 + 4x + 3 > 0$

$x^2 + 3 > -4x$ $(x+3)(x+1) > 0$.

−3 −1.

QUESTION 11

(a) Solve for θ:

(i) $2\sin\theta + 3\cos\theta = 0$ for $\theta \in [-90°; 270°]$ (4)

CP

$2\sin\theta = -3\cos\theta$

$\tan\theta = -\frac{3}{2}$

56,31 56,31

$\theta = -56,31°$ or $\theta = 123,69°$

(ii) Give the general solution of $3 + 3\sin\theta - \cos^2\theta = 0$ (5)

CP

$3 + 3\sin\theta - (1 - \sin^2\theta) = 0$

$\sin^2\theta + 3\sin\theta + 2 = 0$

$(\sin\theta + 1)(\sin\theta + 2) = 0$

$\sin\theta = -1$ or $\sin\theta = -2$

$\theta = 270° + n360°$ no soln.

$n \in \mathbb{Z}$

(b) Show that

$$\frac{\sin\theta}{1+\cos\theta}+\frac{1+\cos\theta}{\sin\theta}=\frac{2}{\sin\theta}$$

(6) CP

Lhs = $\dfrac{\sin^2\theta + (1+\cos\theta)(1+\cos\theta)}{\sin\theta(1+\cos\theta)}$

$= \dfrac{\sin^2\theta + 1 + 2\cos\theta + \cos^2\theta}{\sin\theta(1+\cos\theta)}$

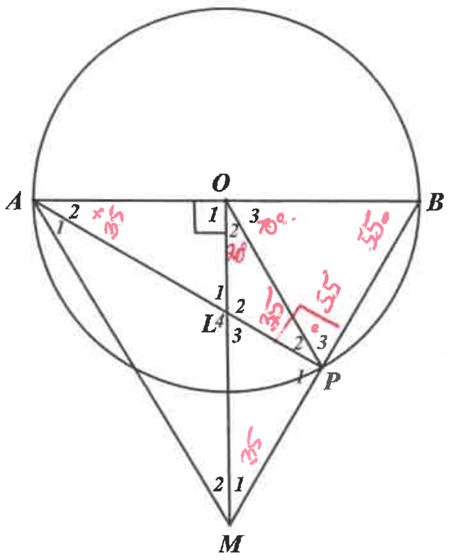
$= \dfrac{2 + 2\cos\theta}{\sin\theta(1+\cos\theta)} = \dfrac{2\cancel{(1+\cos\theta)}}{\sin\theta\cancel{(1+\cos\theta)}}$

= rhs.

[15]

This page has been left blank. It can be used for additional working.

QUESTION 12



In the diagram, O is the centre of circle ABP.

BP is produced to M, such that $MO \perp AB$.

AP intersects OM at L. $\hat{O}_2 = 20°$

(a) Calculate, the following, **stating all necessary reasons:**

(i) $\hat{A}_2$ = 35° ∠ at centre = 2∠ at circum (3)

$\hat{O}_3 = 70°$ circum ∠'s at str line

RP

(ii) $\hat{P}_1$ = 90° $\hat{P}_2 + \hat{P}_3 = 90°$ ∠ in semi circle. (2)

∠'s on str line.

RP

(b) Prove, with reasons, that AOPM is a cyclic quadrilateral. (2)

$\hat{O}_1 = 90°$ given

$\hat{P}_1 = 90°$ proved above.

$\therefore \hat{O}_1 = \hat{P}_1$

$\therefore$ AOPM is cyclic CONV ∠'s in same segment.

CP

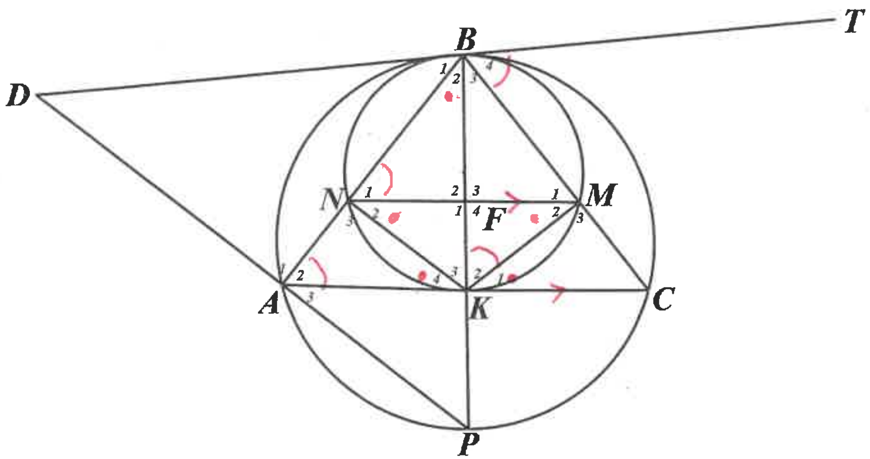
(c) Name, with a reason, ONE other cyclic quadrilateral in the diagram. (1)

OBPL (conv. ext ∠ = int opp ∠'s. or. opp ∠'s suppl.)

CP

[8]

QUESTION 13



In the given diagram:

- DBT is a common tangent to circles BNKM and BAPC, at B.
- AKC is also a tangent to the smaller circle at K.
- MN // CA.

Prove, with reasons:

ΔKMN is isosceles. (4)

$\hat{K}_1 = \hat{N}_2$ tan chord Th. ✓

$\hat{N}_2 = \hat{K}_4$ alt ∠s =; MN // AC ✓

$\hat{K}_4 = \hat{M}_2$ tan chord Theor. ✓

$\therefore \hat{N}_2 = \hat{M}_2$

$\therefore \Delta$KMN is isos. ✓ base ∠s =

CP

[4]

QUESTION 14

$SA_{sphere} = 4\pi r^2$ $\qquad$ $Vol_{sphere} = \frac{4}{3}\pi r^3$

Figure 1

Figure 2

(a) A hemisphere has a total volume of $1000\ cm^3$, as shown in Figure 1.

What is the radius of the hemisphere? (3)

$V = 1000 = \frac{2}{3}\pi r^3$ ✓a

$\frac{3 \times 1000}{2\pi} = r^3$ ✓ca

$= 477,464$

$\therefore r = 7,82$ ✓ca

if used $\frac{4}{3}\pi r^3 = 1000$ RP

$r = 6,2$

max 2/3

(b) A smaller hemisphere is scooped out from the larger hemisphere to create a container, as shown in Figure 2. The diameter of the larger hemisphere is twice that of the smaller hemisphere.

(i) Find the volume of the container. (2)

$V_{smaller} = \frac{2}{3}\pi\left(\frac{7,82}{2}\right)^3$ ✓ca

$= 125\ cm^3$

$\frac{2}{3}\pi(3,1)^2$ RP

(ii) Find the surface area of the container. (4)

$SA_{large} = 2\pi(7,82)^2 = 384,23$ ✓ca

$SA_{small} = 2\pi\left(\frac{7,82}{2}\right)^2 = 96,06$ ✓ca

$SA_{rim} = \pi(7,82)^2 - \pi\left(\frac{7,82}{2}\right)^2 = 192,12 - 48,03$

$= 144,09$ ✓ca

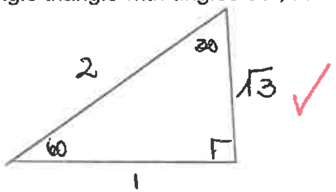
$\therefore$ Total SA $= 624,38\ cm^2$ ✓ca

PS

[9]

QUESTION 15

(a) Draw a special angle triangle with angles $30°, 60°$ *and* $90°$. Show the lengths of the sides. (1)

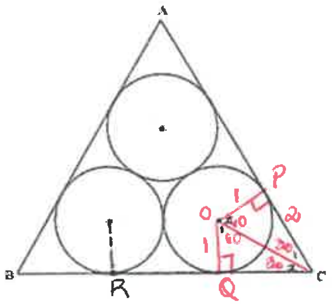


K

(b) Hence, consider the diagram below.

Three circles of radius 1, fit snugly into the equilateral ΔABC and they just touch each other as well as the sides of the triangle, as shown.

Determine the area of ΔABC without the use of a calculator. Leave your answer in simplified surd form. (7)



PS

$\triangle OPC \equiv \triangle OQC$ rhs or SSS

$PC = QC$ equal tang.

$\therefore \hat{C}_1 = \hat{C}_2 = 30°$ $\quad OC$ common

$\hat{O}_1 = \hat{O}_2 = 60°$ $\quad OP = OQ$ equal radii

$\therefore \tan 60° = \dfrac{QC}{1}$

$QC = \sqrt{3}$

$RQ = 2 \quad BR = \sqrt{3}$

$\therefore BC = 2 + 2\sqrt{3}$

$\therefore$ area $\triangle ABC = \frac{1}{2} . AC . BC . \sin C$

$= \frac{1}{2}(2+2\sqrt{3})^2 . \sin 60$

$= (4\sqrt{3} + 6)\ u^2$

[8]

12, 93

This page has been left blank and can be used for additional working