BALLITO

Mathematics
Paper 2

FORM 4
22 November 2019

TIME: 3 hours	**TOTAL: 150 marks**
Examiner: Mrs A Gunning	**Moderators: Miss A Rohrs; Miss M Eastes**

Name

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY BEFORE ANSWERING THE QUESTIONS.

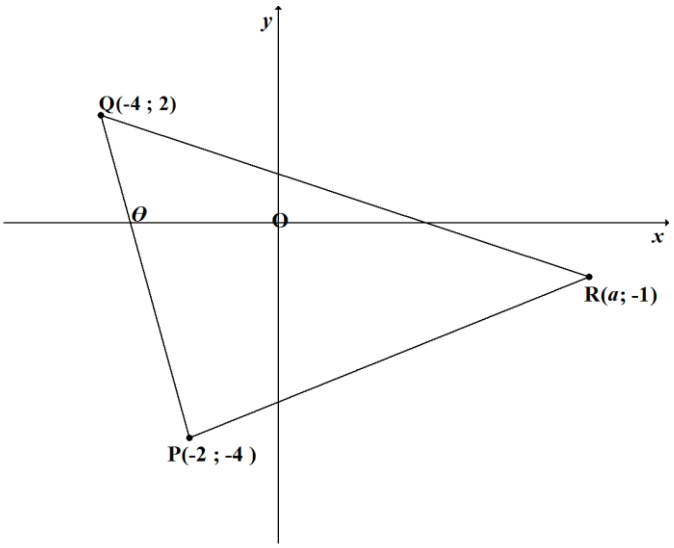
- This question paper consists of 28 pages. A separate coloured formula sheet has been included.
- Please check that your question paper is complete.
- Answer all questions on this question paper
- Read and answer all questions carefully.
- It is in your own interest to write legibly and to present your work neatly.
- All necessary working which you have used in determining your answers **must** be clearly shown.
- Approved non-programmable calculators may be used except where otherwise stated. Where necessary give answers correct to **2 decimal places** unless otherwise stated.
- Ensure that your calculator is in DEGREE mode.
- Diagrams have not necessarily been drawn to scale.
- **Give reasons for all statements used in Geometry.**

Question	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	Total
Out of	19	14	5	7	7	13	12	9	13	7	15	8	4	9	8	150
Mark																

SECTION A

QUESTION 1

Refer to the sketch below:



In the diagram, P(–2 ; –4); Q(–4 ; 2); and R(a; –1) are the vertices of Δ QPR.

(a) Determine the gradient of the line PQ. (2)

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(b) Determine the gradient of the line PR, if PQ $\perp$ PR. (1)

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(c) Hence, determine the value of *a.* (3)

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(d) Calculate the area of Δ QPR. (4)

(e) Determine the coordinates of the midpoint M of QR. (1)

(f) Hence, determine the equation of the line MN passing through M and parallel to PR. (3)

(g) Find the size of $\hat{Q}$. (5)

[19]

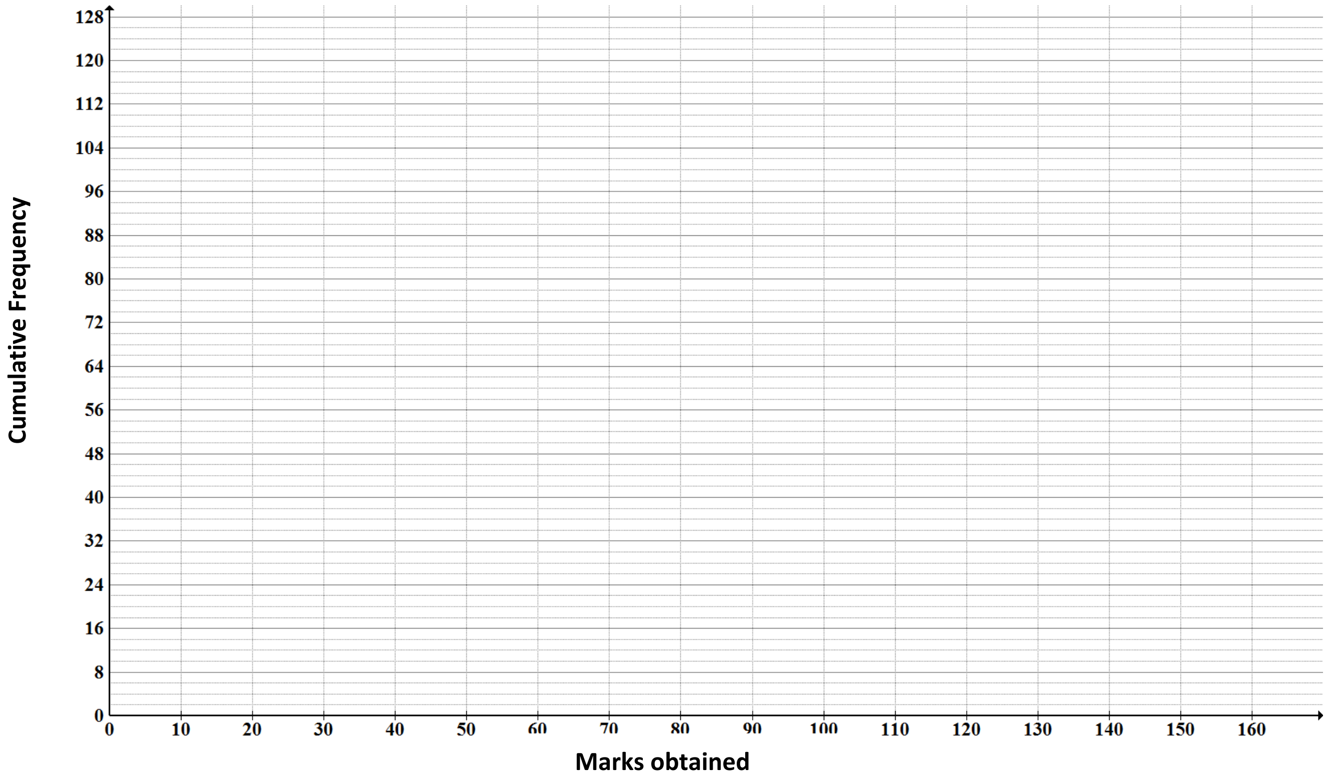
QUESTION 2

The frequency table below represents the marks out of a maximum of 150 marks, obtained by a group of Grade 11 students in a Mathematics examination.

Marks Obtained	**Frequency *f***	**Cumulative Frequency**
$0 < x \leq 30$	6	
$30 < x \leq 60$	12	
$60 < x \leq 90$	46	
$90 < x \leq 120$	42	
$120 < x \leq 150$	14	

(a) Use the table to complete the cumulative frequency column. (2)

(b) On the grid below, draw an Ogive, using the information from the table above. (3)



(c) Use the Ogive

(i) to determine the median. (show where you took your reading) (1)

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(ii) Comment on the skewness of the data, showing all necessary calculations to prove your reasoning. (3)

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(d) Complete the table below: (3)

Marks Obtained	**Midpoint** x_i	**Frequency** f	$f \times x_i$
$0 < x \leq 30$		6	
$30 < x \leq 60$		12	
$60 < x \leq 90$		46	
$90 < x \leq 120$		42	
$120 < x \leq 150$		14	
			$\Sigma(f \times x_i) =$

Using the information from the table above, determine the estimated mean. (2)

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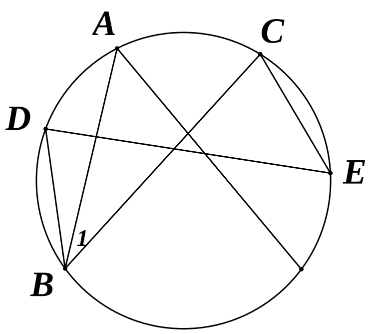
[14]

QUESTION 3

Circle the correct solution only.

(a)



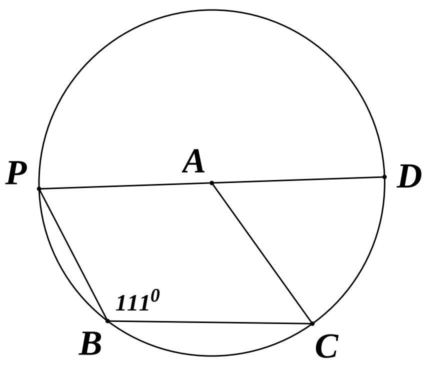
A, B, C, D, and E are points on the circumference of the circle.

Which statement is true? (1)

A. $\hat{A} = \hat{C}$ B. $\hat{D} = \hat{C}$

C. $\hat{A} = \hat{D}$ D. $\hat{B}_1 = \hat{E}$

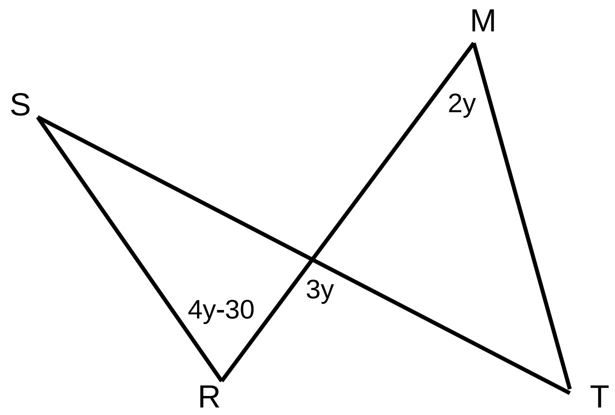
(b)



A is the centre of the circle, PAD is a straight line, and $\hat{B} = 111^\circ$. Determine the magnitude of $C\hat{A}D$. (2)

A. 69° B. 62°

C. 59° D. 42°



S, R, T and M lie on the circumference of a circle.

Determine the numerical value of *y.* (2)

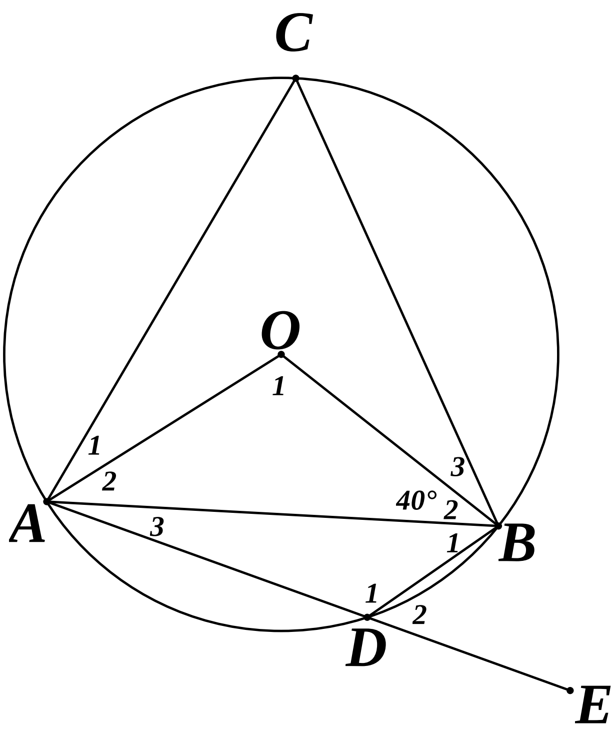
A. $35°$ B. $30°$

C. $20°$ D. $10°$

[5]

QUESTION 4

Refer to the figure below:



O is the centre of the circle CADB. ADE is a straight line. $\hat{B}_2 = 40^\circ$.

Determine the following, **stating all necessary reasons**:

(a) $\hat{C}$. (4)

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(b) $\widehat{D}_2$ (1)

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(c) $\hat{A}_3$ if AO // DB. (2)

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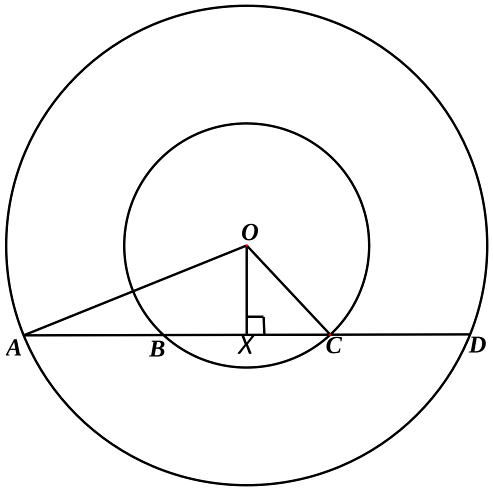
[7]

QUESTION 5

Refer to the diagram below:



In the diagram, O is the centre of two concentric circles.

ABCD is a straight line that intersects the circle as shown.

OX $\perp$ AD; OA = 25 cm; OC = 17 cm; OX = 15 cm.

(a) Determine, with reasons, the length of AC. (4)

(b) Prove, with reasons, that AB = CD. (3)

[7]

QUESTION 6

Simplify, without the aid of a calculator. Show **all** calculations.

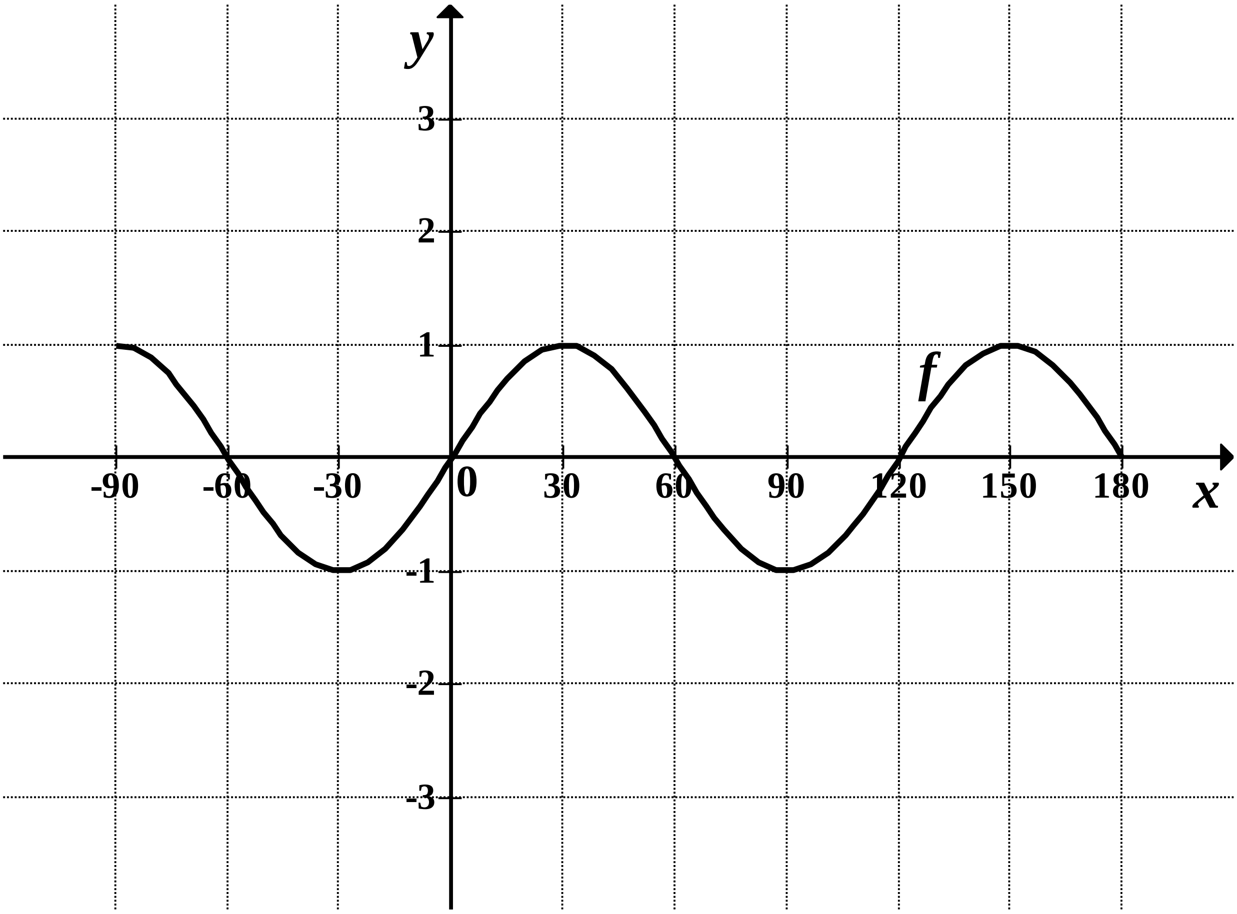
(a) $\dfrac{3.cos\ 150°.\sin 270°}{\tan(-45°)+\cos 600°}$ (6)

(b) $\dfrac{\sin(180°-\theta).\sin(90°+\theta).\sin 310°}{\cos(-\theta).\sin(360°-\theta).\cos 140°}$ (7)

[13]

QUESTION 7

The graph $f(x) = sin\,3x$; $x \in [-90^0 ; 180^0]$, is drawn below.



(a) Write down the value(s) of *x*, which satisfy the equation $\sin 3x = -1$,

in the interval $x \in [-90^0 ; 180^0]$ (2)

(b) Given $h(x) = f(x) - 2$, determine the maximum value of *h*. (1)

(c) Draw the graph of $g(x) = 3\cos x$ for $x \in [-90^0 ; 180^0]$ on the same system of axes, as *f* (3)

(e) Use the graphs to determine the **number of solutions** that exist for the equation

$\frac{\sin 3x}{3} - \cos x = 0$ in the interval $x \in [-90^0 ; 180^0]$. (2)

(f) **Use the graphs** to solve for x if $x \in [-90^0 ; 180^0]$:

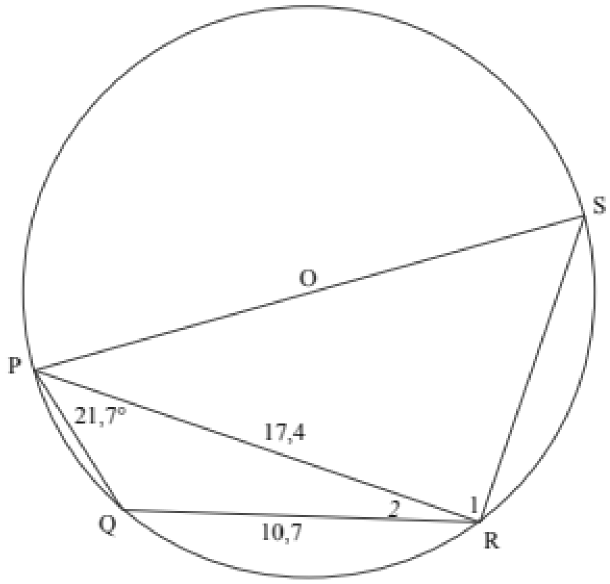
$f(x).g(x) \leq 0$ (3)

(i) If the graph of $f(x)$ is shifted $30°$ to the left, give the new equation. (1)

[12]

QUESTION 8

The accompanying diagram shows a cyclic quadrilateral PQRS with $\hat{Q} > 90°$. PS is a straight line. O is the centre of the circle. PR =17,4 units, QR = 10,7 units and $Q\hat{P}R = 21,7°$.



Determine, giving reasons where relevant:

(a) The size of $\hat{Q}$ (3)

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(b) The size of $\hat{S}$ (2)

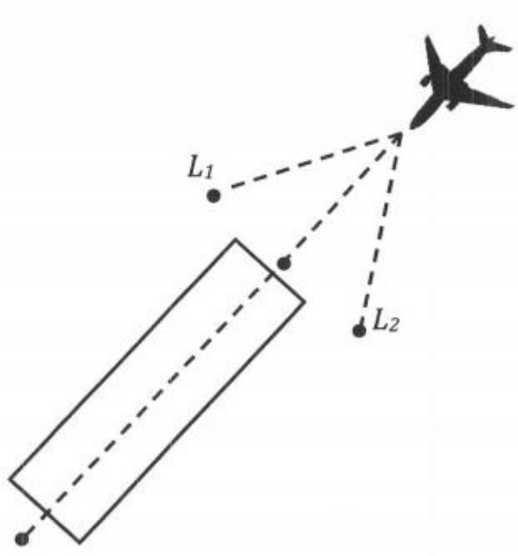
(c) The length of the diameter of the circle. (4)

[9]

SECTION B

QUESTION 9

(a) The pilot of a plane coming in to land has to make sure that his plane is constantly equidistant from the two outer landing lights L_1 and L_2. The line of landing lights is at a right angle to the runway. The coordinates of L_1 and L_2 are $(16; 30)$ and $(20; 25)$ respectively. Find the equation of his flight path in the form $ax + by + c = 0$. (6)



(b) The equation of a straight line AB is given by $y = -\frac{2}{3}x + 2$

The equation of the straight line CD is given by $3x + ry = -2$; $r \neq 0$

Determine the value(s) of r such that :

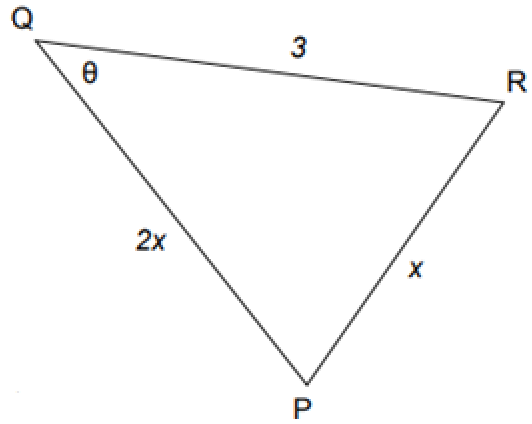
(i) CD // AB (3)

(ii) If the angle of inclination of the line AB is the same as that as the line CD, solve for r. (4)

[13]

QUESTION 10

In ΔPQR, $QR = 3$ units, $PR = x$ units, $PQ = 2x$ units and $\hat{Q} = \theta$



(a) Show that $\cos\theta = \frac{x^2+3}{4x}$ (3)

(b) Hence, or otherwise, calculate the value of x for which a solution to the equation $\cos\theta = \frac{x^2+3}{4x}$ exists. (4)

[7]

QUESTION 11

(a) Solve for θ:

(i) $2\sin\theta + 3\cos\theta = 0$ for $\theta \in [-90°; 270°]$ (4)

(ii) Give the general solution of $3 + 3\sin\theta - cos^2\theta = 0$ (5)

(b) Show that

$$\frac{\sin\theta}{1+\cos\theta}+\frac{1+\cos\theta}{\sin\theta}=\frac{2}{\sin\theta}$$

(6)

[15]

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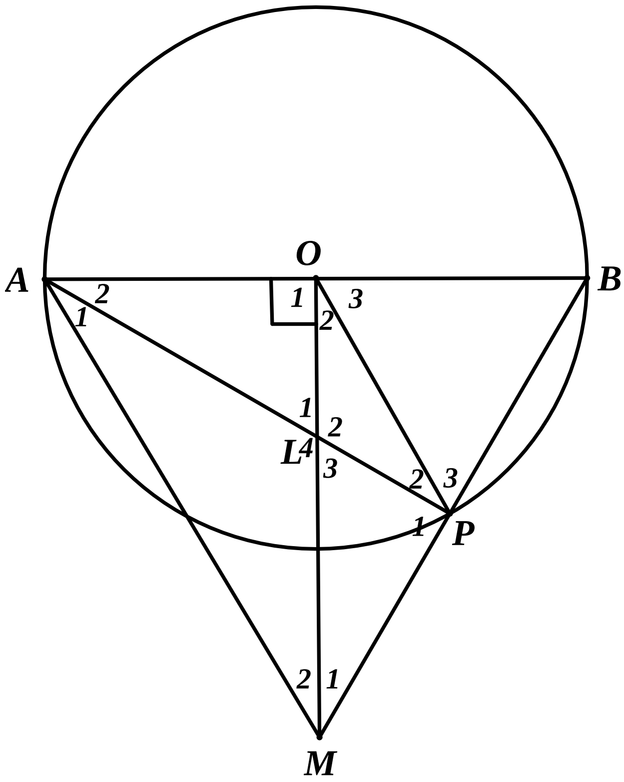
QUESTION 12



In the diagram, O is the centre of circle ABP.

BP is produced to M, such that $MO \perp AB$.

AP intersects OM at L. $\hat{O}_2 = 20^\circ$

(a) Calculate, the following, **stating all necessary reasons**:

(i) $\hat{A}_2$ (3)

(ii) $\hat{P}_1$. (2)

(b) Prove, with reasons, that AOPM is a cyclic quadrilateral. (2)

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(c) Name, with a reason, ONE other cyclic quadrilateral in the diagram. (1)

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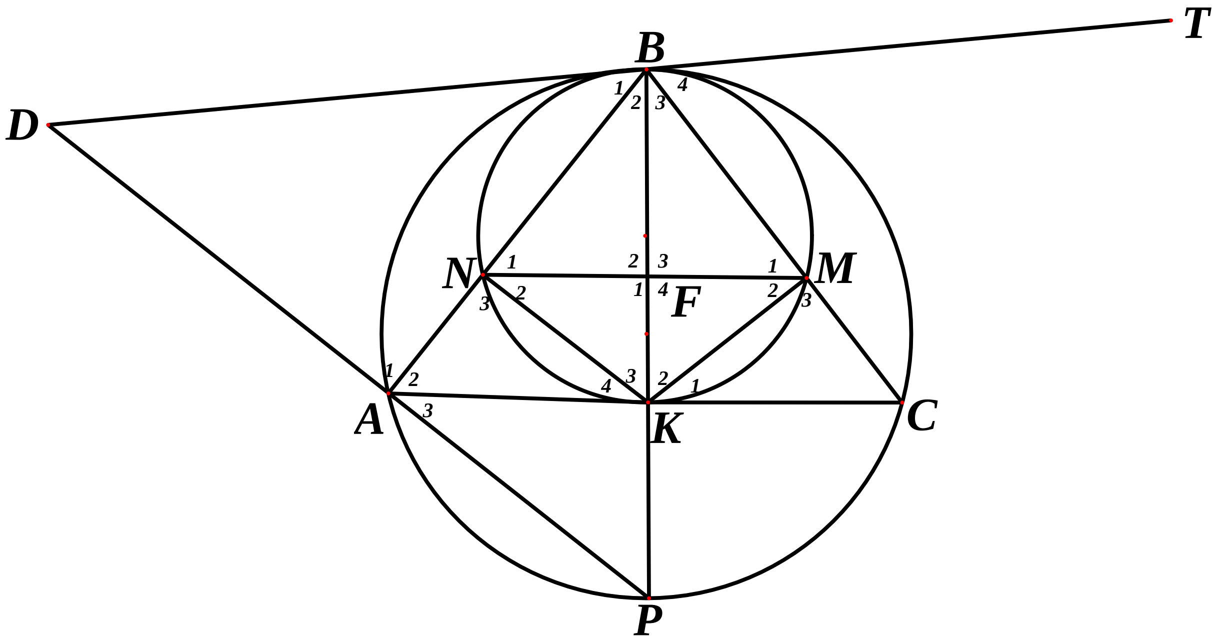
[8]

QUESTION 13



In the given diagram:

- DBT is a common tangent to circles BNKM and BAPC, at B.
- AKC is also a tangent to the smaller circle at K.
- MN // CA.

Prove, with reasons:

ΔKMN is isosceles. (4)

[4]

QUESTION 14

$SA_{sphere} = 4\pi r^2$ $\qquad$ $Vol_{sphere} = \frac{4}{3}\pi r^3$

Figure 1

Figure 2

(a) A hemisphere has a total volume of $1000\ cm^3$, as shown in Figure 1.

What is the radius of the hemisphere? (3)

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(b) A smaller hemisphere is scooped out from the larger hemisphere to create a container, as shown in Figure 2.The diameter of the larger hemisphere is twice that of the smaller hemisphere.

(i) Find the volume of the container. (2)

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(ii) Find the surface area of the container. (4)

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[9]

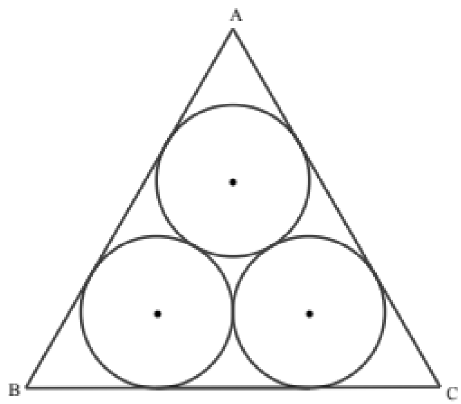
QUESTION 15

(a) Draw a special angle triangle with angles $30°, 60°\ and\ 90°$. Show the lengths of the sides. (1)

(b) Hence, consider the diagram below.

Three circles of radius 1, fit snugly into the equilateral ΔABC and they just touch each other as well as the sides of the triangle, as shown.

Determine the area of ΔABC without the use of a calculator. Leave your answer in simplified surd form. (7)



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