Province of the
EASTERN CAPE
EDUCATION

NATIONAL SENIOR CERTIFICATE
NASIONALE SENIOR SERTIFIKAAT

GRADE/*GRAAD* 11

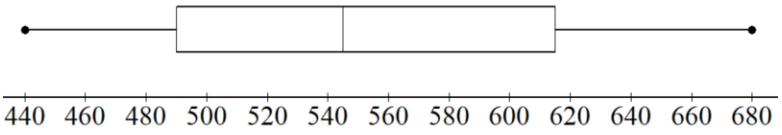
NOVEMBER 2019

MATHEMATICS P2/*WISKUNDE V2*
MARKING GUIDELINE/*NASIENRIGLYN*

MARKS/*PUNTE*: 150

This marking guideline consists of 13 pages.
Hierdie nasien riglyn bestaan uit 13 bladsye.

QUESTION 1/*VRAAG 1*

1.1	$\overline{x}$ = 549,5	✓✓ answer/*antwoord*	(2)
1.2	SD = 69,08	✓ answer/*antwoord*	(1)
1.3	Min/*Min* : 440 Q_1 : 490 Q_2 : 545 Q_3 : 615 Max/*Maks* : 680  	✓ Min and max/ *Min en maks* ✓ Q_1 ✓ Q_2 ✓ Q_3 ✓ box correctly in place / *diagram korrek geteken*	(5)
1.4	Data skewed slighly right as mean > median	✓✓ slightly skewed to the right/ positively skewed *effens skeef na regs / positief skeef*	(2)
1.5	Snack bars have greater variety in energy levels as the SD is greater than that of the cereals which means the data is more widely spread about the mean. *Peuselstaffies het 'n groter verskeidenheid in energievlakke omdat die SA groter as die van die graanpapsoorte, wat beteken dat die data meer wyd verspreid rondom die gemiddelde is.*	✓ snack bars / *peuselstaffies* ✓ greater SD hence / *groter SA* ✓greater spread about the mean / *groter verspreiding om die gemiddelde*	(3)
			[13]

QUESTION 2/*VRAAG 2*

2.1

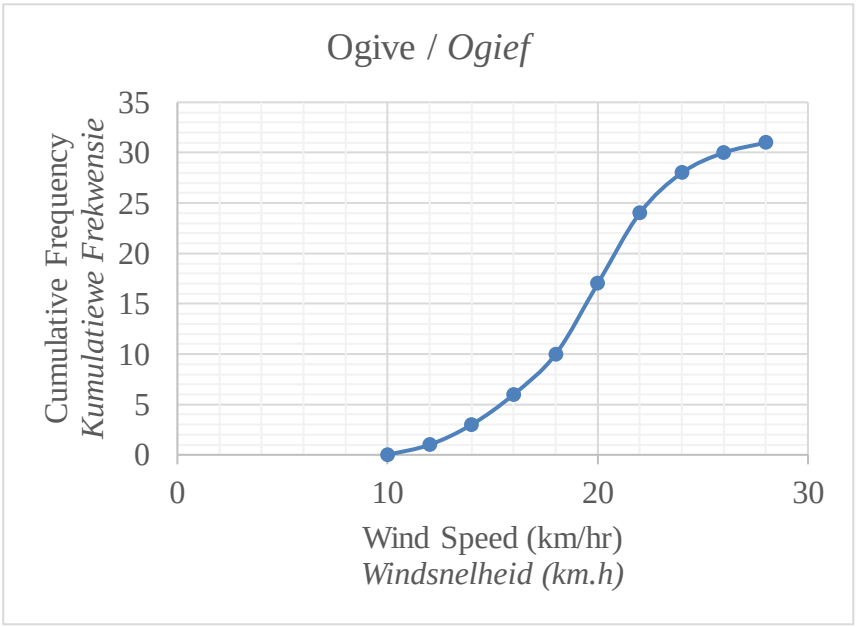
Wind Speed (km/hr) *Windsnelheid(km/h)*	Frequency *Frekwensie*	Cumulative Frequency *Kumulatiewe Frekwensie*
$10 < x \leq 12$	1	1
$12 < x \leq 14$	2	3
$14 < x \leq 16$	**3**	6
$16 < x \leq 18$	4	10
$18 < x \leq 20$	7	**17**
$20 < x \leq 22$	7	24
$22 < x \leq 24$	4	28
$24 < x \leq 26$	**2**	30
$26 < x \leq 28$	1	**31**

✓ complete freq. column
voltooi frekwensie kolom

✓ complete cum. freq. column / *voltooi kum. frekwensie kolom*

(2)

2.2



✓upper boundary values
boonste grens waardes

✓correct points
korrekte punte

✓smooth curve
egalige kurwe

(3)

2.3.1

Median wind speed = 17,5 km/hr

Gemiddelde windsnelheid = 17,5 km/h

✓✓ answer with units
antwoord met eenhede

(2)

2.3.2

31 – 29 = 2 days/*dae*

✓ 29
✓ answer / *antwoord*

(2)

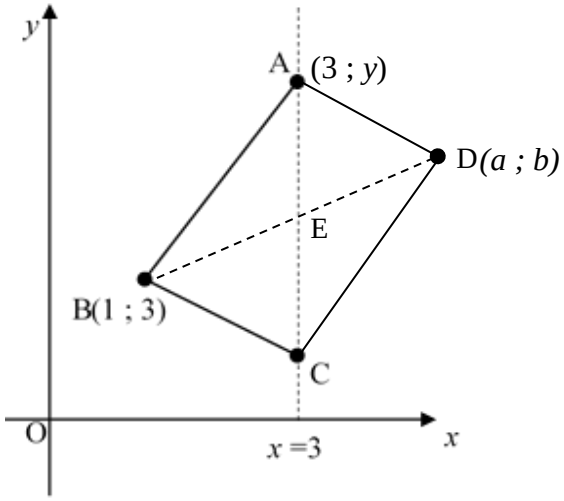
[9]

QUESTION 3 / *VRAAG 3*

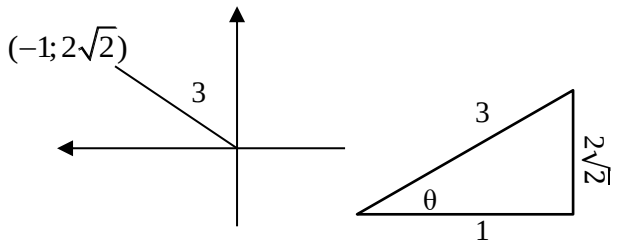
3.1	$3(0) + 2y = 6$ $y = 3$ E (0 ; 3)	✓substitution / *vervanging* $x = 0$ ✓ answer / *antwoord*	(2)
3.2	$y = -\frac{3}{2}x + 3$ $m_{CD} = -\frac{3}{2}$	✓ standard form / *standaardvorm* ✓ answer / *antwoord*	(2)
3.3	$\tan^{-1}(-\frac{3}{2}) = -56.31^0$ $\therefore \alpha = 180^o - 56{,}31^o$ $\therefore \alpha = 123{,}69^o$ OR Ref. angle / Verw. hoek $= \tan^{-1}(\frac{3}{2}) = 56.31^0$ $\therefore \alpha = 180^o - 56{,}31^o$ $\therefore \alpha = 123{,}69^o$	✓ $\tan^{-1}$ ✓answer / *antwoord*	(2)
3.4	$\beta = 123{,}69^o - 63{,}69^o$ $\beta = 60^o$	✓answer / *antwoord*	(1)
3.5	Gradient of/*van* AB : $\tan(60^o) = \sqrt{3}$ E(0 ; 3) $\therefore y = \sqrt{3}x + 3$	✓use of tan / *gebruik van* tan ✓✓ answer / *antwoord*	(3)
3.6	$t = \sqrt{3}(\sqrt{3}) + 3$ $t = 6$	✓ substitute/*vervang* $x = \sqrt{3}$ ✓ answer / *antwoord*	(2)

3.7	$0 = \sqrt{3}x + 3$ $\frac{-3}{\sqrt{3}} = x$ $\therefore x = -\sqrt{3}$, hence/*vervolgens* $A(-\sqrt{3}; 0)$ $3x + 2(0) = 6$ $x = 2$, hence/*vervolgens* D(2 ; 0) Length of AD (base of ΔABD) *Lengte van* AD (*basis van* Δ ABD) $= \sqrt{3} + 2 = 3{,}73$ Height/*Hoogte* = 6 Hence area ΔABD / *Vervolgens is oppervlakte van* ΔABD $= \frac{1}{2} \times 3{,}73 \times 6$ $= 11{,}19$ units^2/eenhede^2 OR Length of AD (base of ΔABD) *Lengte van* AD (*basis van* Δ ABD) $= \sqrt{3} + 2$ Hence area ΔABD / *Vervolgens is oppervlakte van* ΔABD $= \frac{1}{2} \times (\sqrt{3} + 2) \times 6$ $=11{,}20$ units^2	✓ calculation of A / *berekening van* A ✓calculation of D / *berekening van* D ✓height / *hoogte* ✓area formula / *oppervlakte formule* ✓answer / *antwoord*	(5)
			[17]

QUESTION 4 / *VRAAG 4*

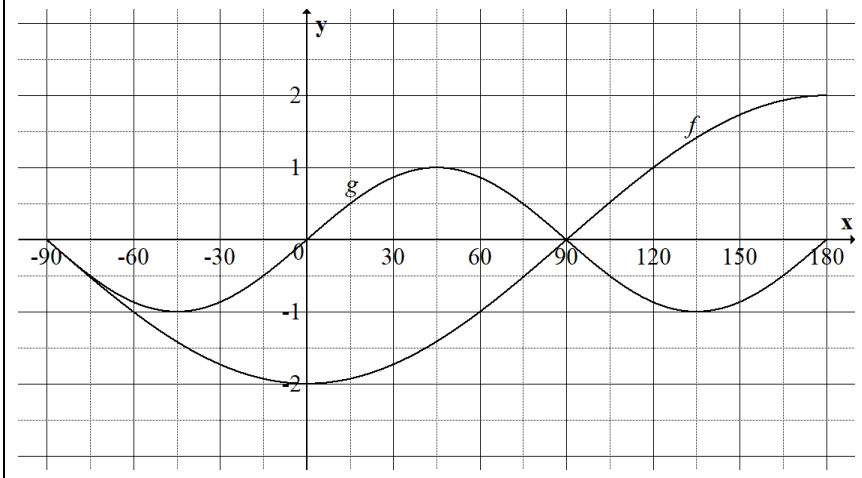
4.1	 $(2\sqrt{5})^2 = (3-1)^2 + (y-3)^2$ $20 = 4 + (y-3)^2$ $16 = (y-3)^2$ **OR/*OF*** $20 = 4 + y^2 - 6y + 9$ $3 \pm 4 = y$ **OR/*OF*** $0 = y^2 - 6y - 7$ $y = 7$ or -1 **OR/*OF*** $0 = (y-7)(y+1)$ A(3 ; 7)	✓distance formula *afstand formule* ✓$x = 3$ and/*en* (y – 3) ✓solving for y *oplossing vir y* ✓coordinates *koördinate*	(4)
4.2	C(3 ; 2)	✓✓ coordinates *koördinate*	(2)
4.3	$\left(\frac{3+3}{2};\frac{7+2}{2}\right) = (3;4,5)$	✓✓substitution in corr formula *vervanging in korrekte formule*	(2)
4.4	$\left(\frac{1+a}{2};\frac{3+b}{2}\right) = (3;4,5)$ $\frac{1+a}{2} = 3 \Rightarrow 1+a = 6 \Rightarrow a = 5$ $\frac{3+b}{2} = 4,5 \Rightarrow 3+b = 9 \Rightarrow b = 6$ D(5;6) **OR/*OF*** using the gradient of AB = $\frac{4}{2}$ = gradient of BC Hence C(3;2) moved up 4 and across 2 $\Rightarrow$ D(5;6)	✓use of midpoint theorem *Gebruik van middelpunt stelling* ✓✓coordinates of D *koördinate van* D ✓gradient/*gradiënt* ✓✓coordinates of D	(3)
4.5	B'(9;0)		(2)
			[13]

QUESTION 5/*VRAAG 5*

5.1		✓diagram / *diagram*	
5.1.1	$\tan(180^0 + \theta)$ $= \tan\theta$ $= -2\sqrt{2}$	✓ reduction / *reduksie* ✓answer / *antwoord*	(3)
5.1.2	$3\sin(\theta - 90^o)$ $=3(-\cos\theta)$ $=3\left(-\frac{1}{3}\right)$ $=-1$	✓reduction / *reduksie* ✓answer / *antwoord*	(2)
5.2.1	$\frac{\sin(-210^0)}{\cos(300^0)} + \frac{\cos(x+90^0)}{\sin(360^0+x)}$ $\frac{\sin 30^0}{\cos 60^0} + \frac{-\sin x}{\sin x}$ $= \frac{\frac{1}{2}}{\frac{1}{2}} + -1$ $=1-1$ $=0$	✓✓sin and cos reduction sin *en* cos *reduksie* ✓✓ cos and sin reduction cos *en* sin *reduksie* ✓answer / *antwoord*	(5)
5.2.2	$\sin(360^o + x) \neq 0$ $360^0 + x \neq 0^o + k.360^0$ or/*of* $360^0 + x \neq 180^0 + k.360^0$ $x \neq -360^0\ ;\ -180^o\ ;\ 0\ ;\ 180^o\ ;\ 360^0$	✓✓answers / *antwoorde*	(2)
5.3	LHS/*LK* : $\tan\theta\sqrt{\frac{1}{\sin^2\theta}-1}$ $= \frac{\sin\theta}{\cos\theta}\sqrt{\frac{1-\sin^2\theta}{\sin^2\theta}}$ $= \frac{\sin\theta}{\cos\theta}\sqrt{\frac{\cos^2\theta}{\sin^2\theta}}$ $= \frac{\sin\theta}{\cos\theta}\times\frac{\cos\theta}{\sin\theta}$ $= 1$ = RHS/*RK*	✓single fraction/*enkel breuk* ✓changing tan/ *verander* tan ✓identity / *identiteit* ✓taking square root / *vierkantswortel*	(4)

5.4	$2\sin^2\theta = 1 + \sin\theta$ $2\sin^2\theta - \sin\theta - 1 = 0$ $(2\sin\theta + 1)(\sin\theta - 1) = 0$ $\sin\theta = -\frac{1}{2}$ or/*of* $\sin\theta = 1$ $\therefore \theta = -30^o + k.360^o$ or/*of* $\theta = 210^o + k.360^o$ **OR/OF** $\theta = 330^o + k.360^o$ or/*of* $\theta = 210^o + k.360^o$ or/of $\theta = 90^o + k.360^o$; $k \in Z$	✓standard form / *standaardvorm* ✓factorising / *faktorisering* ✓solving / *los op* ✓✓✓ solutions / *oplossings* Penalise 1 mark if no $k \in Z$ *Penaliseer 1 punt indien geen* $k \varepsilon Z$	(6)
			[22]

QUESTION 6 / *VRAAG 6*

6.1		*f* ✓turning pts/*draaipunte* ✓*x*-intercepts/*x-afsnitte* ✓*y*-intercept / *y-afsnit* *g:* ✓intercepts / *afsnitte* ✓turning pts/*draaipunte* ✓shape / *vorm*	(6)
6.2.1	$x = 0^o$	✓answer/*antwoord*	(1)
6.2.2	$x \in [-90^o ; 90^o]$ **OR/*OF*** $-90^o \leq x \leq 90^o$	✓✓interval end points / *interval eindpunte*	(2)
6.2.3	$x \in (0^o ; 45^o)$ **OR/*OF*** $0^o < x < 45^o$ $x \in (135^o ; 180^o)$ **OR/*OF*** $135^o < x < 180^o$	✓ answer/*antwoord* ✓ answer/*antwoord*	(2)
6.3	$h(x) = 2\cos(x - 30^o)$	✓positive cos / *positiewe* cos ✓$(x - 30^o)$	(2)
			[13]

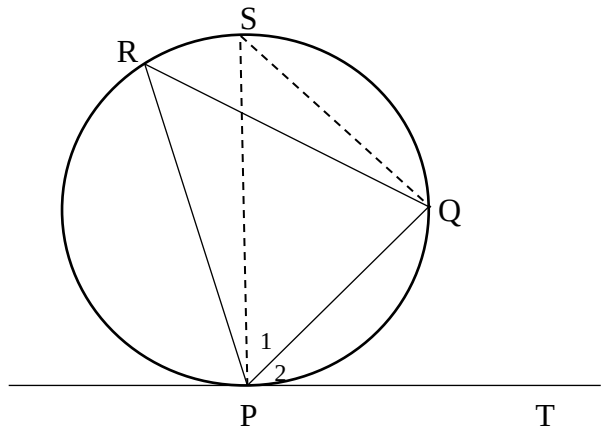
QUESTION 7 / *VRAAG 7*

7.1.1	$B\hat{D}C = 70^o$	✓answer / *antwoord*	(1)
7.1.2	$\frac{BD}{\sin 50^0} = \frac{5}{\sin 20^0}$ $BD = \frac{5\sin 50^0}{\sin 20^0}$ BD = 11,20 m $\sin 70^0 = \frac{BC}{11,2}$ $BC = 11,2 \times \sin 70^0$ BC = 10,52 m	✓sine rule / *sinusreël* ✓correct substitution / *korrekte vervanging* ✓answer for AC / *antwoord vir* AC ✓sine ratio/*sinus verhouding* ✓answer / *antwoord*	(5)
7.2.1	$AB^2 = r^2 + r^2 - 2 \times r \times r \times \cos 45^o$ $AB^2 = 2r^2(1 - \cos 45^o)$ $AB^2 = r^2 (2 - 2 \times \frac{\sqrt{2}}{2})$ $AB = r(\sqrt{2 - \sqrt{2}})$ Perimeter/*Omtrek* = $8 \times AB = 8r\sqrt{2 - \sqrt{2}}$	✓use of cos rule *gebruik van cos-reël* ✓expression for AB *uitdrukking vir* AB ✓answer / *antwoord*	(3)
7.2.2	Area of the octagon / *Oppervlakte van agthoek* $= 8 \times \text{area } \Delta AOB$ $= 8 \times \frac{1}{2} \times r \times r \times \sin 45^0$ $= 8 \times \frac{1}{2} \times r \times r \times \frac{\sqrt{2}}{2}$ $= 2\sqrt{2}r^2$	 ✓×8 ✓angle in triangle/*hoek in driehoek* ✓area formula/*oppervlakte formule* ✓sin 45° value / *waarde*	(4)
			[13]

QUESTION 8 / *VRAAG 8*

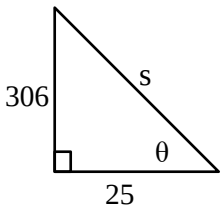
8.1.1	$\hat{A} = 45°$ ($\angle$ at centre = 2×$\angle$ at circum.) (*Middelpunts* $\angle$ = 2 × *Omtreks* $\angle$) $\hat{D} = 45°$ ($\angle$s in the same seg./ $\angle$e *in dieselfde segm.*) $A\hat{C}D = 45°$ OR/*OF* $A\hat{B}D = 45°$(*alt* $\angle$'s ∥ *lines*)	✓ (S) and (R) *(S) en (R)* ✓ S ✓ R	(3)
8.1.2 (a)	R.T.P /*Te Bewys*: CE = DE Proof/*Bewys* : $A\hat{C}D = \hat{D}$ (proved above / *bo bewys*) ∴ CE = DE (equal sides opp equal $\angle$'s) (*gelyke sye tenoor gelyke hoeke*)	✓ S ✓ R	(2)
8.1.2 (b)	R.T.P/*Te Bewys* : CD is the diameter of a circle passing through ECD / *CD is die middellyn van die sirkel deur punte E, C en D gaan* Proof/*Bewys* : $\hat{E} = 90°$ (3 $\angle$'s/*e* Δ) ∴ CD is diameter (line subtends 90°) CD *is die middellyn* (*lyn onderspan* 90°)	✓ S ✓ R	(2)
8.2.1 (a)	$\hat{C}_1$ =70° (equal $\angle$'s opp equal sides / *gelyke ∠e teenoor gelyke sye*)	✓ S ✓ R	(2)
8.2.1 (b)	$\hat{A}_2$ = 35° (exterior $\angle$ of Δ / *buitehoek van driehoek*)	✓ S ✓ R	(2)
8.2.1 (c)	$\hat{B}_2 = 35^0$ ($\angle$'s in same segment / *∠e in dieselde segment*)	✓ S ✓ R	(2)
8.2.2	R.T.P./*Te Bewys* : $\hat{B}_1 = \hat{B}_2$ Proof/*Bewys* : $\hat{B}_2 = 35^0$ (proved above / *bo bewys*) $A\hat{B}C = \hat{C}_1$ =70° (equal $\angle$'s opp equal sides / *gelyke ∠e teenoor gelyke sye*) ∴ $\hat{B}_1 = 70^0 - 35^0$ ∴ $\hat{B}_1 = \hat{B}_2$ ∴ BE bisects $A\hat{B}C$ / BE *halveer* $A\hat{B}C$	✓ S ✓ R ✓ $\hat{B}_1 = \hat{B}_2$	(3)
8.3.1	R.T.P/*Te Bewys* : CDOB is a cyclic quadrilateral / *is 'n koordevierhoek* Proof/*Bewys* : $\hat{O}_3 = 90^0$ (CO ⊥ AB given / *gegee*) $A\hat{D}B = 90^0$ (angle in a semi-circle)/(*hoek in halwe sirkel*) $\hat{D}_3 = 90^0$ (angles on st line) / (*hoeke op 'n reguitlyn*) ∴CDOB is a cyclic quadrilateral / *is 'n koordevierhoek* (∠'s in same seg)	✓ S/R ✓ S/R ✓ S ✓ S	(4)
8.3.2	R.T.P/*Te Bewys.* : $\hat{D}_2 = \hat{C}_1$ Proof/*Bewys* : $\hat{D}_2 = \hat{B}_1 = x$ (equal $\angle$'s opp equal radii)/ (*gelyke ∠e teenoor gelyke radiusse*) $\hat{B}_1 = \hat{C}_1 = x$ ($\angle$'s in the same segment of cyclic quad.) (*∠e in dieselfde segment van koordevierhoek*) ∴ $\hat{D}_2 = \hat{C}_1$	✓ S/R ✓ S/R	(2)
			[22]

QUESTION 9 / *VRAAG 9*

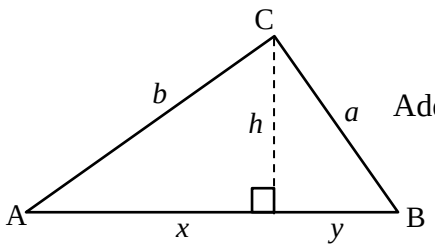
9.1	 R.T.P/*Te Bewys* : $\hat{P}_2 = \hat{R}$ Construction: Draw diameter SP. Join SQ *Konstruksie: Teken middellyn* SP. *Verbind* SQ Proof/*Bewys* : $\hat{P}_1 + \hat{P}_2 = 90^0$ (tang ⊥diameter)/ (*raaklyn ⊥ middellyn*) $S\hat{Q}P=90^0$ (∠ in semi-circle)/(∠ *in halwe sirkel*) $\hat{P}_1+\hat{S}=90^0$ (∠'s in Δ) / (∠*e in* Δ) But/*Maar* $\hat{S} = \hat{R}$ (∠'s in same segment) / (∠e *in dieselfde segment*) $\therefore \hat{P}_1 + \hat{R} = \hat{P}_1 + \hat{P}_2 = 90^0$ Hence/*Vervolgens is* $\hat{R} = \hat{P}_2$	✓construction/ *konstruksie* ✓ S/R ✓ S/R ✓ S/R ✓ S/R	(5)
9.2.1	$\hat{B}_2 = y$ (tan chord thrm) / (*raaklyn koord stelling*)	✓ S ✓ R	(2)
9.2.2	$\hat{D}_3 = x + y$ (ext ∠ of Δ) / (*buite ∠ van* Δ)	✓ S ✓ R	(2)
9.3	R.T.P/*Te Bewys* : AB ∥ EC Proof/*Bewys* : $\hat{C}_2 = \hat{D}_3 = x + y$ (∠'s in same segment) (∠*'e in dieselfde segment*) $B\hat{A}C = x + y$ ∴ AB ∥ EC (equal alt ∠'s) / (*gelyke verw. ∠e*)	✓ S ✓ R ✓ R	(3)
9.4.1	R.T.P/*Te Bewys* : $\hat{C}_3 = \hat{F}$ Proof/*Bewys* : $\hat{C}_3 = \hat{D}_1$ (∠'s in same segment) / (∠*'e in dieselfde segment*) $\hat{D}_1 = \hat{F}$ (corresp ∠'s ∥ lines) / (ooreenk. ∠ : ∥ lyne) $\therefore \hat{C}_3 = \hat{F}$	✓ S ✓ R ✓ S	(3)

9.4.2	R.T.P / *Te Bewys* : $A\hat{C}F = C\hat{D}F$ Proof / *Bewys* : $A\hat{C}F = 180^0 - (\hat{A}_1 + \hat{F})$ (∠'s of Δ)/(∠*e van* Δ) $C\hat{D}F = 180^0 - (\hat{C}_1 + \hat{F})$ (∠'s of Δ) / (∠*e van* Δ) But $\hat{C}_1 = \hat{A}_1$ (tan chord thrm) / (*raaklyn koord stelling*) $\therefore A\hat{C}F = C\hat{D}F$ **OR/*OF*** $A\hat{C}F = \hat{C}_1 + \hat{C}_2$ and/*en* $\hat{C}_1 = \hat{B}_2$ (tan chord thrm)/(*raaklyn koord stelling*) $\hat{C}_2 = \hat{B}_1$ (∠'s in same segment)/(∠*e in dies. segment*) $C\hat{D}F = \hat{B}_1 + \hat{B}_2$ (ext ∠ cyclic quad) / (*buite ∠ van* k.v)	✓ S ✓ S ✓ R ✓ S ✓ S ✓ R	(3)
			[18]

QUESTION 10 / *VRAAG 10*

10.1	Surface area of sides = $\frac{1}{2} \times$ perimeter of the base × slant ht *Buite oppervlakte van sye = ½ × omtrek van basis × skuinshg* Slant height/*Skuinshoogte*: $306^2 = 25^2 + (s)^2$ Pythagoras s = 304,98 m  SA = $\frac{1}{2} \times 4(50) \times 304,98$ = 30498 m^2	✓Pytha eqn *Pyth. verg.* ✓25m ✓substitution in formula *vervanging in formule* ✓answer / *antwoord*	(4)
10.2	$\tan^{-1}\theta = \frac{306}{25}$ $\theta = 85,33^o$	✓arctan ✓answer/ *antwoord*	(2)
			[6]

QUESTION 11 / *VRAAG 11*

 $a^2 = h^2 + y^2$ $b^2 = h^2 + x^2$ Adding/*Optel* : $a^2 + b^2 = 2h^2 + x^2 + y^2$ $= 2(xy) + x^2 + y^2$ $a^2 + b^2 = (x + y)^2$ $BC^2 + AC^2 = AB^2$ Hence triangle is rht ∠'d given : $h^2 = xy$ *Die driehoek is vervolgens reghoekig, gegee dat*: $h^2 = xy$	✓ use of Pythag *gebruik van Pyth.* ✓adding / *optel* ✓substitution / *vervanging* ✓factorising / *faktorisering*	**[4]**

TOTAL/TOTAAL: 150