ASHTON
INTERNATIONAL COLLEGE
BALLITO

Mathematics
Paper 2

FORM 4
11 June 2019
Session 1

TIME: 2 hours	TOTAL: 100 marks
Examiner: Mrs A Gunning	**Moderators: Miss A Rohrs; Miss M Eastes**

Name and Surname:

Teacher: MEMO

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY BEFORE ANSWERING THE QUESTIONS.

- This question paper consists of 16 pages. Formulae are given on page 17. The formulae page can be detached for ease of use.
- Please check that your question paper is complete.
- <u>Answer all questions on this question paper</u>
- Read and answer all questions carefully.
- It is in your own interest to write legibly and to present your work neatly.
- All necessary working which you have used in determining your answers **must** be clearly shown.
- Approved non-programmable calculators may be used except where otherwise stated. Where necessary give answers correct to **2 decimal places** unless otherwise stated.
- Ensure that your calculator is in DEGREE mode.
- Diagrams have not necessarily been drawn to scale.
- **Give reasons for all statements used in geometry.**

Question	1	2	3	4	5	6	7	8	9	10	Total
Out of	9	9	8	15	13	7	14	6	12	7	100
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QUESTION 1

You are given points A(-1;1) and B(5;4) on the Cartesian plane.

(a) Determine the gradient of the line AB. (2)

$$M_{AB} = \frac{4-1}{5+1} \checkmark = \frac{3}{6} = \frac{1}{2} \checkmark$$

K.

(b) Determine the coordinates of M, the midpoint of line AB (2)

$$x = \frac{5-1}{2} \qquad \checkmark \qquad y = \frac{1+4}{2} \checkmark \qquad M\left(2 ; \frac{5}{2}\right)$$
$$= 2 \qquad = \frac{5}{2}$$

K.

(c) Determine the equation of the line perpendicular to AB, and passing through the point M. (3)

$$M_{\perp} = -2 \text{ ca.} \qquad 1\text{pt} \left(2 , \frac{5}{2}\right)$$
$$y - \frac{5}{2} = -2(x - 2) \checkmark m \quad \boxed{OR} \quad y = -2x + C$$
$$y = -2x + \frac{13}{2} \text{ ca} \qquad \text{sub} \left(2; \frac{5}{2}\right) \checkmark m$$
$$\frac{5}{2} = -4 + C$$
$$6\frac{1}{2} = C \checkmark$$

RP.

(d) Determine the length of line AB. Leave your answer in the simplest surd form. (2)

$$AB = \sqrt{(-1-5)^2 + (1-4)^2} \checkmark a$$
$$= \sqrt{36 + 9}$$
$$= \sqrt{45}$$
$$= 3\sqrt{5} \checkmark a.$$

RP.

[9]

QUESTION 2

You are given the coordinates of 3 points: E (4; 3), F (0; − 1) and G (t; 1).

For each of the following, determine the value of t for which:

(a) E, F and G are collinear   (3)

$$M_{EF} = M_{FG}.$$

$$\frac{4}{4} \checkmark = \frac{1+1}{t} \checkmark$$

$$t = 2. \checkmark$$

CP.

(b) ΔFEG is right angled at F   (3)

$$M_{FE} \cdot M_{FG} = \checkmark -1$$

$$1 \cdot \frac{2}{t} \checkmark = -1.$$

$$-2 = t \checkmark$$

CP.

(c) FG = 2√5 units   (3)

$$\left(2\sqrt{5}\right)^2 = \left(t^2 + 2^2\right) \checkmark ma.$$

$$20 - 4 = t^2 \checkmark a.$$

$$16 = t^2$$

$$t = \pm 4 \checkmark ca.$$

CP.

[9]

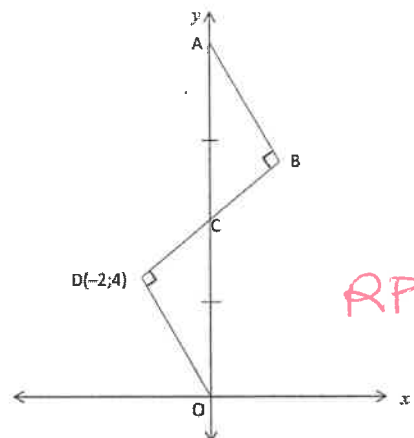Form 4 June 2019 Paper 2

QUESTION 3

In the diagram alongside, right-angled triangles ABC and ODC are drawn. O is the origin. A and C lie on the y-axis. C is the midpoint of OA.

D is the point (−2; 4).

(a) Determine the equation of OD.   (2)

$$y = -2x$$ ✓

$$m = \frac{4-0}{-2-0} = -2$$

RP

(b) Show that the coordinates of C are (0; 5)   (3)

eqn of BD.   1pt $(-2; 4)$ $m = \frac{1}{2}$ ✓

$$y - 4 = \frac{1}{2}(x + 2)$$

$$y = \frac{1}{2}x + 5$$ ✓ $C(0; 5)$ CP.

(c) Determine the equation of AB.   (3)
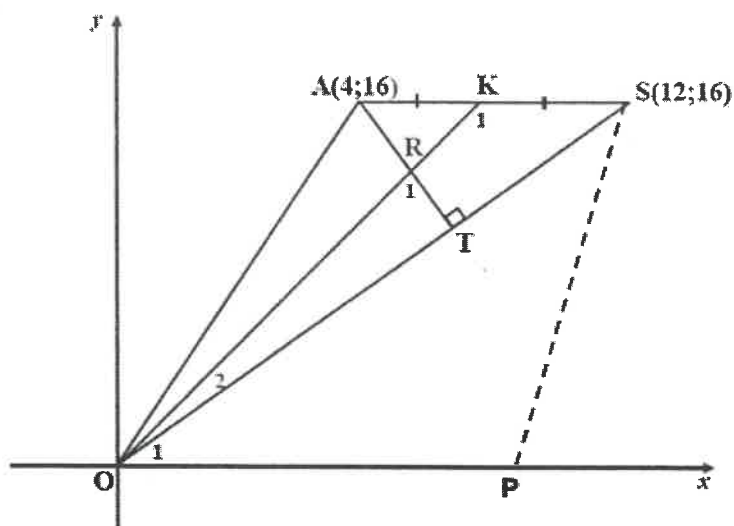
coords of A $(0; 10)$ ✓

$$y - 10 = -2(x - 0)$$

$$y = -2x + 10$$ ✓

RP

[8]

QUESTION 4

In the diagram K is the midpoint of AS and AT is perpendicular to OS.

(a) Determine the gradient of line OK. (3)

$$K\left(\frac{4+12}{2}; 16\right) = K(8; 16)$$

$$m_{OK} = \frac{16}{8} = 2.$$

RP

(b) Find $\hat{O}_2$ if $\hat{O}_1 = 53,13°$ (4)

$$\tan KOP = 2$$

$$K\hat{O}P = \tan^{-1}(2)$$

$$= 63,4349.$$

$$\therefore O_2 = 10,3°.$$

CP.

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(c) If P is a point on the x-axis such that $S\hat{P}O = 108,4°$, find the equation of SP

(to 1 decimal place). (4)

$M_{SP} = \tan(180 - 108,4)$ ✓a. $\tan 71,6°$ ✓a.

$= 3$ ✓a.

$S(12, 16)$

$y - 16 = 3(x - 12)$ ✓ca.   RP.

$y = 3x - 20.$ ✓ca.

(d) Calculate the area of ΔSPO. (4)

area $\triangle SPO = \frac{1}{2} OP \times \overset{m}{\perp} h$ ✓m $P\left(\frac{20}{3}, 0\right)$ ✓ca.

$= \frac{1}{2} \cdot \frac{20}{3} \times 16$ ✓ca.

$= \frac{160}{3} u^2$ ✓ca.

[15]

QUESTION 5

A quadratic number pattern $T_n = an^2 + bn + c$ has a first term equal to 1.

The general term of the **first differences** is given by $4n + 6$

Bald

(a) Determine the first three terms of the first differences sequence and hence find

the first three terms of the quadratic number pattern. (4)

$$T_1 = 4(1) + 6 = 10$$
$$T_2 = 8 + 6 = 14$$
$$T_3 = 12 + 6 = 18 \qquad 10; 14; 18.$$

PS

$$\begin{array}{ccccc} 1 & & 11 & & 25 \\ & 10; & & 14; & & 18 \end{array}$$

(b) Hence determine the value of a in the quadratic number pattern

$$T_n = an^2 + bn + c \qquad (2)$$
$$d_2 = 4 = 2a$$
$$2 = a$$

RP.

K.

(c) Determine the formula of the quadratic number pattern. (3)

$$3a + b = T_1 \text{ of } d_1$$
$$6 + b = 10$$
$$b = 4$$
$$a + b + c = 1$$
$$2 + 4 + c = 1$$
$$c = -5. \qquad T_n = 2n^2 + 4n - 5$$

RP

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(d) Is 153 a term in this quadratic number pattern? Show working to support your answer. (4)

$$2n^2 + 4n - 5 = 153$$
$$2n^2 + 4n - 158 = 0$$
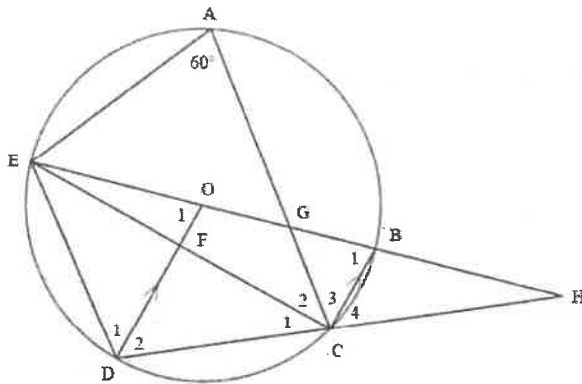$$n = -1 + 4\sqrt{5} \text{ or } -1 - 4\sqrt{5}$$

not integers

∴ 153 not a term.

(red annotations: a, ca, ✓ must see roots, CP.)

[13]

QUESTION 6

In circle ABCDE, centre O, OD // BC and diameter EB is produced to H. $\hat{A}_1 = 60°$.

Find, with reasons, the size of each of the following angles.

Statement	Reasons	
$\hat{B}_1$ $60°$ ✓	$\angle's$ on same segment ✓	(1)
$\hat{O}_1$ $60°$ ✓	corresp $\angle's =$ DO // CB ✓	(1)
$\hat{C}_1$ $30°$ ✓	$\angle$ at centre $= 2\angle \alpha$ circumf ✓	(1)
$\hat{D}_1 + \hat{D}_2$ $120°$ ✓	opp $\angle's$ cyclic quad. ✓	(1)
$\hat{C}_4$ $C_4 = D_2$ $\hat{D}_1 = 60°$ $\therefore D_2 = \hat{C}_4 = 60°$ ✓	corresp $\angle's =$ ✓ OD // BC. $\angle's$ in equilat. $\triangle$ ✓	(3)

[7] RP.

QUESTION 7

GEH is a tangent to the circle at E. F and C are two points on the circle and FB, FE, BC, CE and BE are drawn. $\hat{E}_1 = 32°$ and $\hat{E}_3 = 56°$. BOD is the diameter of the circle.
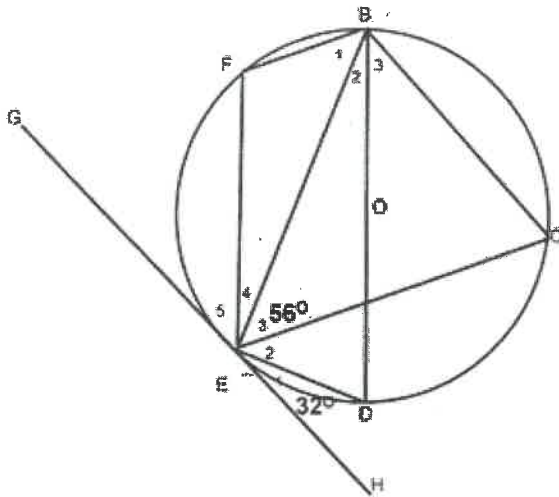
(a) Complete the following table, giving full reasoning for each answer.

	Statement	Reasons	
$\hat{E}_2$	$90 - 56 = 34°$ ✓	$\angle$ in semi circle. ✓	(2)
$\hat{B}_2$	$32°$ ✓	tan chord $\angle$. ✓	(2)
$\hat{B}_3$	$34°$ ✓	$\angle$'s in same seg. ✓	(2)
$\hat{F}$	$= 90 + 32$ ✓ $= 122°$	tan chord $\angle$ /	(2)
$\hat{C}$	$58°$ ✓	opp $\angle$'s cyclic quad. ✓	(2)
$\hat{D}$	$58°$ ✓	opp $\angle$'s cyclic quad OR. $\angle$'s in same seg. /	(2)

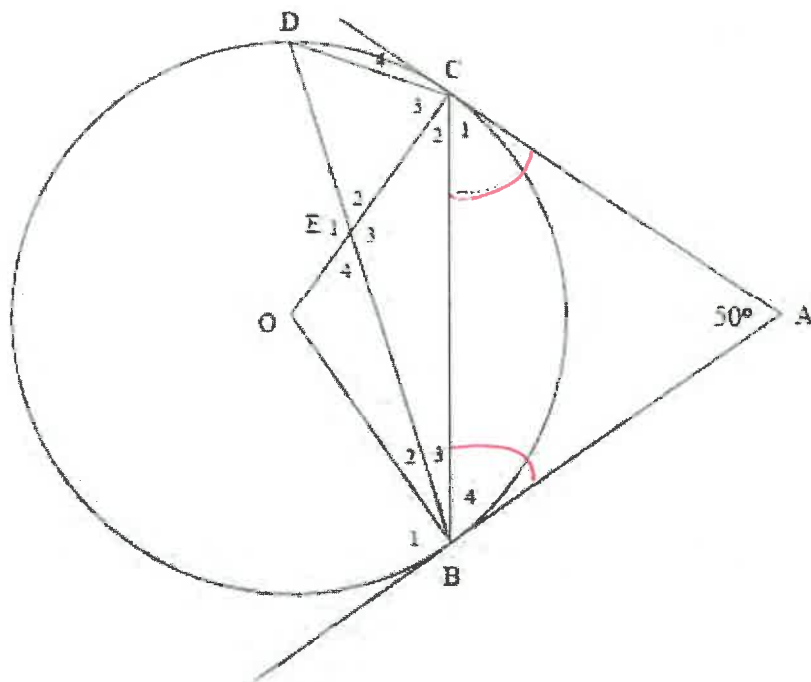(b) Is BD a tangent to the circle BFE? Give a reason for your answer. (2)

$B_2 \neq \hat{F}$ ✓

∴ not a tang ✓

RP.

CP

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QUESTION 8

In the diagram below, $\hat{A} = 50°$. AC and AB are tangents to the circle at C and B respectively. O is the centre of the circle.

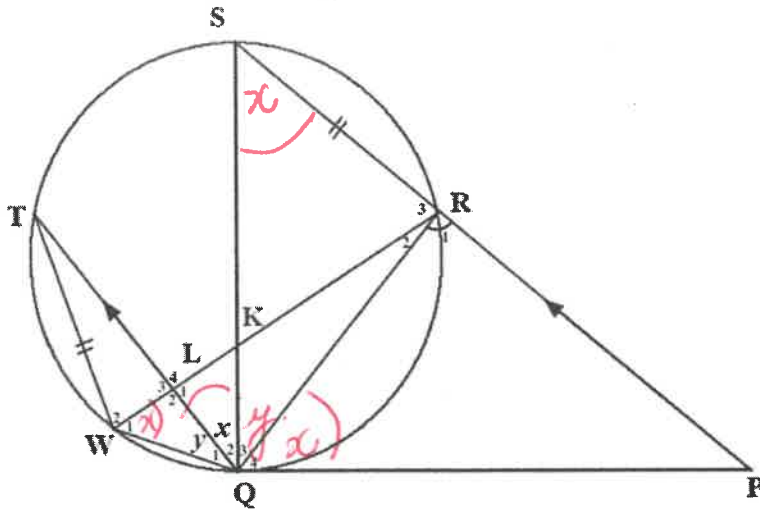| Statements	Reasons	
(a) Determine, with reasons, $B\hat{D}C$ $AC = AB$ $\therefore \hat{C}_1 = \hat{B}_4 = 65°$ $\therefore B\hat{D}C = 65°$	2 equal tangents from same pt $\angle$s in isos $\triangle$. tan chord $\angle$	(4) RP
(b) Determine, with reasons, $B\hat{O}C$ $130°$.	$\angle$ at centre = $2\angle$ on circum	(2) RP

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QUESTION 9

In the diagram below, PQ is a tangent to circle SRQWT at Q. PRS is a straight line. RW cuts SQ and QT at K and L respectively. PS//QT, RS = TW, $\hat{Q}_2 = x$ and $\hat{Q}_1 = y$

(a) State the reason(s) why each of the following angles is equal to x		
$\hat{S} = x$	alt $\angle$'o = ; SP//TQ ✓	(1)
$\hat{Q}_4 = x$	tan chord Th. ✓	(1)
$\hat{W}_1 = x$	$\angle$'o in same segment. ✓	(1)
(b) State the reason(s) why the following angle is equal to y ✓		
$\hat{Q}_3 = y$	equal chords subt = $\angle$'o.	(1)
(c) Hence, prove that $\hat{R}_1 = \hat{L}_3$		(4)

$\hat{R}_1 = \hat{S} + \hat{Q}_3$ ✓ eock $\angle$ ✓ $\triangle SRQ$

$\quad = x + y$

$\hat{L}_3 = \hat{W}_1 + \hat{Q}_1$ ✓ ext $\angle$ $\triangle LWQ$.

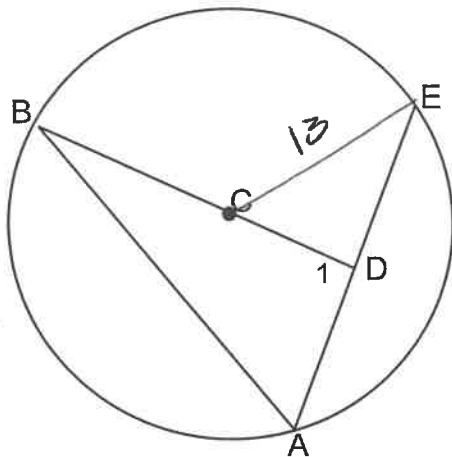$\quad = x + y$ ✓

$\therefore \hat{R}_1 = \hat{L}_3$

RTP: PRKQ cyclic

Proof:
$\hat{R}_3 = \hat{SQW}$ ✓ $\angle$'o in same seg. (A)
$\quad = x + y$. ✓
$\quad = \hat{Q}_3 + \hat{Q}_4$ ✓ CONV ext $\angle$ cyclic quad
$\therefore PRKQ$ cyclic

RP.

CP

PS.

(d) Hence, or otherwise, prove that PRKQ is a cyclic quadrilateral.	(4)

QUESTION 10

Consider the diagram below in which it is given that right angled triangle ABD is drawn with A and B on the circumference of the circle with centre C and D on the chord AE.

AE = 10 units and CE = 13 units.

Calculate, showing full reasoning, the length of AB, leaving your answer in the simplest surd form. (7)

$AE = 10$ given

$\hat{D}_1 = 90°$ given   ✓m

$\therefore AD = 5$ ✓a   line from centre ⊥ to chord.

In $\triangle CED$

$CE^2 = CD^2 + DE^2$ Pythag ✓m

$169 = CD^2 + 5^2$

$12 = CD$ ✓a

$\therefore$ In $\triangle BAD$

$AB^2 = BD^2 + DA^2$ Pythag ✓ca

$= 25^2 + 5^2$ ✓ca

$= 650$

$AB = \sqrt{650}$

$= 5\sqrt{26}$ ✓ca

[7]

CP

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