education

Department:
Education
PROVINCE OF KWAZULU-NATAL

NATIONAL SENIOR CERTIFICATE

GRADE 11

MATHEMATICS P2

COMMON TEST

JUNE 2019

MARKS: **100**

TIME: **2 hours**

This question paper consists of 8 pages and 2 DIAGRAM SHEETS.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 6 questions.
2. Answer ALL the questions.
3. Number the answers correctly according to the numbering system used in this question paper.
4. Clearly show ALL calculations, diagrams, graphs, etc. which you have used in determining your answers.
5. Answers only will NOT necessarily be awarded full marks.
6. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
7. If necessary, round off answers correct to TWO decimal places, unless stated otherwise.
8. Diagrams are NOT necessarily drawn to scale.
9. TWO DIAGRAM SHEETS for QUESTION 2.2, QUESTION 5.1, QUESTION 5.2, QUESTION 6.1 AND QUESTION 6.2 are attached at the end of this question paper. Detach the DIAGRAM SHEETS and hand in together with your ANSWER BOOK.
10. Write neatly and legibly.

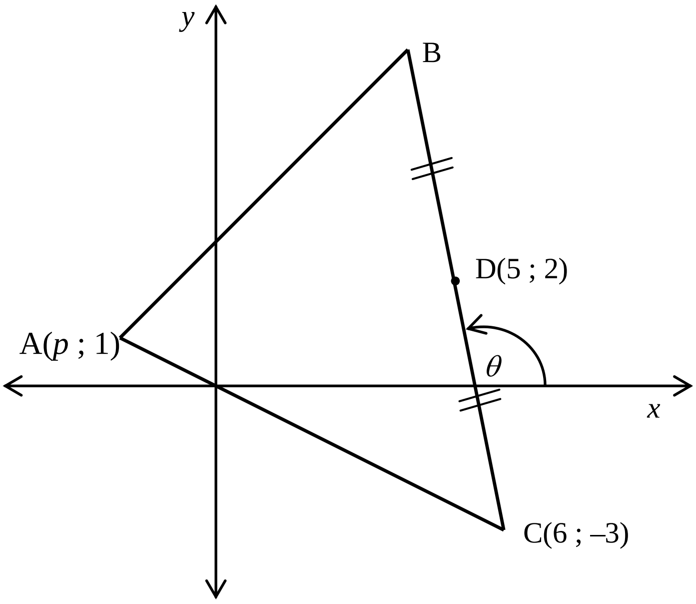
QUESTION 1

1.1 Consider the point K(–8 ; 3) in the Cartesian plane.

1.1.1 Write down the equation of the horizontal line passing through K. (1)

1.1.2 Write down the equation of the vertical line passing through K. (1)

1.2 In the diagram, A(p ; 1), B and C(6 ; –3) are the vertices of ΔABC.
D(5 ; 2) is the midpoint of BC. A lies in the second quadrant.
DC forms an angle θ with the x-axis.



Determine the:

1.2.1 Gradient of BC. (2)

1.2.2 Size of θ, rounded off to ONE decimal place. (3)

1.2.3 Coordinates of B. (3)

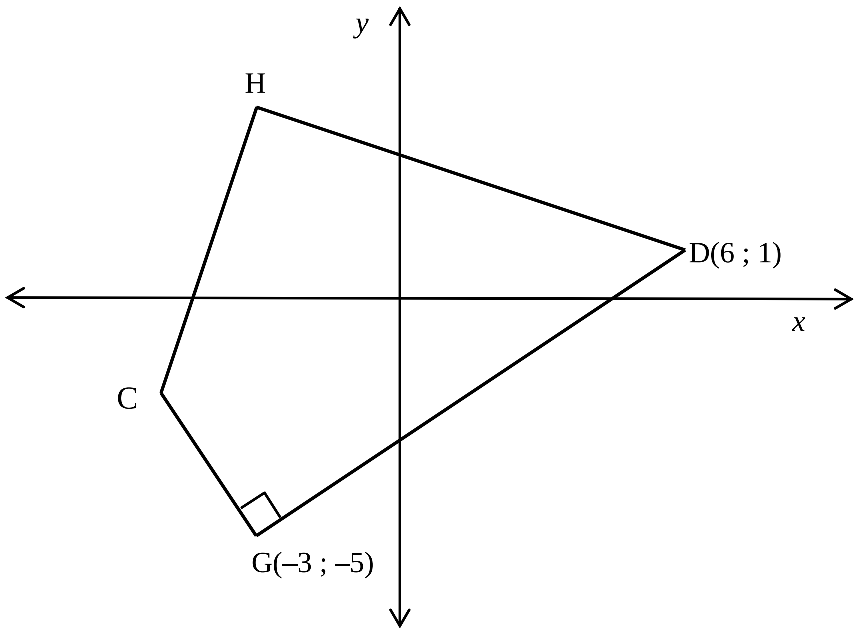
1.2.4 Value of p, if it is given that the length of AC = $4\sqrt{5}$. (5)

[15]

QUESTION 2

2.1 Calculate the value of q if K(–6 ; 9), L(–3 ; q) and M(–2 ; –1) are collinear. (4)

2.2 G(–3 ; –5), D(6 ; 1), H and C are the vertices of quadrilateral GDHC.
CG $\perp$ GD. The equation of CH is $y = 3x + 13$.



2.2.1 Determine the equation of CG. (4)

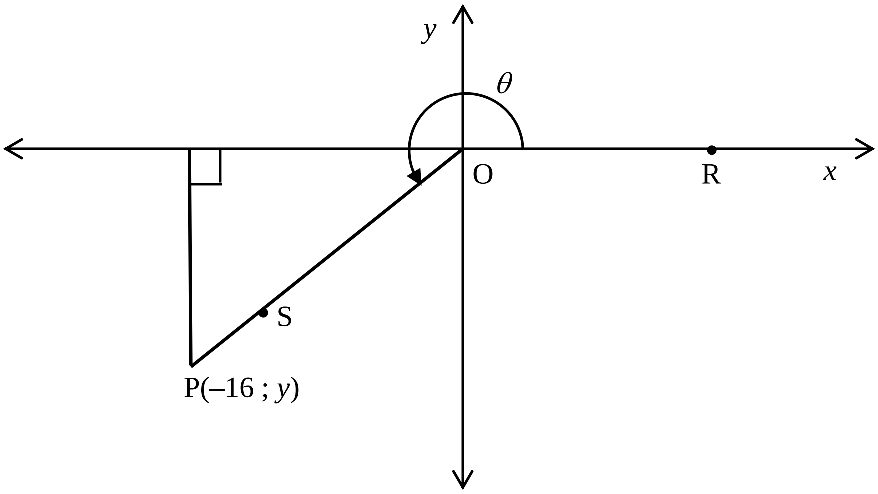
2.2.2 Calculate the coordinates of C. (3)

2.2.3 Calculate the size of $G\hat{C}H$. (5)

[16]

QUESTION 3

3.1 In the diagram below P(–16 ; y) is a point such that OP = 20 units and reflex $R\hat{O}P = \theta$.



3.1.1 Calculate the value of y. (2)

3.1.2 Determine the value of each of the following without using a calculator:

(a) $\sin(180^\circ - \theta)$ (2)

(b) $\cos(180^\circ + \theta)$ (2)

3.1.3 S is a point on OP such that OS = 15.
Determine the coordinates of S, WITHOUT using a calculator. (4)

3.2 Simplify, WITHOUT the use of a calculator: $\dfrac{\cos(-33^\circ).\tan 147^\circ}{2\cos 303^\circ.\sin 240^\circ}$ (7)

[17]

QUESTION 4

4.1 Use trigonometric identities to prove that $\dfrac{\sin^3 x + \sin x.\cos^2 x}{\cos x} = \tan x$ (3)

4.2 Solve for x if $\sin x = 0,412$ and $x \in [0^\circ ; 360^\circ]$. (2)

4.3 Consider the equation: $\tan 3x + 2,64 = 0$.

4.3.1 Determine the general solution of $\tan 3x + 2,64 = 0$ (4)

4.3.2 Hence, or otherwise, solve for x if $-90^\circ \leq x \leq 90^\circ$ (3)

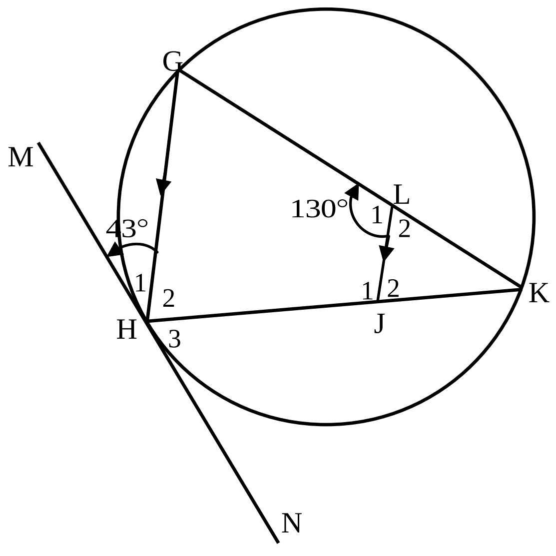
4.4 Solve for x if $4\sin^2 x + 7\cos x - 4 = 0$ and $x \in [0^\circ ; 360^\circ]$. (6)

[18]

GIVE REASONS FOR YOUR STATEMENTS AND CALCULATIONS IN QUESTIONS 5 and 6.

QUESTION 5

5.1 MHN is a tangent to circle GHK at H. L is a point on GK and J a point on HK such that LJ is parallel to GH. $\hat{H}_1 = 43^\circ$ and $\hat{L}_1 = 130^\circ$.

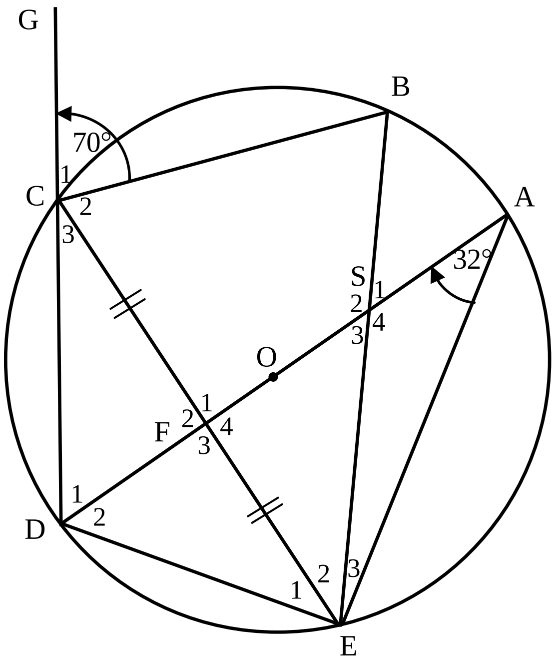


Calculate, with reasons, the size of:

5.1.1 $\hat{K}$ (2)

5.1.2 $\hat{H}_3$ (4)

5.2 A, B, C, D and E are points on the circle having centre O. DC is produced to G. Diameter AOD bisects chord CE in F, and intersects chord BE in S.
$\hat{A} = 32°$ and $G\hat{C}B = 70°$.



Calculate, with reasons, the sizes of the following angles:

5.2.1 $B\hat{E}D$ (2)

5.2.2 $\hat{C}_2$ (4)

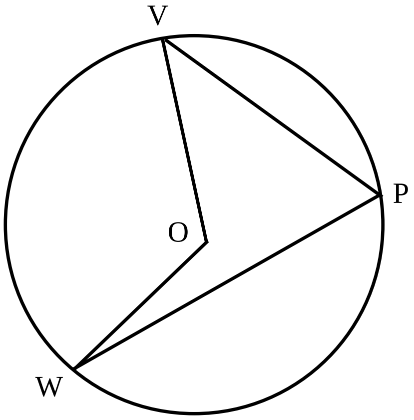
5.2.3 $\hat{D}_1$ (3)

5.2.4 $\hat{E}_3$ (3)

[18]

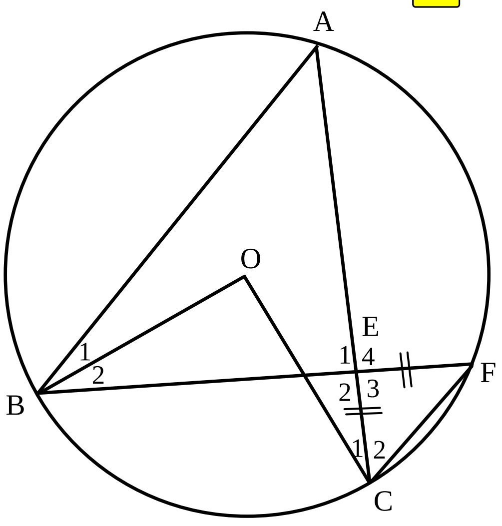
QUESTION 6

6.1 In the diagram, O is the centre of the circle. VP and WP are chords, and VO and WO have been drawn.



Use the diagram on the DIAGRAM SHEET to prove the theorem which states that an angle that an arc subtends at the centre of a circle is twice the size of the angle subtended by the same arc at the circle i.e. $V\hat{O}W = 2\hat{P}$. (6)

6.2 In the diagram, O is the centre of the circle. A, B, C and F are points on the circumference. AC and BF intersect in E and EF = FC.



Prove that:

6.2.1 AB || FC. (5)

6.2.2 OBCE is a cyclic quadrilateral. (5)

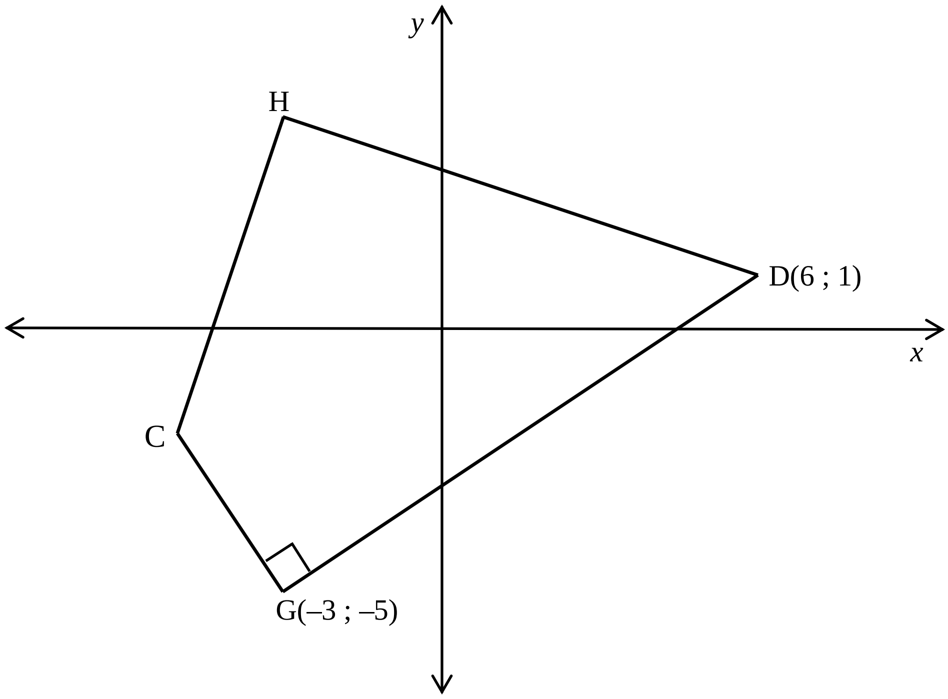
[16]

TOTAL: 100

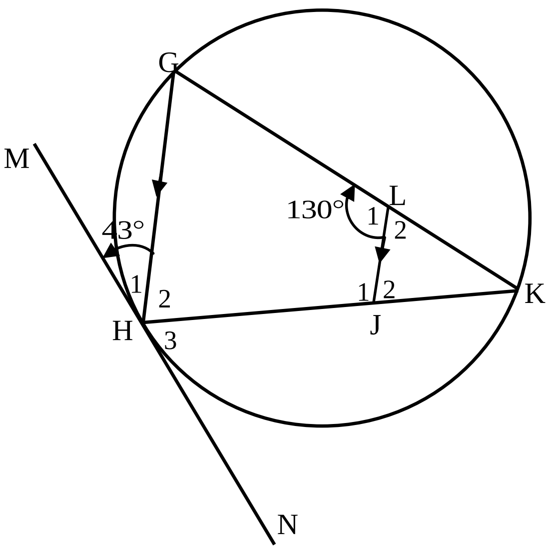
NAME & SURNAME:

DIAGRAM SHEET 1

QUESTION 2.2



QUESTION 5.1



QUESTION 5.2

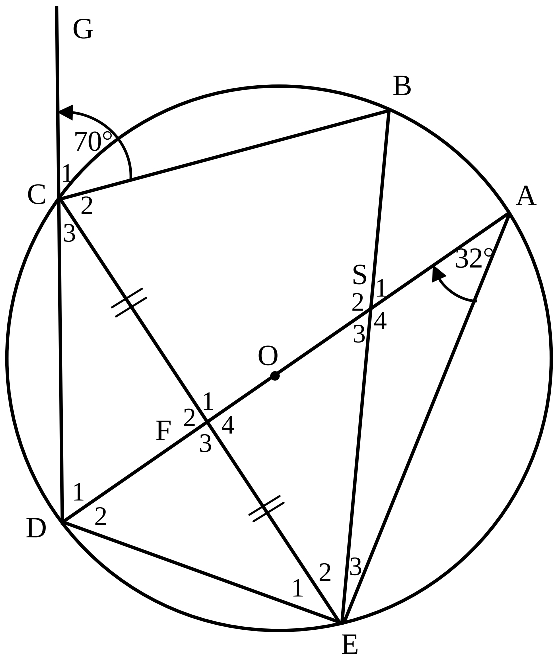


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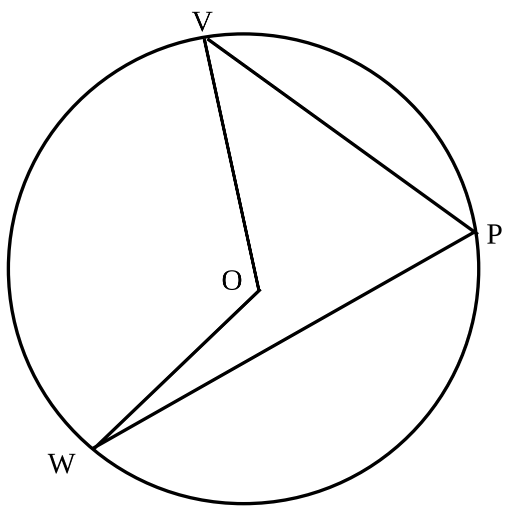
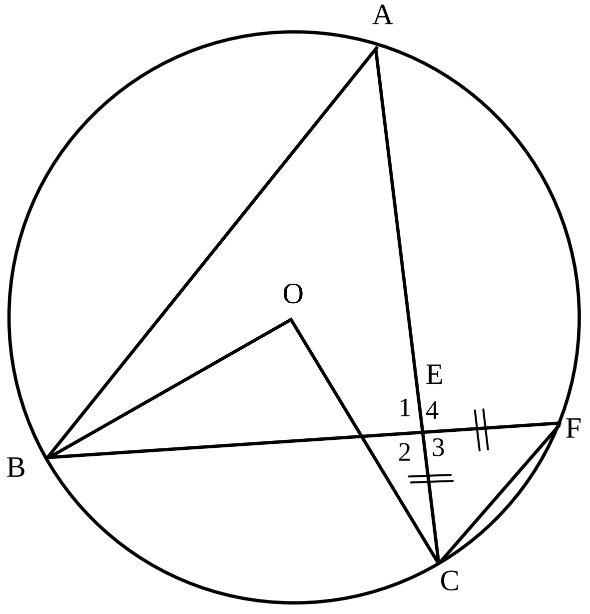
DIAGRAM SHEET 2

QUESTION 6.1



QUESTION 6.2



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MARKING GUIDELINE

NATIONAL
SENIOR CERTIFICATE

GRADE 11

MARKS: 100

This marking guideline consists of 10 pages.

GEOMETRY • *MEETKUNDE*	
S	**A mark for a correct statement** **(A statement mark is independent of a reason)**
	'n Punt vir 'n korrekte bewering ***('n Punt vir 'n bewering is onafhanklik van die rede)***
R	**A mark for the correct reason** **(A reason mark may only be awarded if the statement is correct)**
	'n Punt vir 'n korrekte rede ***('n Punt word slegs vir die rede toegeken as die bewering korrek is)***
S/R	**Award a mark if statement AND reason are both correct**
	Ken 'n punt toe as die bewering EN rede beide korrek is

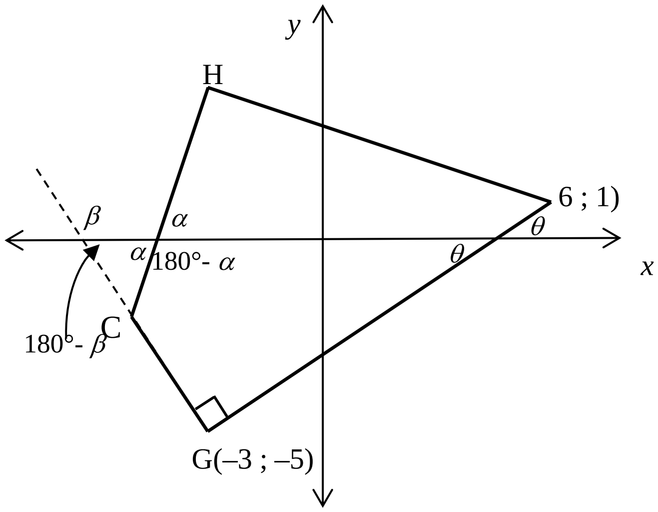
QUESTION 1

1.1.1	$y=3$	✓ answer (1)
1.1.2	$x=-8$	✓ answer (1)
1.2.1	$m=\dfrac{y_2-y_1}{x_2-x_1}$ $=\dfrac{-3-2}{6-5}$ **OR** $=\dfrac{2-(-3)}{5-6}$ $=-5$	✓ correct substitution ✓ answer (2)
1.2.2	$\tan\theta=m$ $\tan\theta=-5$ reference angle: $78,7^\circ$ $\theta=180^\circ-78,7^\circ$ $=101,3^\circ$	✓ $\tan\theta=-5$ ✓ reference angle: $78,7^\circ$ ✓ $101,3^\circ$ (3)
1.2.3	$\dfrac{x+6}{2}=5$ and $\dfrac{y+(-3)}{2}=2$ $x=4$ $y=7$ B(4 ; 7)	✓ method ✓ $x=4$ ✓ $y=7$ (3)
1.2.4	$AC=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ $4\sqrt{5}=\sqrt{(6-p)^2+(-3-1)^2}$ $4\sqrt{5}=\sqrt{36-12p+p^2+16}$ $4\sqrt{5}=\sqrt{p^2-12p+52}$ $80=p^2-12p+52$ $p^2-12p-28=0$ $(p-14)(p+2)=0$ $p=-2$ or $p=14$ $p=-2$ **OR** $AC=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ $(p-6)^2+(1-(-3))^2=\left(4\sqrt{5}\right)^2$ $(p-6)^2+16=80$ $(p-6)^2=64$ $p-6=\pm8$ $p=-2$ or $p=14$ $p=-2$	✓substitution into distance formula ✓ equating to $4\sqrt{5}$ ✓ squaring both sides ✓factors ✓answer (5) ✓substitution into distance formula ✓ equating to $4\sqrt{5}$ ✓ squaring both sides ✓ square rooting both sides ✓answer (5)
		[15]

QUESTION 2

2.1	$m_{KM}=\dfrac{-1-9}{-2-(-6)}$ $=-\dfrac{5}{2}$ $m_{KL}=\dfrac{q-9}{-3-(-6)}$ $=\dfrac{q-9}{3}$ Because the points are collinear: $m_{KM}=m_{KL}$ $-\dfrac{5}{2}=\dfrac{q-9}{3}$ $2(q-9)=-15$ $q=\dfrac{3}{2}$	✓substitution to determine m_{KM} ✓ expression for m_{KL} ✓equating gradients ✓answer (4)
	OR $m_{KM}=\dfrac{-1-9}{-2-(-6)}$ $=-\dfrac{5}{2}$ $m_{LM}=\dfrac{-1-q}{-2-(-3)}$ $=-1-q$ Because the points are collinear: $m_{KM}=m_{LM}$ $-\dfrac{5}{2}=-1-q$ $q=\dfrac{3}{2}$	OR ✓substitution to determine m_{KM} ✓ expression for m_{LM} ✓equating gradients ✓answer (4)
	OR $m_{KL}=\dfrac{q-9}{-3-(-6)}$ $=\dfrac{q-9}{3}$ $m_{LM}=\dfrac{-1-q}{-2-(-3)}$ $=-1-q$ Because the points are collinear: $m_{KL}=m_{LM}$ $\dfrac{q-9}{3}=-1-q$ $q-9=-3-3q$ $q=\dfrac{3}{2}$	OR ✓ expression for m_{KL} ✓ expression for m_{LM} ✓equating gradients ✓answer (4)

2.2.1	$m_{DG}=\frac{y_2-y_1}{x_2-x_1}$ $=\frac{-5-1}{-3-6}$ $=\frac{2}{3}$ $m_{CG}=-\frac{3}{2}$ Equation of CG: $y=-\frac{3}{2}x+c$ **OR** $y-y_1=-\frac{3}{2}(x-x_1)$ $-5=-\frac{3}{2}(-3)+c$ **OR** $y-(-5)=-\frac{3}{2}(x-(-3))$ $c=-\frac{19}{2}$ **OR** $y+5=-\frac{3}{2}x-\frac{9}{2}$ $y=-\frac{3}{2}x-\frac{19}{2}$	✓ gradient of DG ✓ gradient of CG ✓ substitution of $(-3;-5)$ ✓ equation of CG (4)
2.2.2	At C: $-\frac{3}{2}x-\frac{19}{2}=3x+13$ $\frac{9}{2}x=-\frac{45}{2}$ $x=-5$ $y=3(-5)+13=-2$ C(–5 ; –2)	✓ equating equations of CG and CH ✓ x-value ✓ y-value (3)

2.2.3		
	α = angle of inclination of CH $\tan\alpha = 3$ $\alpha = 71,57°$ β = angle of inclination of CG $\tan\beta = -\frac{3}{2}$ $180° - \beta = 56,31°$ $H\hat{C}G = \alpha + (180° - \beta)$ ext angle of triangle $H\hat{C}G = 71,57° + 56,31°$ $= 127,88°$	✓ $\tan\alpha = 3$ ✓ $71,57°$ ✓ $\tan\beta = -\frac{3}{2}$ ✓ $56,31°$ ✓ answer (5)
	OR	
	α = angle of inclination of CH $\tan\alpha = 3$ $\alpha = 71,57°$ $180° - \alpha = 108,43°$ θ = angle of inclination of GD $\tan\theta = \frac{2}{3}$ $\theta = 33,69°$ $H\hat{C}G = 360° - (33,69° + 90° + 108,43°)$ $\angle$s of quad $= 127,88°$	✓ $\tan\alpha = 3$ ✓ $108,43°$ ✓ $\tan\theta = \frac{2}{3}$ ✓ $33,69°$ ✓ answer (5)
		[16]

QUESTION 3

3.1.1	$y^2 = r^2 - x^2$ [Theorem of Pythagoras] $= 20^2 - (-16)^2$ $= 144$ $y = -12$	✓ substitution ✓ answer (2)
3.1.2(a)	$\sin(180^\circ - \theta) = \sin\theta$ $= \frac{-12}{20}$ $= -\frac{3}{5}$	✓ $\sin\theta$ ✓ answer (2)
3.1.2(b)	$\cos(180^\circ + \theta) = -\cos\theta$ $= -\left(\frac{-16}{20}\right)$ $= \frac{4}{5}$	✓ $-\cos\theta$ ✓ answer (2)
3.1.3	$\sin\theta = \frac{-3}{5} = \frac{y}{15}$ $y = -9$ $\cos\theta = \frac{-4}{5} = \frac{x}{15}$ $x = -12$ S(–12 ; –9)	✓ $\frac{-3}{5} = \frac{y}{15}$ ✓ y-coordinate ✓ $\frac{-4}{5} = \frac{x}{15}$ ✓ x-coordinate (4)
3.2	$\frac{\cos(-33^\circ).\tan 147^\circ}{2\cos 303^\circ.\sin 240^\circ}$ $= \frac{\cos 33^\circ.-\tan 33^\circ}{2\cos 57^\circ.-\sin 60^\circ}$ $= \frac{\cos 33^\circ.-\frac{\sin 33^\circ}{\cos 33^\circ}}{2\sin 33^\circ.-\frac{\sqrt{3}}{2}}$ $= \frac{1}{\sqrt{3}}$ or $\frac{\sqrt{3}}{3}$	✓ $\cos 33^\circ$ ✓ $-\tan 33^\circ$ ✓ $\cos 57^\circ$ ✓ $-\sin 60^\circ$ ✓ $\tan 33^\circ = \frac{\sin 33^\circ}{\cos 33^\circ}$ ✓ $\cos 57^\circ = \sin 33^\circ$ ✓ answer (7)
		[17]

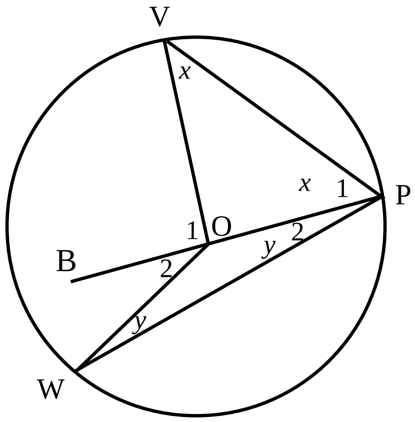
QUESTION 4

4.1	$\dfrac{\sin^3 x+\sin x\cos^2 x}{\cos x}$ $=\dfrac{\sin x\left(\sin^2 x+\cos^2 x\right)}{\cos x}$ $=\dfrac{\sin x(1)}{\cos x}1$ $=\tan x$	✓factors ✓$\sin^2 x+\cos^2 x=1$ ✓ $\dfrac{\sin x}{\cos x}=\tan x$ (3)
4.2	$\sin x=0,412$ $x=24,33°$ or $x=155,67°$	✓✓ answers (2)
4.3.1	$\tan 3x+2,64=0$ $\tan 3x=-2,64$ Reference angle: $69,25°$ $3x=110,75°+k.180°$ $x=36,92°+k.60°;\ k\in Z$ **OR** $\tan 3x+2,64=0$ $\tan 3x=-2,64$ Reference angle: $69,25°$ $3x=110,75°+k.360°$ or $3x=290,75°+k.360°$ $x=36,92°+k.120°$ or $x=96,92°+k.120°;\ k\in Z$	✓ $\tan 3x=-2,64$ ✓ $69,25°$ ✓ $3x=180°-69,25°$ ✓ General solution (4) ✓ $\tan 3x=-2,64$ ✓ $69,25°$ ✓ $3x=180°-69,25°$ or $3x=360°-69,25°$ ✓ General solution (4)
4.3.2	SS: $x\in\{-83,08°\ ;\ -23,08°\ ;\ 36,92°\}$	✓✓✓ answers NOTE: 1 mark for each correct answer (3)
4.4	$4\sin^2 x+7\cos x-4=0$ $4\left(1-\cos^2 x\right)+7\cos x-4=0$ $-4\cos^2 x+7\cos x=0$ $4\cos^2 x-7\cos x=0$ $\cos x\left(4\cos x-7\right)=0$ $\cos x=0$ or $\cos x=\dfrac{7}{4}$ no solution $\therefore x=90°$ or $270°$	✓$\sin^2 x=1-\cos^2 x$ ✓standard form ✓ factors ✓no solution ✓$90°$ ✓$270°$ (6)
		[18]

QUESTION 5

5.1.1	$\hat{K}=\hat{H}_1=43°$ [tan-chord-theorem]	✓S ✓R (2)
5.1.2	$\hat{G}=180°-\hat{L}_1\ =50°$ [co-interior ∠'s; GH ∥ LG] $\hat{H}_3=\hat{G}\ =50°$ [tan-chord-theorem]	✓S ✓R ✓S ✓R (4)
5.2.1	$B\hat{E}D=\hat{C}_1\ =70°$ [ext. ∠ of a cyclic quad.]	✓S ✓R (2)
5.2.2	$\hat{C}_3=\hat{A}\ =32°$ [∠'s in same segment] $\hat{C}_2=180°-(\hat{C}_1+\hat{C}_3)$ [∠'s on a straight line] $=180°-(70°+32°)$ $=78°$	✓S ✓R ✓R ✓S (4)
5.2.3	$\hat{F}_2=90°$ [line from centre to midpoint of chord] $\hat{D}_1=180°-(\hat{F}_2+\hat{C}_3)$ [sum of ∠'s of Δ] $=180°-(90°+32°)$ $=58°$	✓S ✓R ✓S (3)
5.2.4	$A\hat{E}D=90°$ [∠ in a semi-circle] $\hat{E}_3=90°-B\hat{E}D$ $=90°-70°$ $=20°$	✓S ✓R ✓S (3)
		[18]

QUESTION 6

6.1	Construction: Join PO and produce to B.  Proof: Let $\hat{V}=x$ $\hat{P}_1=\hat{V}=x$ [VO = PO = radii; $\angle$'s opp. = sides] $\hat{O}_1=\hat{V}+\hat{P}_1$ [ext. $\angle$ of Δ] $=2x$ Let $\hat{W}=y$ $\hat{P}_2=\hat{W}=y$ [WO = PO = radii; $\angle$'s opp. = sides] $\hat{O}_2=\hat{W}+\hat{P}_2$ [ext. $\angle$ of Δ] $=2y$ $\hat{O}_1+\hat{O}_2=2x+2y$ $V\hat{O}W=2(x+y)$ $=2\hat{P}$	✓ construction ✓ S/R ✓ S/R ✓ S ✓ S ✓ S (6)
6.2.1	Let $\hat{A}=x$ $\hat{F}=\hat{A}=x$ [$\angle$'s in same segment] $\hat{C}_2=\hat{F}=x$ [$\angle$'s opp. = sides] $\hat{A}=\hat{C}_2$ [both $=x$] AB ∥ FC [alt. $\angle$'s are =]	✓ S ✓ R ✓ S ✓ R ✓ R (5)
6.2.2	$\hat{O}=2\hat{A}$ [$\angle$ at centre = 2 × $\angle$ at circum.] $=2x$ $\hat{E}_2=\hat{F}+\hat{C}_2$ [ext. $\angle$ of Δ] $=2x$ $\hat{O}=\hat{E}_2$ [both = $2x$] OBCE is a cyclic quad. [converse: $\angle$'s in same segment]	✓ S ✓ R ✓ S ✓ S ✓ R (5)
		[16]

GRADE 11
Marking Guideline

TOTAL: 100