PREPARATORY EXAMINATION

2017

MARKING GUIDELINES

MATHEMATICS (FIRST PAPER) (10611)

14 pages

GAUTENG DEPARTMENT OF EDUCATION

PREPARATORY EXAMINATION

MATHEMATICS
(First Paper)

MEMORANDUM

	QUESTION 1	
1.1		
1.1.1	$x(x-1) = 12$ $x^2 - x = 12$ $x^2 - x - 12 = 0$ $(x-4)(x+3) = 0$ $x = 4$ **OR** $x = -3$	✓standard form ✓both factors ✓both answers (3)
1.1.2	$2x^2 + 3 = 8x$ $2x^2 - 8x + 3 = 0$ $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $= \dfrac{8 \pm \sqrt{(-8)^2 - 4(2)(3)}}{2(2)}$ $= \dfrac{8 \pm \sqrt{64 - 24}}{4}$ $= \dfrac{8 \pm \sqrt{40}}{4}$ $x = 3{,}58$ **OR** $x = 0{,}42$	✓standard form ✓substitution into correct formulae ✓✓each x answer (4)
1.1.3	$(2x+3)(3-x) > 4$ $6x - 2x^2 + 9 - 3x - 4 > 0$ $-2x^2 + 3x + 5 > 0$ $2x^2 - 3x - 5 < 0$ $(2x-5)(x+1) < 0$ $-1 < x < \dfrac{5}{2}$	✓simplification ✓standard form $2x^2 - 3x - 5 < 0$ ✓factors ✓critical values ✓inequalities (5)
1.1.4	$2^x - 5.2^{x+1} = -144$ $2^x(1 - 5.2\ ) = -144$ $2^x(-9) = -144$ $2^x = 16$ $2^x = 2^4$ $x = 4$	✓common factor ✓simplification $2^x = 2^4$ ✓answer (3)

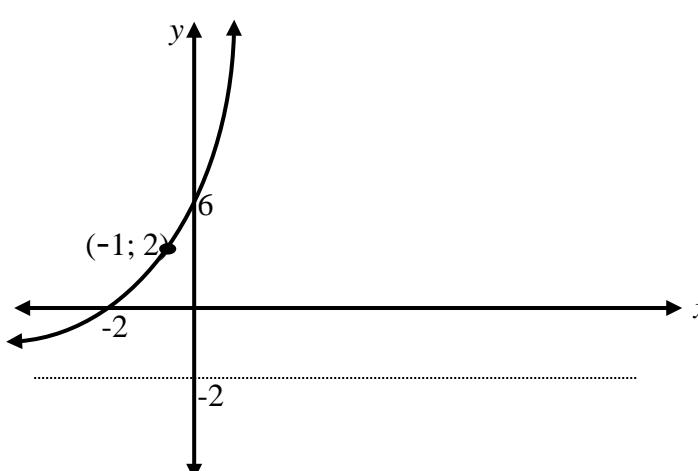| 1.2	$y + 2x - 3 = 0$ (1) $y = x^2 + 2x + 3$ (2) From (1): $y = -2x + 3$ (3) Substitute (3) into (2): $-2x + 3 = x^2 + 2x + 3$ $x^2 + 4x = 0$ $x(x + 4) = 0$ $x = 0 \quad or \quad x = -4$ Substitute $x = 0$ into (3): **OR** Substitute $x = -4$ into (3): $y = -2(0) + 3$ $y = -2(-4) + 3$ $y = 3$ $y = 11$	✓ $y = -2x + 3$ ✓ equating $-2x + 3 = x^2 + 2x + 3$ ✓ factorisation ✓ both x-values ✓ both y – values (5)
		[20]
	QUESTION 2	
2.1	$4a - a - 2 = 6a + 4 - 4a$ $\qquad a = 6$	✓ equating ✓ answer (2)
2.2 2.2.1		✓ 33 ✓ 96 (2)
2.2.2	45 – Term odd 23rd term of the odd sequence: $T_{23} = 5 + (23 - 1)(7)$ $T_{23} = 159$	✓ 45 – Term odd ✓ verifying T_{23} ✓ answer (3)

2.3	$T_8 = 33 = a + 7d$ (1)	✓equation 1
	$T_{11} = 45 = a + 10d$ (2)	✓equation 2
	From equation (2):	
	$\qquad\qquad\qquad a + 10d = 45$ (2)	
	$\qquad\qquad\qquad a + 7d = 33$ (1)	
	$a = 45 - 10d$ (3) **OR** (2) $-$ (1):	
	$\qquad\qquad\qquad 3d = 12$	
	$\qquad\qquad\qquad d = 4$	✓value of d
	Substitute (3) into (1):	
	$33 = 45 - 10d + 7d$	
	$-12 = -3d$	
	$d = 4$	
	$\qquad\qquad$ Substitute into (2)	
	$\qquad\qquad a = 45 - 10(4)$	
	$\qquad\qquad a = 5$	✓value of a
	$\qquad T_{15} = 5 + 14(4)$	
	$\qquad T_{15} = 61$	✓$T_{15} = 61$
	OR	
	$T_8 + 3d = T_{11}$	✓$T_8 + 3d = T_{11}$
	$33 + 3d = 45$	✓substitution
	$\qquad 3d = 12$	
	$\qquad d = 4$	✓value of d
	$T_{15} = T_{11} + 4d$	✓$T_{15} = T_{11} + 4d$
	$T_{15} = 45 + 4(4)$	
	$\qquad = 61$	✓answer
		(5)
2.4	$\qquad 2a = -4$	✓second difference
	$\qquad a = -2$	✓value of a
	$\qquad 3a + b = 0$	
	$\qquad 3(-2) + b = 0$	
	$\qquad b = 6$	
	$a + b + c = 6$	✓value of b
	$-2 + 6 + c = 6$	
	$\qquad c = 2$	✓value of c
	$T_n = -2n^2 + 6n + 2$	
		(4)

2.5 2.5.1	$3;\ \dfrac{3}{2};\ \dfrac{3}{4};\ ...$ $S_\infty = \dfrac{a}{1-r}$ $S_\infty = \dfrac{3}{1-\dfrac{1}{2}}$ $= 6$	✓substitute into the correct formula ✓answer (2)
2.5.2	$S_n = \dfrac{a(1-r^n)}{1-r}$ $S_n = \dfrac{3\left(1-\left(\dfrac{1}{2}\right)^n\right)}{1-\dfrac{1}{2}}$ $= 6\left(1-\left(\dfrac{1}{2}\right)^n\right)$	✓substitute into the correct formula ✓answer (2)
2.5.3	$A - B < \dfrac{1}{36}$ $6 - 6\left(1-\left(\dfrac{1}{2}\right)^n\right) < \dfrac{1}{36}$ $6\left(1 - 1 + \left(\dfrac{1}{2}\right)^n\right) < \dfrac{1}{36}$ $1 - 1 + \left(\dfrac{1}{2}\right)^n < \dfrac{1}{216}$ $\left(\dfrac{1}{2}\right)^n < \dfrac{1}{216}$ $n > \log_{\frac{1}{2}} \dfrac{1}{216}$ $n > 7,75$ $\therefore n = 8;\ 9;\ ...$	✓factorisation ✓$\dfrac{1}{216}$ ✓$\left(\dfrac{1}{2}\right)^n < \dfrac{1}{216}$ ✓$\log_{\frac{1}{2}} \dfrac{1}{216}$ ✓answer (5)
		[25]

	QUESTION 3	
3.1	$\left(1+i_{eff}\right)=\left(1+\dfrac{i_{nom}}{m}\right)^{m}$ $\left(1+i_{eff}\right)=\left(1+\dfrac{0{,}06}{4}\right)^{4}$ $i_{eff}=1{,}06-1$ $i_{eff}=6{,}14\%$	✓correct formula ✓correct substitution ✓answer (3)
3.2	$P=\dfrac{x\left[1-\left(1+i\right)^{-n}\right]}{i}$ $P=\dfrac{2\,000\left[1-\left(1+0{,}02\right)^{-24}\right]}{0{,}02}$ $=R37\,827{,}85$ The gym borrowed R 37 827,85.	✓substitution of x into correct formula ✓ $n=24$ ✓ $i=0{,}02$ ✓ answer (4)
3.3	$F=\dfrac{x\left[\left(1+i\right)^{n}-1\right]}{i}$ $200000=\dfrac{x\left[\left(1+\dfrac{0{,}0625}{12}\right)^{57}-1\right]}{\dfrac{0{,}0625}{12}}$ $x=\dfrac{200000\times\dfrac{0{,}0625}{12}}{\left[\left(1+\dfrac{0{,}0625}{12}\right)^{57}-1\right]}$ $x=3022{,}73$ $\therefore$ The monthly payments are R 3 022,73 per month.	✓formula ✓ value of F ✓ $n=57$ ✓ $i=\dfrac{0{,}0625}{12}$ ✓making x the subject ✓answer (6)
		[13]

	QUESTION 4	
4.1 4.1.1	$f(-1) = 4.2^{-1+1} - 2$ $\quad y = 4 - 2 \qquad$ **OR** $\quad$ f(-1) = 2 $\quad y = 2$	$\checkmark\ y = 2$ (1)
4.1.2	$f(x) = 4.2^{x+1} - 2$ x-intercept $\quad f(x) = 0$ $4.2^{x+1} - 2 = 0$ $\quad 4.2^{x+1} = 2$ $\quad 2^{x+3} = 2^1$ $\quad x + 3 = 1$ $\qquad x = -2$ $(-2; 0)$ **OR** $4.2^{x+1} - 2 = 0$ $\quad 4.2^{x+1} = 2$ $\quad 2^{x+1} = \dfrac{2}{4}$ $\quad 2^{x+1} = 2^{-1}$ $\quad x + 1 = -1$ $\qquad x = -2$ $(-2; 0)$	$\checkmark$ simplification $\left(2^{x+3} = 2^1\right)$ $\checkmark\ x + 3 = 1$ $\checkmark$ answer $\checkmark$ simplification $\left(2^{x+1} = 2^{-1}\right)$ $\checkmark\ x + 1 = -1$ $\checkmark$ answer (3)
4.1.3	y-intercept $\quad x = 0$ $y = 4.2^{0+1} - 2$ $y = 8 - 2$ $y = 6$ $(0; 6)$	$\checkmark$ substitute $x = 0$ $\checkmark$ answer (2)
4.1.4	$y = -2$	$\checkmark\ y = -2$ (1)
4.2		$\checkmark$ asymptote $\checkmark$ shape with any one correct coordinate $\checkmark$ both intercepts with axes (3)

4.3	$g(x) = 4.2^{x-2+1} - 2$ $g(x) = 4.2^{x-1} - 2$ **OR** $g(x) = 2.2^{x} - 2$ **OR** $g(x) = 2^{x+1} - 2$	✓answer ✓answer ✓answer (1)
		[11]
	QUESTION 5	
5.1 5.1.1	$(3; 4)$	✓3 ✓4 (2)
5.1.2	$y \geq -4$ **OR** $y \in [-4; \infty)$	✓critical values ✓inequality ✓critical values ✓correct brackets (2)
5.1.3	$(2; 3)\quad (5; 0)$ $m = \dfrac{0-3}{5-2}$ $m = -1$	✓ substitute with correct coordinates ✓ answer (2)
5.1.4	$y = -(-1) + 5$ $y = 6$ $KM = 6$ $y = -(-1-3)^2 + 4$ $y = -12$ $LM = 12$ $KL = 12 + 6$ $\quad = 18$	✓ KM = 6 **OR** $y = 6$ ✓ LM = 12 **OR** $y = -12$ ✓ KL = 18 (3)
5.1.5	$x \in (2; 5)$ **OR** $2 < x < 5$	✓ critical values ✓ correct brackets ✓ critical values ✓ correct inequality (2)

5.2 5.2.1	$y = \left(\dfrac{4}{3}\right)^x$ $x = \left(\dfrac{4}{3}\right)^y$ $y = \log_{\frac{4}{3}} x$ $g(x) = \log_{\frac{4}{3}} x$	✓ interchange x and y ✓ $g(x) = \log_{\frac{4}{3}} x$ (2)
5.2.2	$h(x) = \left(\dfrac{4}{3}\right)^{-x}$ $= \left(\dfrac{3}{4}\right)^x$	✓ $- x$ ✓ answer (2)
5.2.3 a)	$x = 0$	✓ answer (1)
b)	$0 < x \le 1$ **OR** $x \in (0;1]$	✓ critical values ✓ inequalities ✓ critical values ✓ correct brackets (2)
		[18]

	QUESTION 6	
6.1	$f'(x) = \dfrac{4}{x^2}$ $\therefore x^2 \geq 0$ $\therefore \dfrac{4}{x^2} > 0$	$\checkmark f'(x) = \dfrac{4}{x^2}$ $\checkmark x^2 \geq 0$ $\checkmark \dfrac{4}{x^2} > 0$ (3)
6.2	$m = -4$ *Perpendicular lines* $m = \dfrac{1}{4}$ $f'(x) = \dfrac{4}{x^2}$ $\dfrac{1}{4} = \dfrac{4}{x^2}$ $16 = x^2$ $\therefore x = -4 \qquad y = -\dfrac{4}{-4} + 7$ $\qquad\qquad\qquad y = 8$ Substitute $(-4; 8)$ $y = \dfrac{1}{4}x + c \qquad$ **OR** $\qquad y - 8 = \dfrac{1}{4}(x + 4)$ $8 = \dfrac{1}{4}(-4) + c \qquad\qquad y = \dfrac{1}{4}x + 9$ $c = 9$ $y = \dfrac{1}{4}x + 9$	$\checkmark m = \dfrac{1}{4}$ $\checkmark \dfrac{1}{4} = \dfrac{4}{x^2}$ $\checkmark$ selecting $x = -4$ $\checkmark y = 8$ $\checkmark$ answer (5)
		[8]

	QUESTION 7	
7.1 7.1.1	$f(x+h) = 1 - 4(x+h)^2$ $\qquad = 1 - 4x^2 - 8xh - 4h^2$	$\checkmark f(x+h) =$ $1 - 4x^2 - 8xh - 4h^2$
	$f'(x) = \lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$ $\qquad = \lim\limits_{h \to 0} \dfrac{1 - 4x^2 - 8xh - 4h^2 - (1 - 4x^2)}{h}$ $\qquad = \lim\limits_{h \to 0} \dfrac{-8xh - 4h^2}{h}$ $\qquad = \lim\limits_{h \to 0} \dfrac{h(-8x - 4h)}{h}$ $\qquad = -8x$	$\checkmark$ correct substitution into correct formula $\checkmark$ simplify $(-8xh - 4h^2)$ $\checkmark$ common factor h $\checkmark -8x$ (5)
7.1.2	$f'(2) = -8(2)$ $\qquad = -16$	$\checkmark$ substitute $x = 2$ into $f'(x)$ $\checkmark$ answer (2)
7.2 7.2.1	$y = x^2 - 4x + 4$ $\dfrac{dy}{dx} = 2x - 4$	$\checkmark$ expansion $\checkmark 2x$ $\checkmark -4$ (3)
7.2.2	$f(x) = x^{\frac{2}{3}} + \dfrac{1}{4}x^{-4}$ $f'(x) = \dfrac{2}{3}x^{\frac{-1}{3}} - x^{-5}$	$\checkmark x^{\frac{2}{3}}$ $\checkmark \dfrac{1}{4}x^{-4}$ $\checkmark \dfrac{2}{3}x^{\frac{-1}{3}}$ $\checkmark -x^{-5}$ (4)
		[14]

	QUESTION 8	
8.1	$f(x) = (x+2)(x-1)(x-3)$ $\quad = x^3 - 2x^2 - 5x + 6$	✓factors $(x+2)(x-1)(x-3)$ ✓product of any two factors ✓product of any binomial and trinomial $x^3 + x^2 - 2x - 3x^2 - 3x + 6$ (3)
8.2	$f'(x) = 3x^2 - 4x - 5$ $\quad 0 = 3x^2 - 4x - 5$ $\quad x = \dfrac{4 \pm \sqrt{16 + 60}}{6}$ $\quad x = 2{,}12 \quad$ **OR** $\quad x = -0{,}7$ $y = 8{,}21 \quad$ **OR** $\quad y = -4{,}06$ $A(-0{,}79 ; 8{,}21) \qquad B(2{,}12 ; -4.06)$	✓ $f'(x) = 0$ ✓substitute into quadratic formula ✓x-values ✓y-values (4)
8.3	$y = x^3 - 2x^2 - 5x + 6 - (2x)$ $\quad = x^3 - 2x^2 - 7x + 6$ $\dfrac{dy}{dx} = 3x^2 - 4x - 7$ $\quad = (3x - 7)(x + 1)$ for max $0 = (3x - 7)(x + 1)$ $x = \dfrac{7}{3} \quad$ **OR** $\quad x = -1$ $y = (-1)^3 - 2(-1)^2 - 7(-1) + 6$ $PQ = 10$	✓simplification $x^3 - 2x^2 - 7x + 6$ ✓ differentiation ✓ factors ✓$\dfrac{dy}{dx} = 0$ ✓x-values ✓ answer (6)
8.4	$f'(x) = 3x^2 - 4x - 5$ $f''(x) = 6x - 4$ $6x - 4 < 0$ $\quad x < \dfrac{2}{3}$ $\quad k = \dfrac{2}{3}$	✓ $f''(x)$ ✓ $f''(x) < 0$ ✓answer (3)
		[16]

	QUESTION 9	
9.1	BC = $50 - 2x$	✓answer (1)

9.2	Area = AB × BC
	      = $x(50 - 2x)$
	      = $50x - 2x^2$
	
	Max $A'(x) = 50 - 4x$
	      $0 = 50 - 4x$
	      $x = 12{,}5m$
		✓$x(50 - 2x)$
		✓simplification
		  $(50x - 2x^2)$
		
		✓differentiate
		
		✓answer (4)

		[5]
	QUESTION 10	
10.1		✓first branches ✓$\dfrac{2}{9}; \dfrac{2}{9}; \dfrac{5}{9}$ ✓$\dfrac{3}{9}; \dfrac{1}{9}; \dfrac{5}{9}$ ✓$\dfrac{3}{9}; \dfrac{2}{9}; \dfrac{4}{9}$ ✓all outcomes (5)
10.2 10.2.1	P(B, B) = $\dfrac{\dfrac{5}{10} \times \dfrac{4}{9}}{}$ $= 0{,}22$ **OR** $\dfrac{2}{9}$	✓$\dfrac{5}{10} \times \dfrac{4}{9}$ ✓answer (2)
10.2.2	P(G,P) = $\dfrac{3}{10} \times \dfrac{2}{9} + \dfrac{2}{10} \times \dfrac{3}{9}$ $= 0{,}13$ **OR** $\dfrac{12}{90}$ **OR** $\dfrac{2}{15}$	✓$\dfrac{3}{10} \times \dfrac{2}{9}$ **OR** $\dfrac{6}{90}$ ✓$\dfrac{2}{10} \times \dfrac{3}{9}$ **OR** $\dfrac{6}{90}$ ✓answer (3)
		[10]

	QUESTION 11	
11.1	Letters : $26 - 6 = 20$ Digits: $0 - 9 = 10$ $20^4 \times 10^4$ $= 1\,600\,000\,000$	$\checkmark 20^4$ $\checkmark 10^4$ $\checkmark$ answer (3)
11.2 11.2.1	$P'(A) = 1 - P(A)$ $0,45 = 1 - P(A)$ $P(A) = 0,55$ $P(A \text{ or } B) = P(A) + P(B)$ $\qquad\qquad = 0,55 + 0,35$ $\qquad\qquad = 0,9$	$\checkmark P(A)$ $\checkmark$ substitute in correct formula $\checkmark$ answer (3)
11.2.2	$P(A) \times P(B)$ $= 0,55 \times 0,35$ $= 0,1975 \quad \mathbf{OR} \quad \dfrac{77}{400}$ $P(A \text{ or } B) = P(A) + P(B) - P(A \cap B)$ $\qquad\qquad = 0,55 + 0,35 - 0,1975$ $\qquad\qquad = 0,71 \quad \mathbf{OR} \quad \dfrac{283}{400} \quad \mathbf{OR} \quad \dfrac{71}{100}$	$\checkmark 0,55 \times 0,35$ $\checkmark 0,1975 \ \mathbf{OR} \ \dfrac{77}{400}$ $\checkmark$ substitute in correct formula $\checkmark$ answer (4)
		[10]
		TOTAL: 150