SEPTEMBER 2017 MATHEMATICS PAPER 2 / WISKUNDE VRAESTEL 2
MEMORANDUM

NOTE:
- If a candidate answers a question TWICE, mark only the first one.
- Consistent accuracy applies in ALL aspects of the marking memorandum.

LET WEL:
- Indien 'n kandidaat 'n vraag TWEE keer beantwoord, merk slegs die eerste poging.
- Volgehoue akkuraatheid is deurgaans op ALLE aspekte van die memorandum van toepassing.

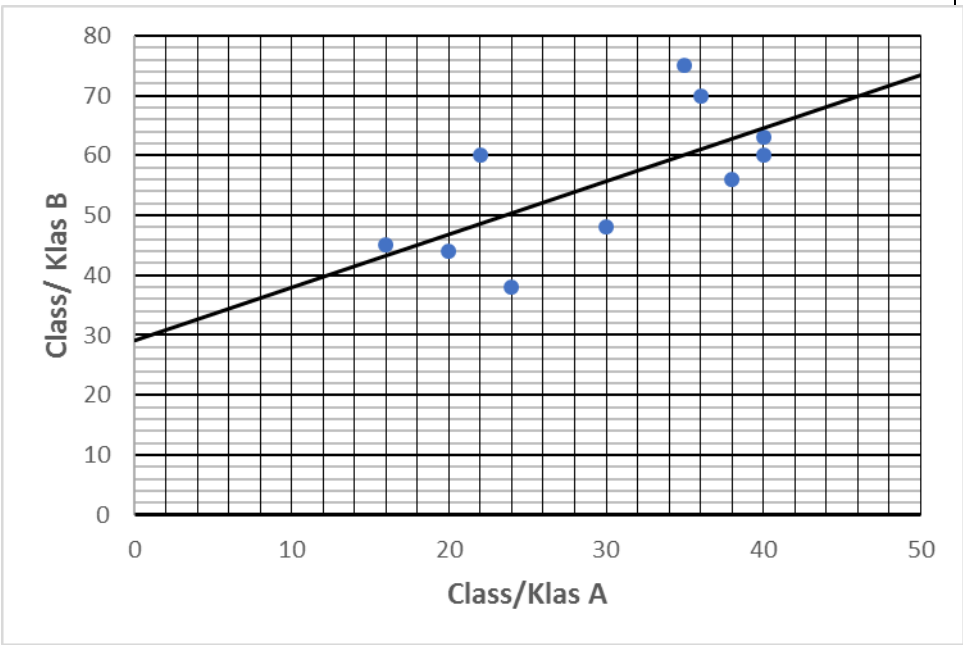
<table>
<tr><th></th><th></th><th>SUGGESTED ANSWER/ VOORGESTELDE ANTWOORD</th><th>DESCRIPTORS/BESKRYWERS</th><th>Marks</th></tr>
<tr><td colspan="5">QUESTION/ VRAAG 1</td></tr>
<tr><td>1.1</td><td></td><td>
<table>
<tr><td>Percentage of income (x)</td><td>Frequency</td><td>Cumulative Frequency</td></tr>
<tr><td>$20 < x \leq 30$</td><td>10</td><td>10</td></tr>
<tr><td>$30 < x \leq 40$</td><td>15</td><td>25</td></tr>
<tr><td>$40 < x \leq 50$</td><td>20</td><td>45</td></tr>
<tr><td>$50 < x \leq 60$</td><td>10</td><td>55</td></tr>
<tr><td>$60 < x \leq 70$</td><td>5</td><td>60</td></tr>
</table>
</td><td>✓ frequency column/ Frekwensie kolom

✓ first 3 points on C. f column/ 1ste 3 punte op Kf kolom

✓ last 2 points on C. f column/ laaste 2 punte op Kf kolom</td><td>(3)</td></tr>
<tr><td></td><td>1.2.1</td><td>$60 - 45$
$= 15$ people spent more than 50% on their income on education</td><td>✓ 45

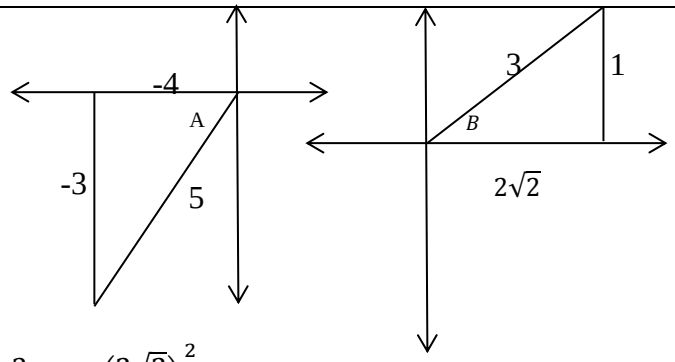
✓ Answer/ Antwoord</td><td>(2)</td></tr>
<tr><td></td><td>1.2.2</td><td>$Median = \approx 42 - 43$</td><td>✓✓Answer/ Antwoord</td><td>(2)</td></tr>
<tr><td></td><td>1.3</td><td>$40 < x \leq 50$
OR
$x \in (40; 50]$</td><td>✓ Answer/ Antwoord

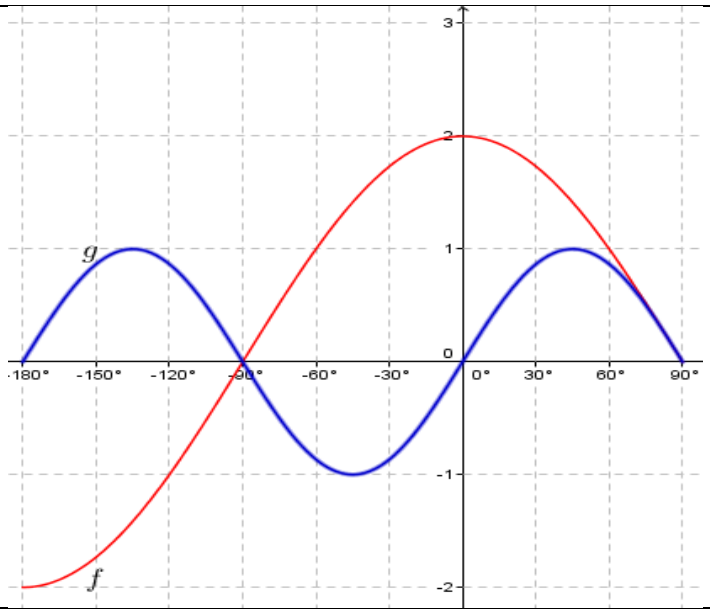
Accept/ Aanvaar: from 40 to 50 / van 40 tot 50</td><td>(1)</td></tr>
<tr><td></td><td colspan="3"></td><td>[8]</td></tr>
</table>

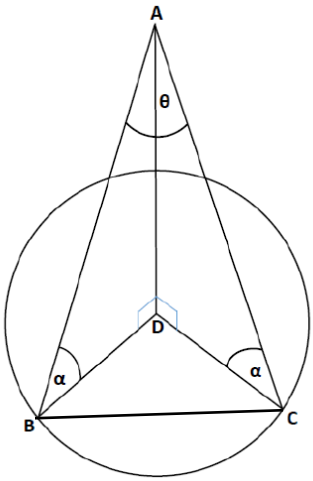
SUGGESTED ANSWER/ VOORGESTELDE ANTWOORD		DESCRIPTORS/BE SKRYWERS	Marks
QUESTION/ VRAAG 2			
2.1 & 2.3		✓ Plotting 5 - 8 points correct/ *Afsteek van 5 – 8 punte korrek* ✓ Plotting 9 -10 points correct/ *Afsteek van 9 – 10 punte korrek* **2.3 Regression line/ *Regressielyn*:** ✓ y -int: ≈ 29 ✓ Passing through ≈ (30; 56)	(2) (2)
2.2	$a = 29{,}22\ \ (29{,}21542\ldots)$ $b = 0.89$ $\therefore y = 29{,}22 + 0{,}89x$	✓ *a* ✓ *b* ✓equation / *vergelyking*	(3)
2.3	On graph/ *Op grafiek*		
2.4	r = 0,66 Moderately positive relationship/ *redelike (matige) positiewe verwantskap*	✓ r ✓ Moderately positive/ *Redelike positiewe*	(2)
2.5	Mean/ *Gemiddelde*, $\bar{x} = \frac{559}{10} = 55{,}90$ Standard deviation/ *Standaardafwyking:* $\sigma = 11.36$	✓ $\frac{559}{10}$ ✓ 55,90 ✓✓ σ = 11.36	(4)
2.6	**44,54** ⟷ **67,26** **55,90** **6** scores lie within one σ/ ***6*** *Punte val binne een σ*	✓ Interval ✓Answer/*Antw*	(2)
			[15]

		SUGGESTED ANSWER/ VOORGESTELDE ANTWOORD	DESCRIPTORS/BESKRYWERS	Marks
QUESTION/ VRAAG 3				
3.1	$m_{TQ} = \frac{4-0}{0-3}$ $= \frac{-4}{3}$		✓ Subst in gradient formula/ *Vervang in gradiënt formule* ✓ Answer/*Antwoord*	(2)
3.2	RQ $= \sqrt{(10-3)^2 + (7-0)^2}$ $= \sqrt{49+49}$ $= \sqrt{98}$ Units/ *eenhede*		✓ Substitute/ *Vervang* ✓ Answer/*Antwoord*	(2)
3.3	$m_{TQ} = \frac{4-0}{0-3} = \frac{-4}{3}$ **or/of** $m_{TF} = m_{TQ}$ $\therefore y = \frac{-4}{3}x + 4$ $\quad \therefore \frac{-8-4}{k} = \frac{-4}{3}$ $\therefore -8 = \frac{-4}{3}(k) + 4$ $\quad -4k = 3(-12)$ $\therefore -12 = \frac{-4}{3}(k)$ $\therefore \boldsymbol{k = 9}$		✓ eqn of/ *vgl van* TQ ***or*** gradients = ✓ Subst/ *vervang* (*k*; -8) **or** gradient of TF ✓ Simplify/ *Vereenv* ***or*** *cross* × ✓ Answer/*Antwoord*	(4)
3.4	$\boldsymbol{S(7;11)}$		✓ $x = 7$ ✓ $y = 11$	(2)
3.5	T$\hat{S}$R = T$\hat{Q}$R .opp ∠ of $\|\|^m$/ *….teenoorst.* ∠ van $\|\|^m$ $m_{TQ} = \frac{-4}{3}$ $\quad m_{RQ} = 1$ $\tan\alpha = \frac{-4}{3}$ $\quad \tan\beta = 1$ RA/ $VH = 53{,}13°$ $\quad \beta = 45°$ $\alpha = 126{,}87°$ $\therefore$ T$\hat{S}$R $= \alpha - \beta$ $\therefore$ **T$\hat{S}$R = 81, 87°**		✓ T$\hat{S}$R = T$\hat{Q}$R ✓ $\tan\alpha = \frac{-4}{3}$ ✓ $\tan\beta = 1$ ✓ $\alpha = 126{,}87°$ ✓ $\beta = 45°$ ✓ Answer/*Antwoord*	(6)
3.6	MQ $= \sqrt{(2)^2 + (2)^2}$ $= \sqrt{8}$ But/*Maar* RQ $= \sqrt{98}$ … *calc. in q* 3.2 $\therefore$ ***Ratio*** $= \frac{\sqrt{8}}{\sqrt{98}}$ $= \frac{2}{7}$		✓ $\sqrt{8}$ ✓ $\frac{\sqrt{8}}{\sqrt{98}}$ ✓ Answer/ *Antwoord*	(3)
				[19]

	SUGGESTED ANSWER/ VOORGESTELDE ANTWOORD	DESCRIPTORS/BESKRYWERS	Marks
QUESTION /VRAAG 4			
4.1	Radius to midpoint of chord/ *radius na midpt van koord*	✓ Answer/ *Antwoord*	(1)
4.2	$m_{TS} = -\frac{3}{3} \Rightarrow -1$ $\therefore m_{PN} = m_{TS} \times -1$ ... ... TS ⊥ NP $\therefore m_{PN} = 1$ $y - y_1 = m(x - x_1)$ OR/ *OF* $y = mx + c$ $\therefore y - 8 = x + 3$ $8 = -3 + c$ $c = 5$ $\therefore \boldsymbol{y = x + 11}$	✓m_{TS} ✓✓m_{PN} ✓Subst (-3; 8) ✓ Equation/ *Vergelyking*	(5)
4.3	*Substitute* $P(0; y)$ $y = 0 + 11$ $\therefore \boldsymbol{y = 11}$ *Radius*: $11 - 5 = 6$ $\therefore \boldsymbol{y = -1}$	✓*Substitute* $P(0; y)$ ✓ Answer/ *Antwoord* ✓*Radius* ✓ Answer/ *Antwoord*	(4)
4.4	Substitute M $(x: 0)$/ *Stel* $M(x; 0)$ $0 = x + 4 \Rightarrow x = -11$ $\boldsymbol{M = (-11; 0)}$ $TO = 5$, $MO = 11$ $TM^2 = 5^2 + 11^2$ …*pyth* $\therefore MT = \sqrt{146}$	✓ Subst $(x; 0)$ / *vervang* $(x; 0)$ ✓ Length of TO/ *Lengte van TO* ✓ Length of MO/ *Lengte van MO* ✓ Answer/ *Antwoord*	(4)
4.5	$(x - a)^2 + (y - b)^2 = r^2$ TM is the diameter/ *is die middellyn* ⊙ TSM $r = \frac{\sqrt{146}}{2} \Rightarrow r^2 = \frac{146}{4}$ Centre of / *midpt* Θ TSM : Midp. of TM $= (\frac{x+x_1}{2} ; \frac{y+y_1}{2})$ $= (\frac{-11+0}{2} ; \frac{0+5}{2})$ $= (\frac{-11}{2} ; \frac{5}{2})$ $\therefore (\boldsymbol{x + \frac{11}{2}})^2 + \left(\boldsymbol{y - \frac{5}{2}}\right)^2 = \frac{146}{4}$ or $\frac{73}{2}$ or 36,5	✓TM is the diameter of/ *is die middellyn van* ⊙ TSM ✓ $r = \frac{\sqrt{146}}{2}$ ✓ centre $(\frac{-11}{2} ; \frac{5}{2})$ ✓ $(\boldsymbol{x + \frac{11}{2}})^2 + \left(\boldsymbol{y - \frac{5}{2}}\right)^2$ ✓ $\frac{146}{4}$ or $\frac{73}{2}$ or 36,5	(5)
			[19]

	SUGGESTED ANSWER/ VOORGESTELDE ANTWOORD	DESCRIPTORS/BESKRYWERS	Marks
QUESTION /VRAAG 5			
5.1	 $5\left(\frac{-3}{5}\right) - 6\left(\frac{2\sqrt{2}}{3}\right)^2$ $= -3 - 6\left(\frac{8}{9}\right)$ $= -\frac{25}{3}$	✓ figure 1 in correct quadrant; $r = 5$ / *figuur 1 in korrekte kwadrant*; $r = 5$ ✓ figure 2 in correct quadrant; $x = 2\sqrt{2}$ / *figuur 2 in korrekte kwadrant*; $x = 2\sqrt{2}$ ✓✓ substitution / *substitusie* ✓ Answer/ *Antwoord*	(5)
5.2.1	$cos\ (360 - 24°)$ $= cos\ 24° = p$	✓ $cos\ 24°$ ✓ Answer/ *Antwoord*	(2)
5.2.2	$cos\ 2(24°)$ $= 2cos^2 24° - 1$ $= 2p^2 - 1$	✓ $cos2(24°)$ ✓$2cos^2 24° - 1$ ✓ Answer/ *Antwoord*	(3)
5.3	$LHS = \tan A$ $\quad RHS = \frac{1-cos2A}{sin2A}$ $= \frac{1-(1-2sin^2A)}{2sinAcosA}$ $= \frac{2sin^2A}{2sinAcosA}$ $= \frac{sinA}{cosA}$ $= tanA$	✓$1 - 2sin^2A$ ✓$2sinAcosA$ ✓ simplify / *vereenvoudig* ✓ Answer/ *Antwoord*	(4)
5.4	$\sin 20° . (\cos 40°) + \cos 20° . \sin 40°$ $= \sin(20° + 40°)$ $= \sin 60°$ $= \frac{\sqrt{3}}{2}$	✓$cos40°$✓ $cos20°$ ✓ $sin40°$ ✓ $sin\ 60°$ ✓$\frac{\sqrt{3}}{2}$	(5)
5.5	$2\cos^2\theta + 5\sin\theta + 1 = 0$ $\therefore 2(1 - \sin^2\theta) + 5\sin\theta + 1 = 0$ $\therefore 2 - 2\sin^2\theta + 5\sin\theta + 1 = 0$ $\therefore -2\sin^2\theta + 5\sin\theta + 3 = 0$ $\therefore 2\sin^2\theta - 5\sin^2\theta - 3 = 0$ $\therefore (2\sin\theta + 1)(\sin\theta - 3) = 0$ $\therefore \sin\theta = -\frac{1}{2}\ ; \sin\theta \neq 3$ $\theta = 180° + 30° + 360°n$ or $/of$ $\theta = 360° - 30° + 360°n,$ $\therefore \theta = 210° + 360°n$ or $/of$ $\theta = 330° + 360°n\ ,\ n \in Z$	✓ $(1 - sin^2\theta)$ ✓ standard form/ *Standaardvorm* ✓ factors *faktore* ✓$\sin\theta = -\frac{1}{2}$ ✓$\sin\theta \neq 3$ ✓ 210° and/ en 330° ✓ $360°n\ , n \in Z$	(7)
			[26]

	SUGGESTED ANSWER/ VOORGESTELDE ANTWOORD	DESCRIPTORS/BESKRYWERS	Marks
QUESTION /VRAAG 6			
6.1	$a = 2$	✓ Answer/ *Antwoord*	(1)
6.2		✓ Max TPs /*Maks DPe* ✓Min TP/*Min DP* ✓x-intercepts / *x-afsnitte*	(3)
6.3	180°	✓180°	(1)
6.4	$x = -90°$ or $x = 90°$	✓ $x = -90°$ ✓ $x = 90°$	(2)
6.5	$3 - 5.2 \sin x \cos x$ $= 3 - 5 \sin 2x$ $= 3 - 5$ (-1) $= 8$	✓ $- 5.2 \sin x \cos x$ ✓$\sin 2x$ ✓Answer/ *Antwoord*	(3)
			[10]

	SUGGESTED ANSWER/ VOORGESTELDE ANTWOORD	DESCRIPTORS/BESKRYWERS	Marks
QUESTION /VRAAG 7			
7.1	 $cos\ \alpha = \frac{r}{AB}$ $AB = \frac{r}{\cos \alpha}$	✓ expression AB / *uitdrukking AB*	(1)
7.2	$cos\ \alpha = \frac{r}{AC}$ $AC = \frac{r}{cos\ \alpha}$ $BC^2 = AB^2 + AC^2 - 2AB.ACcosB\hat{A}C$ $BC^2 = \left(\frac{r}{cos\ \alpha}\right)^2 + \left(\frac{r}{cos\ \alpha}\right)^2 - 2\left(\frac{r}{cos\ \alpha}\right)\left(\frac{r}{cos\ \alpha}\right)cos\theta$ $BC^2 = \frac{2r^2}{cos^2\alpha} - \frac{2r^2cos\theta}{cos^2\alpha}$ $BC^2 = \frac{2r^2}{cos^2\alpha}(1 - cos\theta)$ $BC = \frac{r\sqrt{2(1-cos\theta)}}{cos\alpha}$ **OR** $cos\ \alpha = \frac{r}{AC}$ $AC = \frac{r}{cos\ \alpha}$ $\therefore AB = AC$ $BC^2 = 2AB^2 - 2AB^2cos\ \theta$ $BC^2 = AB^2(2 - 2cos\ \theta)$ $BC = AB\sqrt{2 - 2cos\theta}$ $BC = \frac{r}{cos\alpha}\sqrt{2(1-cos\theta)}$	✓ expression AC / *uitdrukking AC* ✓cosine rule / *cos reël* ✓substitution / *substitusie* ✓common factor / *gemeenskaplike faktor* ✓ expression AC / *uitdrukking AC* ✓cosine rule / *cos reël* ✓substitution $AB = AC$ / *vervanging* $AB = AC$ ✓common factor / *gemeenskaplike faktor*	(4)

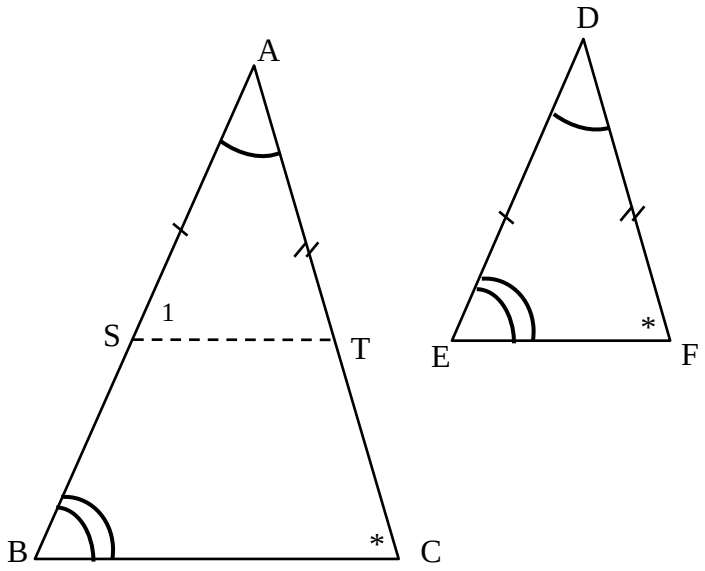
7.3	$100 = \frac{50\left(\sqrt{2(1-cos30°}\right)}{cos\alpha}$ $cos\alpha = \frac{50\left(\sqrt{2(1-cos30°}\right)}{100}$ $\alpha = 75°$	✓ substituting values of BC, r and θ / *substitusie van waardes vir BC, r and* θ ✓ $cos\alpha = \frac{50\left(\sqrt{2(1-cos30°}\right)}{100}$ ✓ $\alpha = 75°$	(3)
			[8]

GEOMETRY/ *MEETKUNDE*

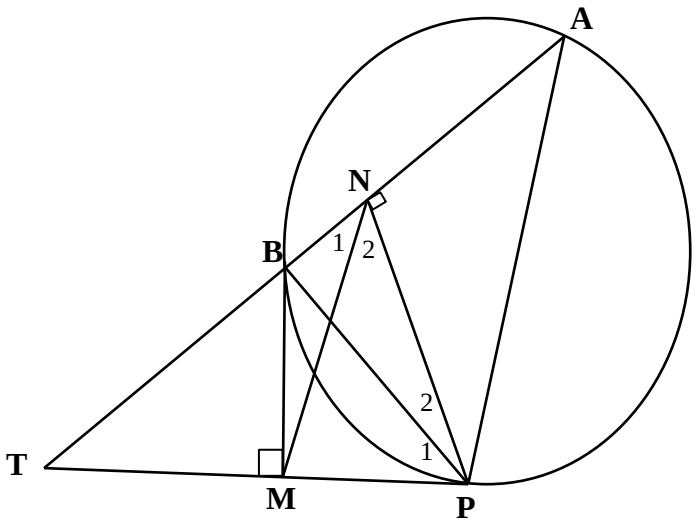
S: Statement/ *Bewering*
R: Reason/ *Rede*
S/R: Both Statement and Reason/ *Beide Bewering en Rede*

Allow for alternative methods/ *Kyk vir Alternatiewe metodes*

	SUGGESTED ANSWER/ VOORGESTELDE ANTWOORD	DESCRIPTORS/BESKRYWERS	Marks
QUESTION /VRAAG 8			
8.1.1	90°	✓ S	(1)
8.1.2	Supplementary/ *supplementêr*	✓ S	(1)
8.2.1	$\hat{E}_1 = 54°$ ... tan-chord thm/ *raaklyn-koordstelling*	✓ S ✓ R	(2)
8.2.2	$\hat{C}_1 = 36°$ ... angle in semi-circle/ *hoek in halfsirkel* OR angle subt. By diameter/ *hoek onderspan deur midlyn*	✓ S ✓ R	(2)
8.2.3	$\hat{C}_3 = 35°$ ... alt. ∠s; AB//CD / *verw.∠e; AB//CD*	✓ S ✓ R	(2)
8.2.4	$A\hat{E}D = 55°$ ...opp ∠s of cyclic quad/ *teenoorst ∠e van kvh*	✓ S ✓ R	(2)
8.2.5	$\hat{E}_4 = 35°$ ... subt by AC/ *ondersp deur AC* $\therefore \hat{E}_3 = 20°$	✓ S ✓ R ✓ S	(3)
8.3	$B\hat{A}C = 90°$ ... tangent- radius/ *raaklyn- radius* And/ *en* $B\hat{E}C = 90°$ ... tangent- radius/ *raaklyn- radius* ∴ BECA is a cyclic quad/ *is 'n kvh* ... opp.∠s suppl./ *teenoorst ∠e suppl.* $\therefore D\hat{B}E = A\hat{C}E$... external ∠ of cyclic quad/ *Buite ∠ van kvh*	✓ S ✓ R ✓ S ✓ R ✓ R can only get this mark if BECA is proved a cyclic quad.	(5)
			[18]

	SUGGESTED ANSWER/ VOORGESTELDE ANTWOORD	DESCRIPTORS/BESKRYWERS	Marks
QUESTION /VRAAG 9			
9.1			
	Constr./ *Konstr:* Draw ST with S on AB and T on AC such that AS = DE and AT = DF/ *Trek ST met S op AB en T op AC sodat AS = DE en AT = DF* $\Delta AST \equiv \Delta DEF$ …S, ∠, S $\therefore \hat{S}_1 = \hat{E}$ … from/ *vanaf* congruency/ *kongruensie* $= \hat{B}$ … given / *gegee* ∴ ST // BC …corresp. ∠s =/ *ooreenk. ∠e* = $\therefore \frac{AB}{AS} = \frac{AC}{AT}$ $\therefore \frac{AB}{DE} = \frac{AC}{DF}$ (Constr/ *Konstr*: AS = DE and/ *en* AT = DF	✓ Constr./ *Konstr* – can be on diagram/ *kan op diagram wees* ✓ S/R ; Congruency/ *Kongruensie* ✓ $\hat{S}_1 = \hat{E}$ $= \hat{B}$ ✓ S: ST // BC ✓ R ✓ S: $\frac{AB}{AS} = \frac{AC}{AT}$ ✓ R	(7)
9.2.1	$\hat{Q}_1 = \hat{Q}_2$… given/ *gegee* $= \hat{N}_2$ … alt. ∠s; MN // QR / *verw. ∠e; MN // QR* ∴MN = MQ … sides opp = ∠s / *sye teenoor = ∠e* ∴ΔQMN is isosceles/ *gelykbenig*	✓ S: $\hat{Q}_1 = \hat{Q}_2$ $= \hat{N}_2$ ✓ R ✓ R	(3)
9.2.2	<u>In ΔPMN and/ en ΔPQR:</u> 1. P is common/ P *is gemeen* 2. $\hat{N}_1 = \hat{R}$ ... corr./ *ooreenk* ∠s; MN // QR ΔPMN /// ΔPQR … (∠, ∠, ∠) or 3rd ∠ given	 ✓ S ✓ S ✓ R ✓ R (∠, ∠, ∠) or 3rd ∠ given	(4)

9.2.3	From/ vanaf /// in 9.2.2: $\frac{PM}{PQ} = \frac{MN}{QR} = \frac{PN}{PR}$ $\therefore \frac{PM}{5} = \frac{MN}{7,5}$ $\therefore \frac{PM}{MN} = \frac{5}{7,5} = \frac{2}{3}$ $\therefore \frac{PM}{MQ} = \frac{2}{3}$ … MQ = MN from/ *vanaf* 9.2.1 $\therefore \frac{PN}{NR} = \frac{2}{3}$ … Prop thm/ *ewer.stelling*; MN // QR or/ *of* line // to 1 side of Δ / *lyn // aan 1 sy van Δ*	✓ Proportionality/ *Eweredigheid* ✓ Subst/*verv* PQ = 5 and/ *en* QR = 7,5 ✓ $\frac{PM}{MN} = \frac{2}{3}$ ✓ $\frac{PM}{MQ} = \frac{2}{3}$ ✓ $\frac{PN}{NR} = \frac{2}{3}$	(5)
			[19]

	SUGGESTED ANSWER/ VOORGESTELDE ANTWOORD	DESCRIPTORS/BESKRYWERS	Marks
QUESTION /VRAAG 10			
			
10.1	$B\hat{M}T = 90° = B\hat{N}P$ ∴ BMPN is a cyclic quad/ *is 'n kvh* ... ext ∠ = int opp∠ / *buite∠ = teenoorst binne∠*	✓ S ✓ R	(2)
10.2	$\hat{N}_1 = \hat{P}_1$ …subtended by/ *onderspan deur* BM But/ *Maar* $\hat{P}_1 = \hat{A}$ … tan-chord thm/ *raakl-koordstelling* ∴NM // AP … corresp/ *ooreenk.* ∠s = $\therefore \frac{TN}{AN} = \frac{TM}{MP}$	✓ S ✓ R ✓ S ✓ R ✓ S ✓ R	(6)
			[16]

Total/Totaal: [150]