GAUTENG PROVINCE
EDUCATION
REPUBLIC OF SOUTH AFRICA

GAUTENG DEPARTMENT OF EDUCATION

PREPARATORY EXAMINATION

2017

10611

MATHEMATICS

FIRST PAPER

TIME: **3 hours**

MARKS: **150**

11 pages + 1 information sheet and 1 answer sheet

MATHEMATICS: Paper 1
1061E

10611E

X05

1

P.T.O.

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(First Paper)

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INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 11 questions.
2. Answer ALL the questions.
3. Clearly show ALL calculations, diagrams, graphs, etc. which were used in determining the answers.
4. Answers only will not necessarily be awarded full marks.
5. Use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
6. Where necessary, answers should be rounded-off to TWO decimal places, unless stated otherwise.
7. Diagrams are NOT necessarily drawn to scale.
8. An ANSWER SHEET for Question 4.2 is located on Page 13 of the question paper.

   This page must be submitted together with your ANSWER BOOK.
9. An information sheet is included on Page 12 of the question paper.
10. Number the questions correctly according to the numbering system used in the question paper.
11. Write neatly and legibly.

QUESTION 1

1.1 Solve for x:

1.1.1 $x(x-1)=12$ (3)

1.1.2 $2x^2+3=8x$ (Correct to TWO decimal places) (4)

1.1.3 $(2x+3)(3-x)>4$ (5)

1.1.4 $2^x-5.2^{x+1}=-144$ (3)

1.2 Solve for x and y simultaneously:

$y+2x-3=0$ and $y=x^2+2x+3$ (5)

[20]

QUESTION 2

2.1 If $a+2$; $4a$; $6a+4$ are the first three terms of an arithmetic progression, determine the value of a. (2)

2.2 Given the sequence: 5; 6; 12; 12; 19; 24; 26; 48; …

2.2.1 Write down the next 2 terms. (2)

2.2.2 Determine the value of the 45th term. (3)

2.3 The eighth term of an arithmetic pattern is 33 and the eleventh term is 45. Determine the 15^{th} term. (5)

2.4 Consider the sequence: 6; 6; 2; –6; –18; …
Determine the n^{th} term of the sequence. (4)

2.5 Given: $A=\sum_{r=1}^{\infty} 3\left(\frac{1}{2}\right)^{r-1}$

$B=\sum_{k=1}^{n} 3\left(\frac{1}{2}\right)^{k-1}$

2.5.1 Calculate the value of *A*. (2)

2.5.2 Determine *B* in terms of *n*. (2)

2.5.3 For which values of *n* is $A-B<\frac{1}{36}$? (5)

[25]

QUESTION 3

3.1 Joseph opens a savings account for 5 years at an interest rate of 6% per annum, compounded quarterly. Determine the effective interest rate. (3)

3.2 A loan is required to purchase equipment for a gym. The owner of the gym can pay R2 000 per month, starting one month after the loan is granted. The repayments continue for 2 years at 24% p.a. compounded monthly.
Calculate how much money the owner intends to borrow. (4)

3.3 On 1 August 2017, a company purchased a new vehicle which they will replace in exactly 5 years' time. The company needs R200 000 to cover the replacement cost of this vehicle and sets up a sinking fund for this purpose.
Calculate the monthly deposits that must be made into the sinking fund at an interest rate of 6,25% p.a. compounded monthly. The first deposit is made 4 months after the original vehicle was purchased. (6)

[13]

QUESTION 4

Given: $f(x) = 4.2^{x+1} - 2$

4.1 Determine:

4.1.1 $f(-1)$ (1)

4.1.2 The x-intercept (3)

4.1.3 The y-intercept (2)

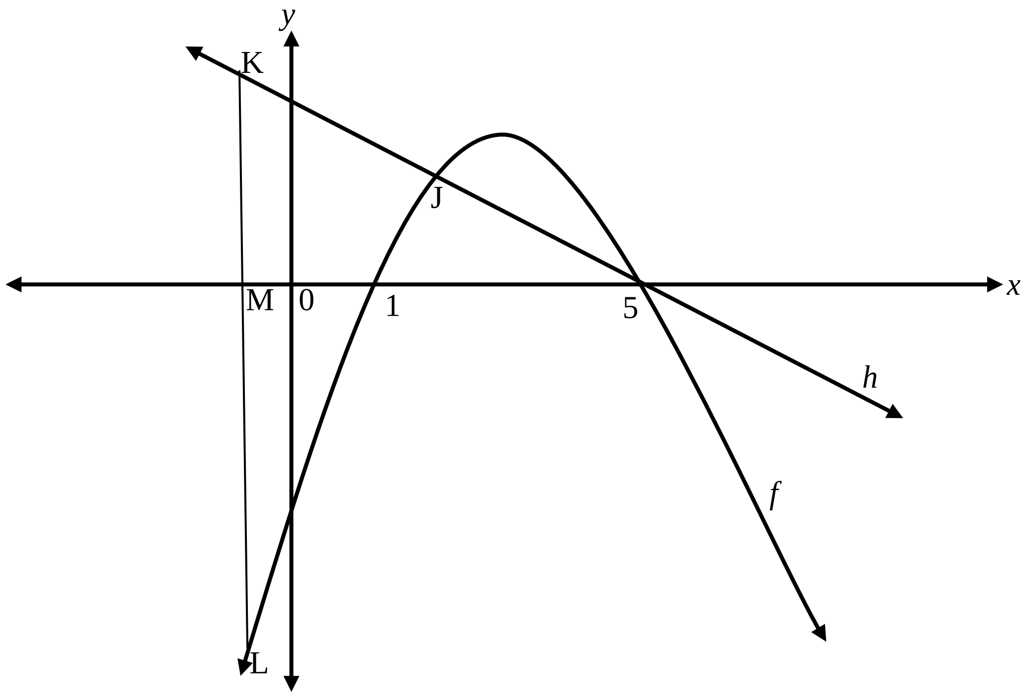
4.1.4 The equation of the asymptote (1)

4.2 Sketch the graph of f on the ANSWER SHEET provided on Page 13.
Indicate the intercepts with the axes, asymptote and coordinates of any ONE point on the graph which is not on an axis. (3)

4.3 Write down the equation of graph g, if g represents the graph of f shifted 2 units to the right. (1)
[11]

QUESTION 5

5.1 The graphs of $f(x) = -(x-3)^2 + 4$ and $h(x) = -x + 5$ are sketched below.
The points (1; 0) and (5; 0) are the x-intercepts of f. The point J (2 ; 3) is a point of intersection of f with h. KML is parallel to the y-axis.



5.1.1 Write down the coordinates of the turning point of f. (2)

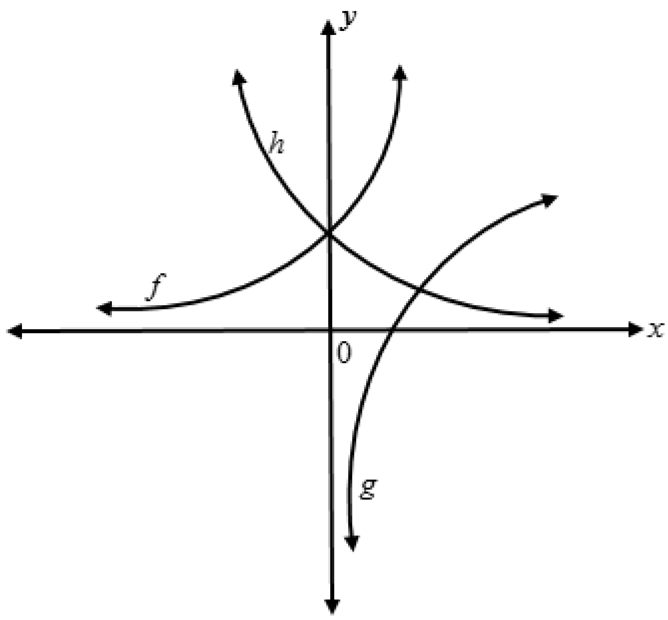
5.1.2 Write down the range of g, the reflection of f in the x-axis. (2)

5.1.3 Determine the average gradient of the curve of f between $x = 2$ and $x = 5$. (2)

5.1.4 If OM = 1 unit, determine the length of KL. (3)

5.1.5 For which values of x is $f(x) > h(x)$? (2)

5.2 The diagram below shows the graphs of g, h and $f(x)=\left(\frac{4}{3}\right)^x$.



5.2.1 Write down the equation of g if $g(x) = f^{-1}(x)$. (2)

5.2.2 Write down the equation of h, the reflection of f in the y-axis. (2)

5.2.3 Determine, with the aid of the sketch above, the x-value(s) for which :

a) $f(x) - h(x) = 0$ (1)

b) $\log_{\frac{4}{3}} x \leq 0$ (2)

[18]

QUESTION 6

Given the equation: $f(x) = -\frac{4}{x} + 7$; $x \neq 0$

6.1 Show that the gradient of all tangents to the curve f will ALWAYS be positive. (3)

6.2 Determine the equation of a tangent to the curve f which is perpendicular to the line $y = -4x$ for values of $x < 0$. (5)

[8]

QUESTION 7

7.1 Given: $f(x) = 1 - 4x^2$

7.1.1 Determine $f'(x)$ from first principles. (5)

7.1.2 Hence, calculate the gradient of the tangent to f at $x = 2$. (2)

7.2 Determine:

7.2.1 $\frac{dy}{dx}$ if $y = (2 - x)^2$ (3)

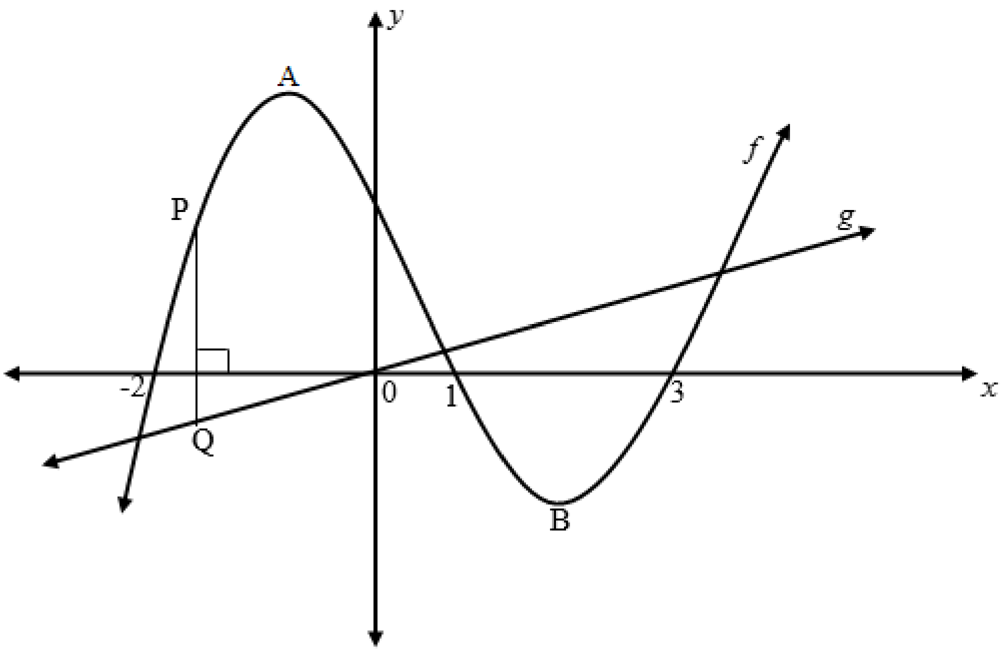
7.2.2 $f'(x)$ if $f(x) = \sqrt[3]{x^2} + \frac{1}{4x^4}$ (4)

[14]

QUESTION 8

Given: $f(x) = x^3 + bx^2 + cx + d$ and $g(x) = 2x$

The graph of f intersects the x-axis at $x = -2$; $x = 1$ and $x = 3$.
The turning points of f are at points A and B respectively, where $x_B > x_A$.
Line PQ is perpendicular to the x-axis, with point P on f and point Q on g.



8.1 Show that the equation of f can be given as:
$f(x) = x^3 - 2x^2 - 5x + 6$. (3)

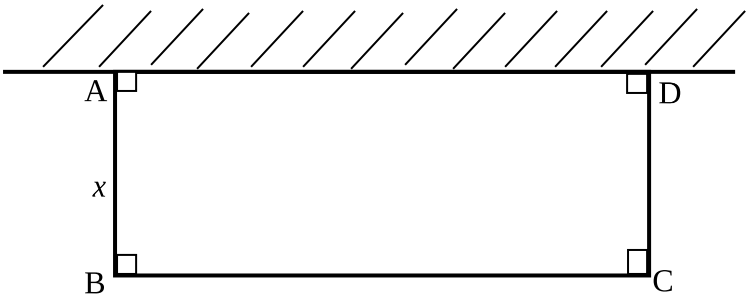
8.2 Calculate the coordinates of points A and B. (4)

8.3 Calculate the maximum length of line PQ, for the interval $-2 < x < 3$. (6)

8.4 The graph of f is concave down for $x < k$. Calculate the value(s) of k. (3)
[16]

QUESTION 9

A 50m long fence is used to enclose a rectangular garden ABCD, using the wall (AD) as one of its boundaries. Let AB = x .



9.1 Express the length of BC in terms of x. (1)

9.2 Hence determine the value of x such that the enclosed area will be maximum. (4)
[5]

QUESTION 10

A packet of sweets has 3 pink, 2 green and 5 blue sweets. Two sweets are removed from the packet, one at a time, without replacement.

10.1 Draw a tree diagram to determine ALL possible outcomes. Indicate on your diagram the probability associated with each branch of the tree diagram. (5)

10.2 Determine the probability that ...

10.2.1 both sweets are blue. (2)

10.2.2 a green and a pink sweet are selected. (3)
[10]

QUESTION 11

11.1 A school requires a new set of codes to classify library books uniquely. All combinations in the current system have been exhausted. Vowels (A; E; I; O; U) and the letter Q may NOT be used. All digits from 0 to 9 can be used. Repetition of letters and numbers is allowed.

BCD 012	BCDF 0123
CURRENT	NEW

How many different combinations are possible if 4 letters and 4 digits are used to generate new codes? (3)

11.2 If $P'(A) = 0{,}45$ and $P(B) = 0{,}35$. Calculate $P(A\ or\ B)$ if ...

11.2.1 A and B are mutually exclusive events. (3)

11.2.2 A and B are independent events. (4)

[10]

TOTAL: 150

END

INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $\quad$ $A = P(1-ni)$ $\quad$ $A = P(1-i)^n$ $\quad$ $A = P(1+i)^n$

$\sum_{i=1}^{n} 1 = n$ $\quad$ $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ $\quad$ $T_n = a + (n-1)d$ $\quad$ $S_n = \frac{n}{2}\left(2a + (n-1)d\right)$

$T_n = ar^{n-1}$ $\quad$ $S_n = \frac{a\left(r^n - 1\right)}{r-1}$; $\quad r \neq 1$ $\quad$ $S_\infty = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x\left[(1+i)^n - 1\right]}{i}$ $\quad$ $P = \frac{x[1-(1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad$ $\mathrm{M}\left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad$ $y - y_1 = m(x - x_1)$ $\quad$ $m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad$ $m = \tan\theta$

$$(x-a)^2 + (y-b)^2 = r^2$$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $\quad$ $a^2 = b^2 + c^2 - 2bc.\cos A$ $\quad$ $area\ \Delta ABC = \frac{1}{2}ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\quad$ $\sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\quad$ $\cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$ $\quad$ $\sin 2\alpha = 2\sin\alpha.\cos\alpha$

$$(x ; y) \to (x\cos\theta - y\sin\theta ; y\cos\theta + x\sin\theta)$$

$\bar{x} = \frac{\sum fx}{n}$ $\quad$ $\sigma^2 = \frac{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $\quad$ $P(A\ of\ B) = P(A) + P(B) - P(A\ en\ B)$

$\hat{y} = a + bx$ $\quad$ $b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$

ANSWER SHEET

NAME: ____ **GRADE 12:** ____________

QUESTION 4.2