



# **PREPARATORY EXAMINATION**

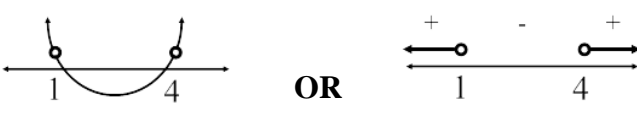
## **2020**

### **MARKING GUIDELINES**

**MATHEMATICS P1 (10611)**

**17 pages**

## QUESTION 1

1.1.1	$3x^2 + 5x = 7$ $3x^2 + 5x - 7 = 0$ $x = \frac{-5 \pm \sqrt{5^2 - 4(3)(-7)}}{2(3)}$ $x = \frac{-5 \pm \sqrt{109}}{6}$ $x = 0,91 \text{ or } x = -2,57$ <p><b>PENALIZE 1 MARK FOR INCORRECT ROUNDING IN THIS QUESTION ONLY.</b></p>	<ul style="list-style-type: none"> <li>✓ standard form</li> <li>✓ subst. into</li> <li>correct formula</li> <li>✓ ✓ answers</li> </ul> <p style="text-align: right;">(4)</p>
1.1.2	$2x^2 = 9x + 5$ $2x^2 - 9x - 5 = 0$ $(2x+1)(x-5) = 0$ $x = -\frac{1}{2} \text{ or } x = 5$ <p><b>ANY OTHER VALID METHOD</b></p>	<ul style="list-style-type: none"> <li>✓ standard form</li> <li>✓ factors</li> <li>✓ both answers correct</li> </ul> <p style="text-align: right;">(3)</p>
1.1.3	$x^2 - 5x > -4$ $x^2 - 5x + 4 > 0$ $(x-4)(x-1) > 0$ <div style="text-align: center;">  </div> $x < 1 \text{ or } x > 4$	<ul style="list-style-type: none"> <li>✓ standard form</li> <li>✓ factors</li> <li>✓ critical values</li> <li>✓ both correct answers</li> </ul> <p style="text-align: right;">(4)</p>

1.1.4	$x - 3x^2 = 4$ $x - 4 = 3\sqrt{x}$ $x^2 - 8x + 16 = 9x$ $x^2 - 17x + 16 = 0$ $(x - 16)(x - 1) = 0$ $x = 16 \text{ OR } x = 1$ <p style="text-align: center;">N/A</p>	<ul style="list-style-type: none"> <li>✓ isolating <math>3\sqrt{x}</math></li> <li>✓ squaring both sides</li> <li>✓ standard form</li> <li>✓ factors</li> <li>✓ critical values</li> <li>✓ selection/rejection</li> </ul> <p style="text-align: right;">(6)</p>
1.2	$2^{2^{x+1}} + 7 \cdot 2^x - 4 = 0$ <p>let <math>2^x = k</math></p> $\therefore 2k^2 + 7k - 4 = 0$ $(2k - 1)(k + 4) = 0$ $k = \frac{1}{2} \text{ OR } k = -4$ $\therefore 2^x = 2^{-1} \quad 2^x \neq -4$ $x = -1 \quad \text{Not a solution}$	<ul style="list-style-type: none"> <li>✓ standard form</li> <li>✓ factors</li> <li>✓ critical values</li> <li>✓ rejection of 1 answer</li> </ul> <p style="text-align: right;">(4)</p>
1.3	$x = y - 13 \dots (1) \quad \text{and} \quad \sqrt{2 - x} = y - 3 \dots (2)$ <p style="text-align: center;">sub (1) into (2)</p> $\sqrt{2 - (y - 13)} = y - 3$ $\sqrt{15 - y} = y - 3$ $15 - y = y^2 - 6y + 9$ $0 = y^2 - 5y - 6$ $0 = (y - 6)(y + 1)$ $y = 6 \text{ OR } y = -1$ $x = -7$ <p style="text-align: center;"><b>OR</b></p>	<ul style="list-style-type: none"> <li>✓ substitution</li> <li>✓ square both sides</li> <li>✓ standard form</li> <li>✓ factors</li> <li>✓ y-values</li> <li>✓ x-value</li> </ul> <p style="text-align: center;"><b>OR</b></p>

	$x = y - 13 \quad \text{and} \quad \sqrt{2 - x} = y - 3$ $\therefore y = x + 13 \quad \dots (1) \qquad \sqrt{2 - x} = y - 3 \quad \dots (2)$ <p>Sub for y into equation 2</p> $\sqrt{2 - x} = x + 13 - 3$ $\sqrt{2 - x} = x + 10$ $2 - x = x^2 + 20x + 100$ $0 = x^2 + 21x + 98$ $0 = (x + 14)(x + 7)$ $x = -14 \quad \mathbf{OR} \quad x = -7$ $y \neq -1 \quad \mathbf{OR} \quad y = 6$	<ul style="list-style-type: none"> <li>✓ substitution</li> <li>✓ square both sides</li> <li>✓ standard form</li> <li>✓ factors</li> <li>✓ x-values</li> <li>✓ y-values</li> </ul> <p style="text-align: right;">(6)</p>
		[27]

## QUESTION 2

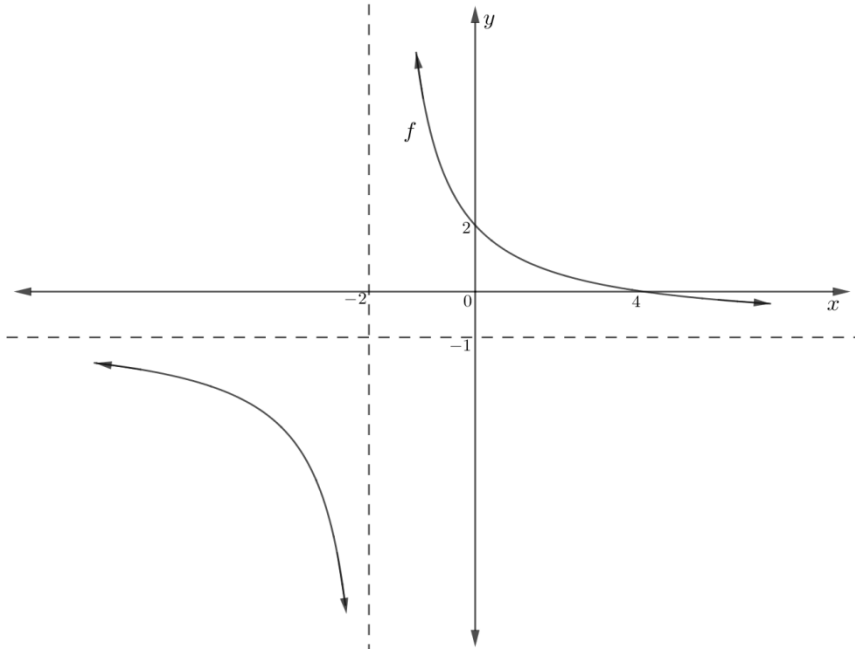
2.1.1	37	✓ answer (1)
2.1.2	$  \begin{array}{c}  1 \quad ; \quad 7 \quad ; \quad 15 \\  \quad \diagdown \quad \diagup \quad \diagdown \quad \diagup \\  \quad 6 \quad \quad \quad 8 \\  \quad \quad \diagdown \quad \diagup \\  \quad \quad \quad 2  \end{array}  $ $2a = 2$ $a = 1$ $3a + b = 6$ $3(1) + b = 6$ $b = 3$ $a + b + c = 1$ $1 + 3 + c = 1$ $c = -3$ $T_n = n^2 + 3n - 3$	✓ second difference  ✓ $a = 1$  ✓ $b = 3$  ✓ $c = -3$  (4)
2.1.3	$W_n = 2n + 4$ $50 = 2n + 4$ $46 = 2n$ $n = 23$ $T_{23} = 23^2 + 3(23) - 3$ $= 595$	✓ $W_n = 2n + 4$ ✓ equating to 50  ✓ $n = 23$ ✓ substitution ✓ answer (5)
2.2.1	$T_{191} = 0$	✓ answer (1)
2.2.2	$-\frac{1}{2}; \frac{1}{2}; \frac{3}{2} \dots 250 \text{ terms}$ $d = 1$ $S_{250} = \frac{250}{2} \left[ 2 \left( -\frac{1}{2} \right) + (250 - 1)(1) \right]$ $S_{250} = 31000$ $\therefore 0 + -\frac{1}{2} + 0 + \frac{1}{2} + 0 + \frac{3}{2} \dots \text{to } 500 \text{ terms}$ $S_{500} = 310$	✓ value of $d$  ✓ correct sub. into correct formula ✓ $S_{250} = 31\ 000$  ✓ answer (4)

2.3	$r = 2 \left( \frac{1-k}{5} \right)$ $\therefore -1 < 2 \left( \frac{1-k}{5} \right) < 1$ $-\frac{1}{2} < \left( \frac{1-k}{5} \right) < \frac{1}{2}$ $-\frac{5}{2} < 1-k < \frac{5}{2}$ $-\frac{7}{2} < -k < \frac{3}{2}$ $-\frac{3}{2} < k < \frac{7}{2}$	<p>✓ <math>r</math></p> <p>✓ <math>-1 &lt; r &lt; 1</math></p> <p>✓ correct critical values</p> <p>✓ answer</p> <p style="text-align: right;">(4)</p>
		<b>[19]</b>

## QUESTION 3

3.1	$S_n > 31$ $\therefore \frac{16 \left( 1 - \left( \frac{1}{2} \right)^n \right)}{1 - \left( \frac{1}{2} \right)} > 31$ $\left( 1 - \left( \frac{1}{2} \right)^n \right) > \frac{31}{32}$ $-\left( \frac{1}{2} \right)^n > -\frac{1}{32}$ $-\left( \frac{1}{2} \right)^n > -\left( \frac{1}{2} \right)^5$ $\therefore n > 5 \text{ or } n \geq 6$ <p><b>ANSWER ONLY: FULL MARKS</b></p>	<p>✓ correct substitution into correct formula</p> <p>✓ simplification <math>\left(-\frac{1}{32}\right)</math></p> <p>✓ correct answers</p> <p>(3)</p>
3.2	$S_\infty = \frac{a}{1-r}$ $S_\infty = \frac{16}{1-\frac{1}{2}}$ $S_\infty = 32$	<p>✓ correct substitution into correct formula</p> <p>✓ answer</p> <p>(2)</p>
		<b>[5]</b>

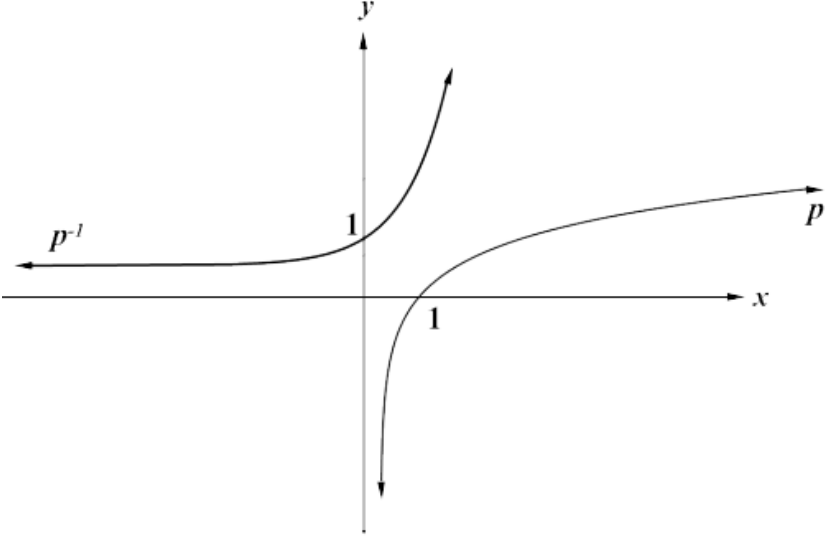
## QUESTION 4

4.1	$x = -2$ $y = -1$	✓ answer ✓ answer (2)
4.2.1	$y = \frac{6}{0+2} - 1$ $y = 2$	✓ $x = 0$ ✓ answer (2)
4.2.2	$0 = \frac{6}{x+2} - 1$ $1 = \frac{6}{x+2}$ $x + 2 = 6$ $x = 4$	✓ $y = 0$ ✓ answer (2)
4.3		✓ shape ✓ asymptotes ✓ x -and y- intercepts (3)



4.4	$y = -(x + 2) - 1$ $y = -x - 3$ <p><b>OR</b></p> $y = mx + c$ $-1 = -1(-2) + c$ $-3 = c$ $\therefore y = -x - 3$	✓ $m = -1$ ✓ correct subst. of point $(-2; -1)$  ✓ answer  <b>OR</b>  ✓ $m = -1$ ✓ correct subst. of point $(-2; -1)$  ✓ answer (3)
		<b>[12]</b>

## QUESTION 5

5.1	$p(x) = \log_a x$ $x = \log_a y$ $\therefore p^{-1}: y = 3^x$	✓ ✓ answers (2)
5.2		✓ ✓ shape $p$ ✓ point on $p$ ✓ ✓ shape $p^{-1}$ ✓ point on $p^{-1}$ (6)
5.3	$2 = \log_a x$ $x = 9$ $0 < x \leq 9$	✓ value of $x$ ✓ ✓ answer (3)
5.4	$x$ -intercept of $p$ is $(1 ; 0)$ $x$ -intercept of $h$ is $(-1 ; 0)$	✓ ✓ answer (2)
		<b>[13]</b>

## QUESTION 6

6.1	$y = a(x - p)^2 + q$ $5 = a(0 - 2)^2 + 3$ $5 = 4a + 3$ $2 = 4a$ $\therefore a = \frac{1}{2}$ $y = \frac{1}{2}(x - 2)^2 + 3$ $y = \frac{1}{2}(x^2 - 4x + 4) + 3$ $y = \frac{1}{2}x^2 - 2x + 2 + 3$ $y = \frac{1}{2}x^2 - 2x + 5$	<p>✓ ✓ sub. point A and B correctly</p> <p>✓ value of <math>a</math></p> <p>✓ sub. <math>a, p</math> and <math>q</math></p> <p>✓ simplification</p> <p>(5)</p>
6.2	<p>The graph does NOT cut the <math>x</math>-axis.</p> <p><math>\therefore</math> No real roots</p> <p><math>\therefore \Delta &lt; 0</math></p>	<p>✓ explanation</p> <p>✓ <math>\Delta &lt; 0</math></p> <p>(2)</p>
6.3	$\frac{1}{2}x^2 - 2x + 5 = k$ <p><math>\therefore k &gt; 3</math></p>	<p>✓ ✓ answer</p> <p>(2)</p>
6.4	$y = \frac{1}{2}x^2 - 2x + 5 - 5$ $y = \frac{1}{2}x^2 - 2x$	<p>✓ answer</p> <p>(1)</p>
<b>[10]</b>		

## QUESTION 7

7.1	$1 + i_{eff} = \left(1 + \frac{0,11}{2}\right)^2$ $i_{eff} = \left(1 + \frac{0,11}{2}\right)^2 - 1$ $i_{eff} = 11,30\%$ <p><math>\therefore</math> Mary has secured the better rate.</p>	<ul style="list-style-type: none"> <li>✓ correct sub. into correct formula</li> <li>✓ answer</li> <li>✓ conclusion</li> </ul> <p style="text-align: right;">(3)</p>
7.2.1	$FV = \frac{10\,000 \left[ \left(1 + \frac{0,0772}{12}\right)^{114} - 1 \right]}{\frac{0,0772}{12}}$ $= R1\,674\,501,44$	<ul style="list-style-type: none"> <li>✓ value of <math>i</math></li> <li>✓ value of <math>n</math></li> <li>✓ correct sub. into correct formula</li> <li>✓ answer</li> </ul> <p style="text-align: right;">(4)</p>
7.2.2	$R1\,674\,501,44 = \frac{30\,000 \left[ 1 - \left(1 + \frac{0,1}{12}\right)^{-n} \right]}{\frac{0,1}{12}}$ $0,46513... = \left[ 1 - \left(1 + \frac{0,1}{12}\right)^{-n} \right]$ $0,53486... = \left(1 + \frac{0,1}{12}\right)^{-n}$ $\log_{\left(1 + \frac{0,1}{12}\right)} 0,53486... = -n$ $n = 75,4$ <p>She will be able to receive the money in 75 full months.</p>	<ul style="list-style-type: none"> <li>✓ subst. of <math>P</math>, <math>x</math> and <math>i</math> into correct formula</li> <li>✓ simplification</li> <li>✓ correct use of logs</li> <li>✓ answer</li> </ul> <p style="text-align: right;">(4)</p>
7.2.3	$Pv = \frac{30\,000 \left[ 1 - \left(1 + \frac{0,1}{12}\right)^{-55} \right]}{\frac{0,1}{12}}$ $Pv = R\,1\,319\,260,60$ <p><math>\therefore</math> No</p>	<ul style="list-style-type: none"> <li>✓ subst. <math>x</math> and <math>i</math> into correct formula</li> <li>✓ correct value of <math>n</math></li> <li>✓ answer</li> <li>✓ conclusion</li> </ul> <p style="text-align: right;">(4)</p>

[15]

## QUESTION 8


8.1	$f(x) = -2x^2 + 6x$ $f(x+h) = -2(x+h)^2 + 6(x+h)$ $f(x+h) = -2(x^2 + 2xh + h^2) + 6x + 6h$ $f(x+h) = -2x^2 - 4xh - 2h^2 + 6x + 6h$ $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ $f'(x) = \lim_{h \rightarrow 0} \frac{-2x^2 - 4xh - 2h^2 + 6x + 6h - (-2x^2 + 6x)}{h}$ $f'(x) = \lim_{h \rightarrow 0} \frac{-2x^2 - 4xh - 2h^2 + 6x + 6h + 2x^2 - 6x}{h}$ $f'(x) = \lim_{h \rightarrow 0} \frac{-4xh - 2h^2 + 6h}{h}$ $f'(x) = \lim_{h \rightarrow 0} \frac{h(-4x - 2h + 6)}{h}$ $f'(x) = \lim_{h \rightarrow 0} -4x - 2h + 6$ $f'(x) = -4x + 6$	<p>✓ value of <math>f(x+h)</math></p> <p>✓ correct sub. into correct formula</p> <p>✓ factorise</p> <p>✓ answer</p> <p>(4)</p>
8.2.1	$f(x) = 2x^2 + \frac{1}{2}x^4 - 3$ $f'(x) = 4x + 2x^3$	<p>✓ <math>4x</math></p> <p>✓ <math>2x^3</math></p> <p>(2)</p>
8.2.2	$f(x) = \frac{x^3 - 5x^2 + 4x}{x - 4}$ $f(x) = \frac{x(x^2 - 5x + 4)}{x - 4}$ $f(x) = \frac{x(x-4)(x-1)}{x-4}$ $f(x) = x^2 - x$ $f'(x) = 2x - 1$	<p>✓ factorising</p> <p>✓ factors</p> <p>✓ simplification of <math>f</math></p> <p>✓ answer</p> <p>(4)</p>

8.3	$y = 2x^2 - 3x - 5$ $\frac{dy}{dx} = 4x - 3$ $dx$ gradient at $x = 2$ $y = 2x^2 - 3x - 5$ $m = 4(2) - 3$ $m = 5$ $y = 5x - 5$	✓ derivative  ✓ substitution ✓ value of $m$ ✓ answer  (4)
		<b>[14]</b>

## QUESTION 9

9.1	<p>For <math>y = \frac{4}{x}</math> the gradient of the tangent to the curve is –</p> <p>1.  <math>y = 4x^{-1}</math>  <math>\frac{dy}{dx} = -4x^{-2} = \frac{-4}{x^2}</math>  <math>\frac{-4}{x^2} = -1</math>  <math>4 = x^2</math></p> <p><math>\therefore x = -2</math> <b>OR</b> <math>x = 2</math>  <math>\therefore y = -2</math>      <math>y = 2</math>  <math>(-2; -2)</math>      <math>(2; 2)</math></p>	<p>✓ exponential form</p> <p>✓ derivative</p> <p>✓ derivative = -1</p> <p>✓ x-values</p> <p>✓ y-values</p> <p>(5)</p>
9.2.1	<p><math>y = (x-1)(x-4)^2</math>  <math>y = (x-1)(x^2 - 8x + 16)</math>  <math>y = x^3 - 9x^2 + 24x - 16</math></p>	<p>✓ <math>(x-1)(x-4)^2</math></p> <p>✓ squaring binomial</p> <p>(2)</p>
9.2.2	<p><math>y = x^3 - 9x^2 + 24x - 16</math>  <math>\frac{dy}{dx} = 3x^2 - 18x + 24 = 0</math>  <math>x^2 - 6x + 8 = 0</math>  <math>(x-2)(x-4) = 0</math>  <math>x = 2</math> <b>OR</b> <math>x = 4</math>  <math>y = 4</math> <b>OR</b> <math>y = 0</math>    <b>B(2 ; 4)</b></p>	<p>✓ derivative = 0</p> <p>✓ factors</p> <p>✓ y-values</p> <p>✓ coordinates of B</p> <p>(4)</p>
9.2.3	<p><math>k &lt; -16</math></p>	<p>✓ ✓ answer</p> <p>(2)</p>
9.2.4	<p><math>f''(x) = 6x - 18</math>  <math>6x - 18 &gt; 0</math>  <math>x &gt; 3</math></p>	<p>✓ <math>6x - 18</math></p> <p>✓ answer</p> <p>(2)</p>
		[15]

## QUESTION 10

10.1	$P[6 - x ; (6 - x)^2]$	✓ answer (1)
10.2	<p style="text-align: center;"> <math>P(6-x ; (6-x)^2)</math> <span style="margin-left: 100px;"><math>S</math></span>    <math>Q(6-x ; 0)</math> <span style="margin-left: 100px;"><math>R(6 ; 0)</math></span> </p> <p> <math>A=L \times B</math>  <math>A = x[(6 - x)^2]</math>  <math>A = x(36 - 12x + x^2)</math>  <math>A = x^3 - 12x^2 + 36x</math> </p> <p> <math>\frac{dA}{dx} = 3x^2 - 24x + 36</math>  <math>x^2 - 8x + 12 = 0</math>  <math>(x - 6)(x - 2)</math>  <math>x \neq 6</math> <b>OR</b> <math>x = 2</math>  <math>y = 16</math> </p> <p> <math>A_{\max} = 16 \times 2</math>  <math>= 32</math> </p>	<p>✓ correct substitution into area formula</p> <p>✓ formula for area in terms of <math>x</math></p> <p>✓ derivative = 0</p> <p>✓ choice of <math>x</math>-value to determine <math>y</math>-value</p> <p>✓ answer (5)</p>
		<b>[6]</b>



## QUESTION 11

11.1	$(x + 0, 2) \times 0,5 = 0, 2$ $x + 0, 2 = 0, 4$ $x = 0, 2$  $0, 2 + 0, 2 + 0,3 + y = 1$ $y = 0,3$	✓ $P(A) \times P(B) = P(A \text{ and } B)$  ✓ $x$ - value  ✓ sum of probabilities = 1 ✓ $y$ -value  (4)
11.2	$P(X \text{ and } Y \text{ together}) = \frac{2!5!}{6!}$ $= \frac{1}{3}$ $P(X \text{ and } Y \text{ not together}) = 1 - \frac{1}{3}$ $= \frac{2}{3}$	✓ $\frac{2!5!}{6!}$  ✓ $\frac{1}{3}$  ✓ answer  (3)
11.3.1	4 digit numbers (with repetition)  $= 9 \times 10 \times 10 \times 10$ $= 9\ 000$	✓ product ✓ answer  (2)
11.3.2	4 digit numbers (without repetition)  $= 9 \times 9 \times 8 \times 7$ $= 4\ 536$	✓ $9 \times 9$ ✓ $8 \times 7$ ✓ answer  (3)
11.3.3	4 digit numbers (with repetition and last digit = 0)  $= 9 \times 8 \times 7 \times 1$ $= 4\ 536$	✓ product ✓ answer  (2)
		[14]

TOTAL: 150