education

Department:
Education
PROVINCE OF KWAZULU-NATAL

MATHEMATICS P1

SEPTEMBER 2020

PREPARATORY EXAMINATION

MARKING GUIDELINE

NATIONAL SENIOR CERTIFICATE

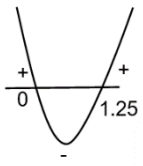
GRADE 12

MARKS: 150

TIME: 3 hours

This marking guideline consists of 13 pages.

QUESTION 1

1.1.1	$x = 0$	A✓✓ 0	(2)
1.1.2	$-3x^2 + 8x + 7 = 0$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $x = \frac{-(8) \pm \sqrt{(8)^2 - 4(-3)(7)}}{2(-3)}$ $x = -0{,}69 \quad or \quad 3{,}36$	A✓standard form CA✓substitution in correct formula CA✓CA✓answers (penalize 1 mark if rounding off is incorrect-once here for entire paper)	(4)
1.1.3	$\sqrt{11 - x} - x = 1$ $\sqrt{11 - x} = x + 1$ $\left(\sqrt{11 - x}\right)^2 = (x + 1)^2$ $11 - x = x^2 + 2x + 1$ $x^2 + 3x - 10 = 0$ $(x + 5)(x - 2) = 0$ $x = -5 \; or \; x = 2$ $x = -5$ *n/a*	A✓Isolating surd A✓ squaring both sides CA✓standard form CA✓factors CA✓answers and rejecting	(5)
1.1.4	$2.3^x = 57 - 3^{x-2}$ $2.3^x + 3^{x-2} = 57$ $3^x(2 + 3^{-2}) = 57$ $3^x\left(2\frac{1}{9}\right) = 57$ $3^x = 27 = 3^3$ $x = 3$	A✓Removing common factor CA✓Simplifying bracket CA✓$3^x = 27$ CA✓answer	(4)
1.1.5	$4x^2 - 5x \leq 0$ $x(4x - 5) \leq 0$  $0 \leq x \leq \frac{5}{4}$	AA✓✓ factors CA✓ end points A✓ interval	(4) (4)

1.2	$2x + y = 3 \quad \rightarrow (1)$ $y^2 = x^2 + y + x \quad \rightarrow (2)$ From (1): $y = 3 - 2x \quad \rightarrow (3)$ Substituting (3) into (2): $(3 - 2x)^2 = x^2 + (3 - 2x) + x$ $9 - 12x + 4x^2 = x^2 + 3 - 2x + x$ $3x^2 - 11x + 6 = 0$ $(3x - 2)(x - 3) = 0$ $x = \frac{2}{3} \quad or \quad x = 3$ $y = \frac{5}{3} \quad or \quad y = -3$	A✓making y/*x* the subject CA✓substitution CA✓standard form CA✓*x*– values CA✓ *y* values	(5)
			[24]

QUESTION 2

2.1	$T_k = 6k + 96 = 2022$ $k = 321$ Between the 321st and 322nd terms	A✓ equating k^{th} term to 2022 CA✓*k* value CA✓answer	(3)
2.2	**T1 T2 T3 T4** **1D 102 108 114** **2D 6 6** $2a = 6 \quad a = 3$ $3a + b = 102 \quad b = 93$ $9a + 3b + c = 310 \quad c = 4$ $T_n = 3n^2 + 93n + 4$ **OR** $2a = 6 \quad a = 3$ $3a + b = 102 \quad b = 93$ $T_2 = 310 - 108 = 202$ $T_1 = 202 - 102 = 100$ $a + b + c = 100 \quad c = 4$ $T_n = 3n^2 + 93n + 4$	A✓*a* value CA✓*b* value CA✓*c* value CA✓answer **OR** A✓*a* value CA✓*b* value CA✓*c* value CA✓answer	(4) (4)

	OR $2a = 6 \qquad a = 3$ $3a + b = 102 \qquad b = 93$ $T_2 = 310 - 108 = 202$ $T_1 = 202 - 102 = 100$ $T_n = T_1 + (n-1)d_1 + \frac{(n-1)(n-2)}{2}d_2$ $T_n = 100 + (n-1)(102) + \frac{(n-1)(n-2)}{2}(6)$ $T_n = 100 + 102n - 102 + 3n^2 - 9n + 6$ $T_n = 3n^2 + 93n + 4$	**OR** A✓a – value A✓b – value CA✓formula CA✓answer	(4)
	OR $2a = 6 \qquad a = 3$ $3a + b = 102 \qquad b = 93$ $T_2 = 310 - 108 = 202$ $T_1 = 202 - 102 = 100$ $T_n = \frac{n-1}{2}[2a + (n-2)d] + T_1$ $T_n = \frac{n-1}{2}[2(102) + (n-2)(6)] + 100$ $T_n = \frac{n-1}{2}[204 + 6n - 12] + 100$ $T_n = \frac{n-1}{2}[6n + 192] + 100$ $T_n = (n-1)[3n + 96] + 100$ $T_n = 3n^2 + 93n + 4$	**OR** A✓a – value A✓b – value CA✓formula CA✓answer	(4)
			[7]

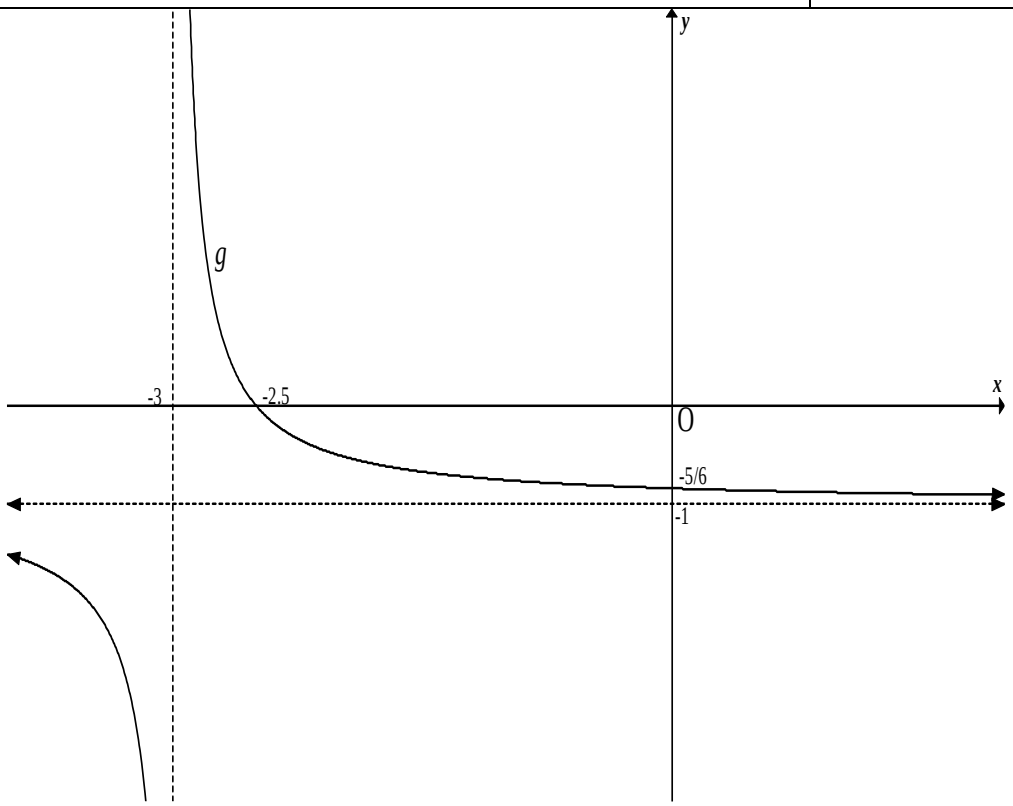
QUESTION 3

3.1.1	$T_n = \frac{1}{5}n + \frac{1}{5}$	A✓ common difference CA✓answer	(2)
3.1.2	$S_n = \frac{n}{2}[2a + (n-1)d]$ $S_{30} = \frac{30}{2}\left[2\left(\frac{2}{5}\right) + (30-1)\left(\frac{1}{5}\right)\right]$ $= 99$	CA✓$a = \frac{2}{5}$ and $d = \frac{1}{5}$ CA✓substitution into formula CA✓answer	(3)
3.2	$S_n = \frac{n}{2}[2a + (n-1)d]$ $72710 = \frac{n}{2}[2(2) + (n-1)(3)]$ $72710 = n/2\,[3n + 1)]$ $3n^2 + n - 145420 = 0$ $n = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $n = \frac{-1 \pm \sqrt{(1)^2 - 4(3)(-145420)}}{2(3)}$ $n = 220 \quad or \quad -220.33$ *n/a*	A✓a – value and d – value CA ✓simplifying bracket CA ✓standard form CA ✓substitution into formula CA ✓answer and rejecting	(5)
			[10]

QUESTION 4

4.1	$a\,;ar\,;ar^2\,;ar^3\,;\ldots$ $a + ar = 4(ar^2 + ar^3)$ $a(1+r) = 4ar^2(1+r)$ $a = 4ar^2 \qquad r \neq -1$ $r^2 = \frac{1}{4} \quad ; \quad a > 0$ $r = \frac{1}{2} \quad ; \quad a > 0$	A✓ $a + ar$ A✓ $4(ar^2 + ar^3)$ A✓factorising A✓simplifying	(4)
4.2	$S_\infty = \frac{a}{1-r} = 3$ $S_\infty = \frac{a}{1-\frac{1}{2}} = 3$ $a = \frac{3}{2}$	A✓equating sum to infinity to 3 CA✓answer	(2)
			[6]

QUESTION 5

5.1	$x = -3 \; and \; y = -1$	A✓$x = -3$ A✓$y = -1$	(2)
5.2	$y-$ intercept $: \left(0 \; ; -\frac{5}{6}\right)$ $x-$ intercept: $\frac{1}{2(x+3)} - 1 = 0$ $2x + 6 = 1$ $x = -2\frac{1}{2}$ $\left(-2\frac{1}{2} \; ; \; 0\right)$	A✓y – intercept A✓$2x + 6 = 1 \;\; or$ A✓$x + 3 = \frac{1}{2}$ CA✓x – intercept (co-ordinate form not needed)	(3)
5.3		CA✓x -intercepts CA✓y - intercept CA✓both asymptotes A✓shape	(4)
			[9]

QUESTION 6

6.1	$A(0;2)$	A✓answer (Must be in coordinate form)	(1)
6.2	$-x^2+x+2=\frac{1}{2}x^2-x$ $0=\frac{3}{2}x^2-2x-2$ $0=3x^2-4x-4$ $(3x+2)(x-2)=0$ $x=-\frac{2}{3}$ or $x=2$ $y=\frac{8}{9}$ or $y=0$ $C\left(-\frac{2}{3};\frac{8}{9}\right)$ & $D(2;0)$	A✓equating CA✓standard form CA✓x – values CA✓y – values CA✓Writing in coordinate form	(5)
6.3	$x\leq-\frac{2}{3}$ or $x\geq 2$	CA✓ $x\leq-\frac{2}{3}$ CA✓ $x\geq 2$	(2)
6.4	Length of PQ $=-x^2+x+2-\left(\frac{1}{2}x^2-x\right)$ $L=-\frac{3}{2}x^2+2x+2$ $x=-\frac{b}{2a}=-\frac{2}{2\left(\frac{-3}{2}\right)}=\frac{2}{3}$ OR $L'=-3x+2=0\ \therefore x=\frac{2}{3}$ Maximum value of PQ $=-\frac{3}{2}\left(\frac{2}{3}\right)^2+2\left(\frac{2}{3}\right)+2$ $=\frac{8}{3}=2\frac{2}{3}=2{,}67$ units	A✓subtraction of both graphs CA✓equating in standard form CA✓Axis of symmetry value or CA✓Axis of symmetry value CA✓AO value	(4)
6.5	$f(x)=-x^2+x+2$ $f'(x)=-2x+1=3$ $x=-1$	AA✓✓derivative and equating to 3 A✓answer	(3)
6.6	Axis of symmetry: $x=\frac{1}{2}$ Maximum value: $y=-\left(\frac{1}{2}\right)^2+\frac{1}{2}+2=2\frac{1}{4}$ $2<k<2\frac{1}{4}$	A✓ Axis of symmetry value CA✓ maximum value of f CA✓end points A✓interval	(4)
			[19]

QUESTION 7

7.1	Inverse: $x = \log_3 y$ $y = 3^x$	A✓ $x = \log_3 y$ A✓ $y = 3^x$ Answer only full marks	(2)
7.2	$y > 0$ or $y \in (0\,;\,\infty)$	A✓answer	(1)
7.3	$2log_3 x = -6$ $log_3 x = -3$ $x = 3^{-3} = \frac{1}{27}$ $0 < x \leq \frac{1}{27}$	A✓dividing by 2 A✓writing in exponential form CA✓end points A✓ interval **Can be solved by log inequalities**.	(4)
			[7]

QUESTION 8

8.1	$A = P(1+i)^n$ $3P = P\left(1+\frac{i}{12}\right)^{72}$ $i = 12\left(\sqrt[72]{3}-1\right)$ $i = 0{,}1845$ Annual interest rate is 18,45 % p.a.	A✓for using 3P and P A✓n =72 CA✓making i the subject CA✓ answer	(4)
8.2.1	$P = \frac{x[1-(1+i)^{-n}]}{i}$ $192\,000 = \frac{x\left[1-\left(1+\frac{0.12}{12}\right)^{-60}\right]}{\frac{0.12}{12}}$ $x = R4270{,}93$	A✓value of n A✓value of i CA✓substitution into correct formula CA✓answer	(4)
8.2.2	$P = \frac{x[1-(1+i)^{-n}]}{i}$ $= \frac{4270{,}93\left[1-\left(1+\frac{0.12}{12}\right)^{-15}\right]}{\frac{0.12}{12}}$ $= R59216{,}72421$	A✓Present value formula A✓value of n CA✓substitution into correct formula CA✓answer	

	OR $A = P(1+i)^n$ $A = 192\,000\left(1+\frac{0.12}{12}\right)^{45}$ =R300 443,6635 $F = \frac{x[(1+i)^n - 1]}{i}$ $F = \frac{4270{,}934\left[\left(1+\frac{0.12}{12}\right)^{45} - 1\right]}{\frac{0.12}{12}}$ =R241 226,9424 Balance on Loan = R300 443,6635 −R241 226,9424 = R59216,7211	**OR** A✓Substitution into Compound Interest Formula CA✓substitution into Future Value Formula CA✓*A – F* CA✓answer	(4) (4)
			[15]

QUESTION 9*(penalize 1 mark once for incorrect notation in this question)*

9.1	$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ $f'(x) = \lim_{h\to 0}\frac{(x+h)^2 - b(x+h) - (x^2 - bx)}{h}$ $f'(x) = \lim_{h\to 0}\frac{x^2 + 2xh + h^2 - bx - bh - x^2 + bx}{h}$ $f'(x) = \lim_{h\to 0}\frac{2xh + h^2 - bh}{h}$ $f'(x) = \lim_{h\to 0}\frac{h(2x + h - b)}{h}$ $f'(x) = 2x - b$ **OR** $f(x+h) = (x+h)^2 - b(x+h)$ $f(x+h) = x^2 + 2xh + h^2 - bx - bh$ $f(x+h) - f(x) = 2xh + h^2 - bh$ $\frac{f(x+h)-f(x)}{h} = \frac{2xh + h^2 - bh}{h}$ $\frac{f(x+h)-f(x)}{h} = \frac{h(2x + h - b)}{h}$	A✓formula A✓substitution CA✓ simplification of numerator CA✓factorization CA✓answer **OR** A✓value of *f(x+h)* CA✓ simplification CA✓factorization	(5)

	$f'(x) = \lim_{h\to 0}(2x + h - b)$ $f'(x) = 2x - b$	A✓formula CA✓answer	(5)
9.2.1	$\frac{d}{dx}\left[\frac{x^4}{4} - 3\sqrt[3]{x} + 7\right]$ $\frac{d}{dx}\left[\frac{x^4}{4} - 3x^{\frac{1}{3}} + 7\right]$ $= x^3 - x^{-\frac{2}{3}}$	A✓rewriting in exponential form CA✓CA✓derivatives	(3)
9.2.2	$y = (x^{\frac{1}{3}} - 2x^{\frac{2}{3}})^2$ $y = x^{\frac{2}{3}} - 4x + 4x^{\frac{4}{3}}$ $\frac{dy}{dx} = \frac{2}{3}x^{-\frac{1}{3}} - 4 + \frac{16}{3}x^{\frac{1}{3}}$	A✓ simplification CA✓CA✓ CA✓each term	(4)
			[12]

QUESTION 10

10.1			
	10.1.1 $x = -5$ or $x = 1$	A✓A✓	(2)
	10.1.2 $x = -4$ or $x = 0$	A✓A✓	(2)
10.2	$y = a(x + 5)(x - 1)$ $-15 = a(0 + 5)(0 - 1)$ $-15 = -5a$ $3 = a$ $y = 3(x + 5)(x - 1) = 3x^2 + 12x - 15$ **OR** $y = a(x + p)^2 + q$ $y = a(x + 2)^2 + q$ $(0\ ;\ -15)$: $-15 = a(0 + 2)^2 + q$ $-15 = 4a + q$ .... (1) $(1\ ;\ 0)$: $0 = a(1 + 2)^2 + q$ $0 = 9a + q$ .... (2) (2) – (1) : $15 = 5a \quad \therefore \quad a = 3$ $0 = 27 + q \quad \therefore \quad q = -27$ $y = 3(x + 2)^2 - 27$ $y = 3(x^2 + 4x + 4) - 27$ $y = 3x^2 + 12x - 15$	A✓substitution of intercepts A✓ a – value A✓substitution of a – value and intercepts into equation **OR** A✓equation (1) and (2) A✓a – value A✓q – value	(3) (3)
10.3	$f(x) = ax^3 + bx^2 + cx + d$ $f'(x) = 3ax^2 + 2bx + c$ $f'(x) = 3x^2 + 12x - 15$ Equating coefficients of equal polynomials $3a = 3 \ and \ 2b = 12 \ and \ c = -15$ $a = 1 \ and \ b = 6 \ and \ c = -15$ $f(x) = x^3 + 6x^2 - 15x + d$	A✓ derivative A✓values of a,b and c	(4)

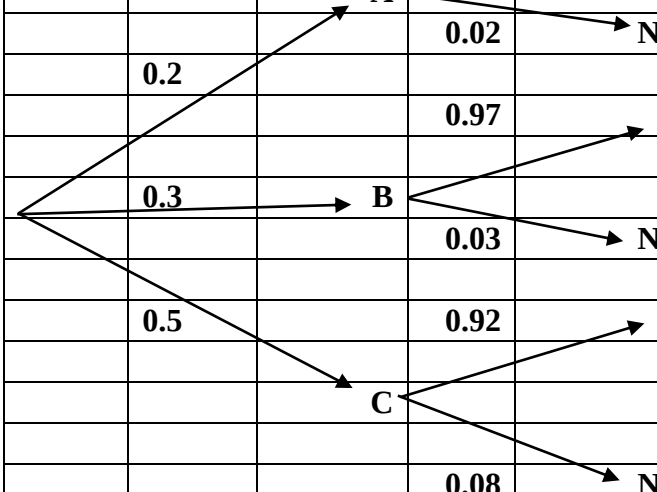
	$f(-3) = (-3)^3 + 6(-3)^2 - 15(-3) + d$ $f(-3) = -27 + 54 + 45 + d$ $0 = 72 + d$ $-72 = d$ **OR** $f(x) = x^3 + 6x^2 - 15x + d$ $f(-3) = (-3)^3 + 6(-3)^2 - 15(-3) + d$ $f(-3) = -27 + 54 + 45 + d$ $0 = 72 + d$ $-72 = d$	A✓substitution of $x = -3$ A✓d – value A✓ anti derivative A✓substitution A✓simplification A✓d – value	(4)
10.4	$x = -5:$ $y = (-5)^3 + 6(-5)^2 - 15(-5) - 72 = 28$ $(-5\,;28)$ maximum point $x = 1:$ $y = (1)^3 + 12(1)^2 - 15(1) - 72 = -74$ $(1\,;-74)$ minimum point	CA✓y – value CA✓maximum point CA✓y – value CA✓ minimum point	(4)
10.5	$3x^2 + 12x - 15 = t$ $3x^2 + 12x - 15 - t = 0$ $\Delta = b^2 - 4ac = 0$ $\Delta = (12)^2 - 4(3)(-15 - t) = 0$ $144 + 180 + 12t = 0$ $12t = -324$ $t = -27$	A✓equating derivative to gradient of tangent A✓standard form A✓discriminant = 0 A✓substitution A✓t – value	(5)
			[20]

QUESTION 11

11.1	$Area = x^2 + 3x^2 + 4xh$ $Area = 4x^2 + 4xh$ $V = x^2h = 1000$ $h = \frac{1000}{x^2}$ $A = 4x^2 + 4x\left(\frac{1000}{x^2}\right)$ $A = 4x^2 + \frac{4000}{x}$	A ✓Total Surface Area A✓$h = \frac{1000}{x^2}$ A✓Substitution for h	(3)

11.2	$A = 4x^2 + 4000x^{-1}$ $A^{/} = 8x - 4000x^{-2} = 0$ $8x = \frac{4000}{x^2}$ $x^3 = 500$ $x = \sqrt[3]{500} = 7{,}94\ cm$ $h = \frac{1000}{(7{,}94)^2} = 15{,}86\ cm$ Alternatively if learners used: $A = 4x^2 + 400x^{-1}$ $A^{/} = 8x - 400x^{-2} = 0$ $8x = \frac{400}{x^2}$ $x^3 = 50$ $x = \sqrt[3]{50} = 3{,}68\ cm$ $h = \frac{1000}{(3{,}68)^2} = 73{,}84\ cm$	CA✓derivative CA✓derivative and equal to 0 CA✓$x^3 = 500$ CA✓x – value CA✓substitution of x CA✓h – value CA✓derivative CA✓derivative and equal to 0 CA✓$x^3 = 50$ CA✓x – value CA✓substitution of x CA✓h – value	(6)
			[9]

QUESTION 12



					OUTCOMES	PROBABILITIES
			0.98	**P**	**AP**	**49/250 = 0.196**
		A				
			0.02	**NP**	**ANP**	**1/250 = 0.004**
	0.2					
			0.97	**P**	**BP**	**291/1000 = 0.291**
	0.3	**B**				
			0.03	**NP**	**BNP**	**9/1000 = 0.009**
	0.5		**0.92**	**P**	**CP**	**23/50 = 0.46**
		C				
			0.08	**NP**	**CNP**	**1/25 = 0.04**
A ✓			**A ✓**		**A ✓**	**A ✓** (4)

12.2.1	$\frac{9}{1000} = 0{,}009$	A✓Answer	(1)
12.2.2	P(NP) = P(ANP) + P(BNP) + P(CNP) = 0,004 + 0,009 + 0,04 = $0{,}053 = \frac{53}{1000} = 5{,}3\ \%$	A✓ formula CA✓substitution CA✓answer	(3)
			[8]

QUESTION 13

	Like ice-cream (L)	Do not like ice-cream(D)	Total
Boys (B)	65	30	**95**
Girls (G)	70	55	**125**
Total	**135**	**85**	**220**

13.1	68,42 %	AA✓✓answer	(2)
13.2	$P(BL) = \frac{65}{220} = \frac{13}{44} = 0{,}2955$	A A ✓✓ $\frac{65}{220}$ or $\frac{13}{44} or\ 0{,}2955$	(2)
13.3	$P(B) = \frac{95}{220} = \frac{19}{44} = 0{,}4318$ $P(B) \times P(L) = \frac{95}{220} \times \frac{135}{220}$ $= \frac{513}{1936} = 0{,}2650$ $P(B) \times P(L) \neq P(BL)$ Events are not independent.	CA✓$\frac{95}{220} or\ \frac{19}{44}\ or\ 0{,}4318$ CA✓probability of product CA✓conclusion	(3)
			[7]

Total: 150