



LIMPOPO
PROVINCIAL GOVERNMENT
REPUBLIC OF SOUTH AFRICA

DEPARTMENT OF
EDUCATION

**NATIONAL
SENIOR CERTIFICATE**

GRADE/GRAAD 12

**MATHEMATICS PAPER 1/ WISKUNDE V1
SEPTEMBER 2020
MEMORANDUM**

MARKS: 150

This memorandum consists of 16 pages/Hierdie memorandum bestaan uit 16 bladsye

QUESTION 1		
1.1.1	$x = 2$ or $x = -1$	✓ $x = 2$ ✓ $x = -1$ (2)
1.1.2	$x^2 - x + 5 = 0$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $= \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(5)}}{2(1)}$ $= \frac{1 \pm \sqrt{-19}}{2}$ No real solution	✓ standard form ✓ correct substitution in correct formula $\sqrt{-19}$ ✓ simplification /vereenvoudiging ✓ answer /antwoord (4)
1.1.3	$(3\sqrt{x-2})^2 = (x-6)^2$ $9(x-2) = x^2 - 12x + 36$ $0 = x^2 - 21x + 54$ $0 = (x-18)(x-3)$ $x = 18 \text{ or } x = 3$ $\therefore x = 18, x \neq 3$	✓ $9(x-2) = x^2 - 12x + 36$ ✓ standard form = 0 ✓ factorisation ✓ $x = 18$ or $x = 3$ ✓ selection $x = 18$ (5)
1.1.4	$-3x^2 + 5x \geq -2$ $-3x^2 + 5x + 2 \geq 0 \Rightarrow 3x^2 - 5x - 2 \leq 0$ $(3x+1)(x-2) \leq 0$ $-\frac{1}{3} \leq x \leq 2$	✓ standard form/standaard vorm ✓ factors/faktore ✓ ✓ $-\frac{1}{3} \leq x \leq 2$ (4)

1.2	$x^2 + p = (p+1)x \Rightarrow x^2 - (p+1)x + p = 0$ $\Delta = b^2 - 4ac$ $= [-(p+1)]^2 - 4p$ $= p^2 + 2p + 1 - 4p$ $= p^2 - 2p + 1$ $= (p-1)^2$ <p>$\therefore \Delta$ a perfect square/volkome vierkant roots are rational/wortels is rasonaal</p>	<p>✓ substitution</p> $[-(p+1)]^2 - 4p$ <p>✓ $p^2 - 2p + 1$</p> <p>✓ $(p-1)^2$</p> <p>✓ explanation/verduideliking (4)</p>
1.3	$\frac{1}{x} + \frac{1}{y} = 3$ <p>$\therefore y + x = 3xy$-----equation 1 (eq 1)</p> $x = y + \frac{1}{2}$ -----equation 2 (eq 2) <p>eq 2 in eq 1 :</p> $y + y + \frac{1}{2} = 3y\left(y + \frac{1}{2}\right)$ $2y + \frac{1}{2} = 3y^2 + \frac{3}{2}y$ $4y + 1 = 6y^2 + 3y$ $(\times 2) \quad 6y^2 - y - 1 = 0$ $(3y+1)(2y-1) = 0$ $y = \frac{-1}{3} \text{ or } y = \frac{1}{2}$ <p>Substitute in eq 2:</p>	<p>✓ $\therefore y + x = 3xy$-</p> <p>✓ substitute eq 2 in eq 1</p> <p>✓ simplification/vereenvoudiging</p> $2y + \frac{1}{2} = 3y^2 + \frac{3}{2}y$ <p>✓ standard form =0</p> <p>✓ $y = \frac{-1}{3}$ or $y = \frac{1}{2}$</p>

$x = \left(\frac{-1}{3}\right) + \frac{1}{2} \quad \text{or} \quad x = \left(\frac{1}{2}\right) + \frac{1}{2}$ $\therefore x = \frac{1}{6} \quad \quad \quad x = 1$ <p style="text-align: center;">ALTERNATIVE:</p> $\therefore y + x = 3xy \text{-----equation 1 (eq 1)}$ $y = x - \frac{1}{2} \text{-----eq 2}$ $x - \frac{1}{2} + x = 3x\left(x - \frac{1}{2}\right) \text{-----eq 2}$ $2x - \frac{1}{2} = 3x^2 - \frac{3}{2}x$ $(\times 2) \quad 4x - 1 = 6x^2 - 3x$ $6x^2 - 7x + 1 = 0$ $(6x - 1)(x - 1) = 0$ $x = \frac{1}{6} \text{ or/of } x = 1$ <p>Substitute in eq 1:</p> $y = 3y\left(\frac{1}{6}\right) - \frac{1}{6} \quad \text{or/of} \quad y = 3y(1) - 1$ $y = \frac{-1}{3} \text{ or } y = \frac{1}{2}$	$\checkmark x = \frac{1}{6} \text{ or } x = 1$ <p style="text-align: right;">Both x-values (6)</p> $\checkmark \therefore y + x = 3xy \text{-----}$ <p>\checkmark substitute eq 2 in eq 1</p> $x - \frac{1}{2} + x = 3x\left(x - \frac{1}{2}\right) \text{-----}$ <p>\checkmark simplification</p> <p>\checkmark standard form</p> <p>x-values</p> $\checkmark x = \frac{1}{6} \text{ or/of } x = 1$ <p>\checkmark both y-values</p> $y = 1 \text{ or } y = \frac{1}{2} \quad \quad \quad (6)$
	[25]

QUESTION 2/VRAAG 2		
<p>2.1</p>	$ \begin{array}{cccc} -6 & 2-x & 4 & 2x+5 \\ \swarrow & \swarrow & \swarrow & \\ 8-x & 2+x & 2x+1 & \text{first difference} \\ \swarrow & \swarrow & & \\ 2x-6 & 2x-1 & & \text{second difference} \\ 2x-6 = 2x-1 & & & \\ x=5 & & & \end{array} $	$ \begin{array}{l} 8-x \quad 2+x \quad 2x+1 \\ \checkmark \quad \text{first differences} \\ \\ \checkmark \text{equating second} \\ \text{differences} \\ \checkmark x\text{-value} \qquad (3) \end{array} $
<p>2.2</p>	<p>Row/ry: $-6; -3; 4; 15$</p> $T_n = an^2 + bn + c$ $2a = 4$ $a = 2$ $T_n = n^2 + bn + c$ $3a + b = T_2 - T_1$ $3(2) + b = 6$ $b = -3$ $T_1 = a + b + c$ $-6 = 2 - 3 + c$ $-5 = c$ $\therefore T_n = 2n^2 - 3n - 5$	$\checkmark a = 2$ $\checkmark b = -3$ $\checkmark c = -5$ $\checkmark \therefore T_n = 2n^2 - 3n - 5 \quad (4)$
		<p>[7]</p>

QUESTION 3/VRAAG 3		
3.1.1	$T_3 = \frac{a^2}{-2}$	✓✓ $\frac{a^2}{-2}$ (2)
3.1.2	$T_3 = 2a + 2$	✓✓ $2a + 2$ (2)
3.2	$S_3 = \frac{a^2}{-2}$ $\frac{3}{2}(2(-2) + (3-1)(a+2)) = \frac{a^2}{-2}$ $3(2a) = -a^2$ $a^2 + 6a = 0$ $a(a+6) = 0$ $a = 0 \text{ of } a = -6$	✓✓ equating ✓ $a^2 + 6a = 0$ ✓ factors ✓ $a = 0$ or/of $a = -6$ (5)
3.3	$r = \frac{a}{-2}$ $r = \frac{-6}{-2} = 3$ $\therefore \text{Nee / No } r > 1$	✓ $\frac{a}{-2}$ ✓ substitution $r = 3$ ✓ no ; $r > 0$ (3)
		[12]

QUESTION 4		
4.1	$\sum_{p=1}^{\infty} (-5+2)^p$ $= \sum_{p=1}^{\infty} (-3)^p$ $= (-3) + (9) + (-27) + \dots$ <p>GS/MR $a = -3$ $r = -3$</p> <p>DOES NOT EXIST/BESTAAN NIE</p> <p>For convergence/vir konvergensie $-1 < r < 1$</p>	✓ $(-3) + (9) + (-27) + \dots$ ✓ DOES NOT EXIST/BESTAAN NIE (2)

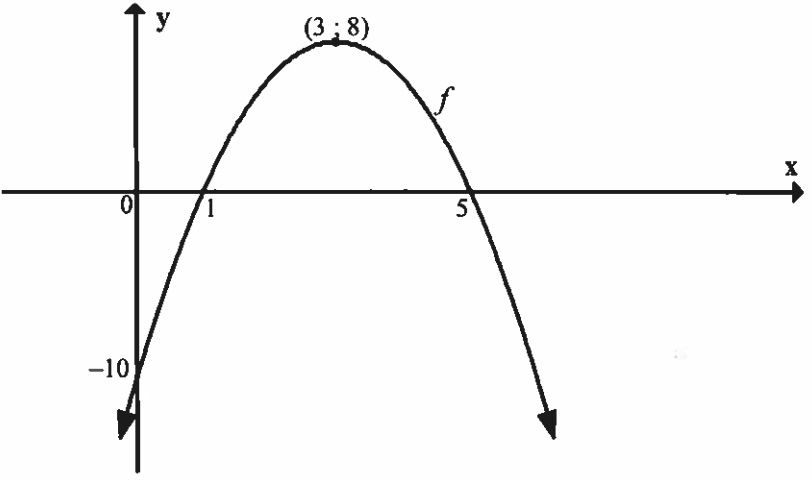
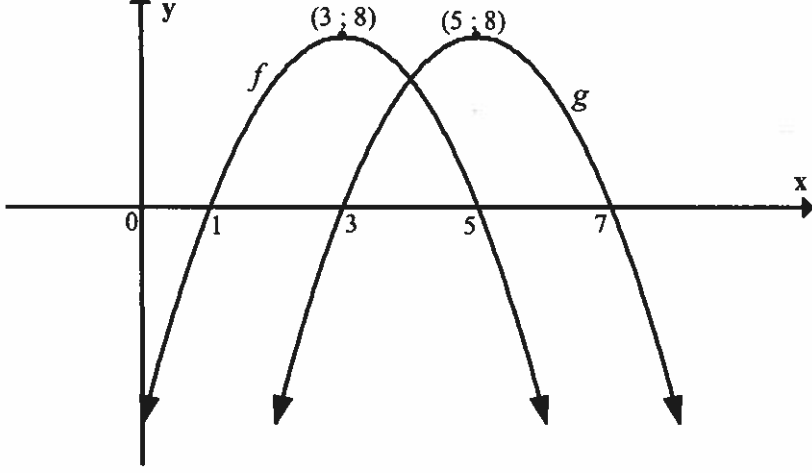
<p>4.2</p>	<p>AS/RR: $a = 1 \quad l = 2000 \quad n = 2000$</p> $S_n = \frac{n}{2}(a+l)$ $S_{2000} = \frac{2000}{2}(1+2000) = 2\,001\,000$ <p>Powers of 2 / magte van 2:</p> $2^0; 2^1; 2^2; \dots; 2^{10}$ <p>[$2^{11} = 2048 > 2000$]</p> <p>GS/MR: $a = 1; r = 2; n = 11$</p> $S_n = \frac{a(r^n - 1)}{r - 1}$ $\therefore S_{11} = \frac{1(2^{11} - 1)}{2 - 1} = 2^{11} - 1 = 2047$ <p>sum of remaining integers/som van oorblywende heelgetalle</p> $2\,001\,000 - 2047 = 1\,998\,953$	$\checkmark S_{2000} = \frac{2000}{2}(1+2000)$ <p>\checkmark answer 2 001 000</p> $\checkmark 2^0; 2^1; 2^2; \dots; 2^{10}$ $\checkmark S_{11} = \frac{1(2^{11} - 1)}{2 - 1}$ <p>Substitution in correct formula</p> <p>\checkmark 1 998 953</p> <p style="text-align: right;">(5)</p>
		[7]

QUESTION 5		
5.1	$y \in R; y \neq -1$	$\checkmark\checkmark$ answer (2)
5.2	$y = \frac{2}{x+p} - 1$ <p>OB = OE</p> <p>$\therefore E(2; 0)$</p> $\therefore y = \frac{2}{x-2} - 1$	<p>\checkmark substitution</p> <p>\checkmark substitute $E(2;0)$ (2)</p>

5.3	$x\text{-intercept } \frac{2}{x-2} - 1 = 0$ $2 = x - 2$ $x = 4$ $y\text{-intercept } y = \frac{2}{-2} - 1$ $= -2$	\checkmark $x\text{-intercept}$ \checkmark $y\text{-intercept}$	(2)
5.4	$f : y = \log_3 x$ $A(t;1)$ in $y = \log_3 x$ $1 = \log_3 t$ $\therefore 3^1 = t$ $A(3;1)$ Substitute in $1 = \frac{2}{3+p} - 1$ $2(3+p) = 2$ $p = -2$ $\therefore y = \frac{2}{x-2} - 1$	$\checkmark\checkmark$ substitution of $A(t;1)$ in correct formula \checkmark	(3)
5.5	$f^{-1} x = \log_3 x$ $y = 3^x$	$\checkmark\checkmark$	(2)
5.6	$0 < x < 1$	$\checkmark\checkmark$ answer	(2)
5.7	$-x + 1 = \log_3 x$ $0 = \log_3 x + (x - 1)$ $\therefore x = 1$	$\checkmark\checkmark$ $-x + 1 = \log_3 x$ \checkmark $x = 1$	(3)
			[16]

QUESTION 6/VRAAG 6

6.1	(3;8)	\checkmark $x = 3$ \checkmark $y = 8$ (2)
6.2	$y = -2(x-3)^2 + 8 = -2x^2 + 12x - 10$ $x\text{-intercepts:}$ $\therefore x^2 - 6x + 5 = 0$ $(x-5)(x-1) = 0$	

	<p>$x = 5$ of/or $x = 1$</p> 	<p>✓ (shape)form /vorm ✓ y- intercept ✓ x - afsnitte/ intercepts ✓ turning point/draaipunt (4)</p>
<p>6.3</p>	<p>$x < 0$ or/of $1 < x < 5$</p>	<p>✓ $x < 0$ ✓ $1 < x < 5$ (2)</p>
<p>6.4</p>	<p>$f(-1) = -2(-1)^2 + 12(-1) - 10 = -23$ $f(0) = -2(0)^2 + 12(0) - 10 = -10$ $m = \frac{f(-1) - f(0)}{-1 - 0}$ $m = \frac{-23 - (-10)}{-1 - 0} = 13$</p>	<p>✓ $f(-1) = -23$ ✓ $f(0) = -10$ ✓ $m = 13$ (3)</p>
<p>6.5</p>		<p>✓ graph/grafiek ✓ turning point/draaipunt (5;8) ✓✓ x-intercepts (4)</p>
		<p>[15]</p>
<p>QUESTION 7</p>		

<p>7.1.</p>	$A = P(1 - ni)$ $1000 = 11\,500(1 - 3i)$ $3i = 1 - \frac{1000}{11500}$ $\therefore i = 30,4\%$	<p>✓ correct substitution into correct formula</p> <p>✓ simplification</p> <p>✓ 30,4 %</p> <p>(3)</p>
<p>7.2.1</p>	$P = \frac{x[1 - (1+i)^{-n}]}{i}$ $950\,000 = \frac{x \left[1 - \left(1 + \frac{0,10}{12} \right)^{-108} \right]}{\frac{0,10}{12}}$ $7196,666\dots = 0,591911\dots x$ $\therefore x = R13374,76 \text{ is the monthly instalment}$	<p>✓ - 108</p> <p>✓ $\frac{0,10}{12}$</p> <p>✓ simplification</p> <p>✓ instalment</p> <p>(4)</p>
<p>7.2.2</p>	$P = \frac{x[1 - (1+i)^{-n}]}{i}$ $P = \frac{13\,374,76 \left[1 - \left(1 + \frac{0,10}{12} \right)^{-12} \right]}{\frac{0,10}{12}}$ $= R152\,131,32 \text{ is the outstanding balance}$	<p>✓ - 12</p> <p>✓ R 13 374,76</p> <p>✓ balance</p> <p>(3)</p>
<p>7.2.3</p>	$R152\,131,32 \left(1 + \frac{0,1}{12} \right)$ $= R15\,3399,08$ $\therefore \text{interest} = 15\,3399,08 - 152\,131,32$ $= R1\,267,76$ $\text{Percentage} = \frac{1\,267,76}{13\,374,76} \times 100$ $= 9,48\%$	<p>✓ R 152 131,32</p> <p>✓ R 15 3399,08</p> <p>✓ Interest</p> <p>✓ 13 374,76</p> <p>✓ 9,48 %</p> <p>(5)</p>
		<p>[15]</p>

QUESTION 8/VRAAG 8		
<p>8.1</p>	$f(x) = -x^2 - 3x$ $f'(x) = \frac{f(x+h) - f(x)}{h}$ $= \lim_{h \rightarrow 0} \frac{-(x+h)^2 - 3(x+h) - (-x^2 - 3x)}{h}$ $= \lim_{h \rightarrow 0} \frac{-x^2 - 2hx - h^2 - 3x - 3h + x^2 + 3x}{h}$ $= \lim_{h \rightarrow 0} \frac{-2hx - h^2 - 3h}{h}$ $= \lim_{h \rightarrow 0} \frac{h(-2x - h - 3)}{h}$ $= \lim_{h \rightarrow 0} (-2x - h - 3)$ $= -2x - 3$ <p>ANSWER ONLY: 0 marks SLEGS antwoord: 0 punte</p>	<p>✓ simplifying of $f(x + h)$</p> <p>✓ simplifying</p> $\frac{-2hx - h^2 - 3h}{h}$ <p>✓ common factor</p> <p>✓ answer (4)</p>
<p>8.2</p>	$D_x \left[x + \frac{1}{x} \right]^2$ $= D_x \left[x^2 + 2 + \frac{1}{x^2} \right]$ $= D_x \left[x^2 + 2 + x^{-2} \right]$ $= 2x - 2x^{-3}$ $= 2x - \frac{2}{x^3}$	<p>✓ $x^2 + 2 + \frac{1}{x^2}$</p> <p>✓ x^{-2}</p> <p>✓ $2x$</p> <p>✓ $-2x^{-3}$ (4)</p>
<p>8.3</p>	$y = -3x^2 - \frac{3}{\sqrt[3]{x}} + 3^{-1}$ $y = -3x^2 - 3x^{-1/3} + 3^{-1}$ $\frac{dy}{dx} = -6x + x^{-4/3}$	<p>✓ $-3x^{-1/3}$</p> <p>✓ $-6x$</p> <p>✓ $x^{-4/3}$ (4)</p>

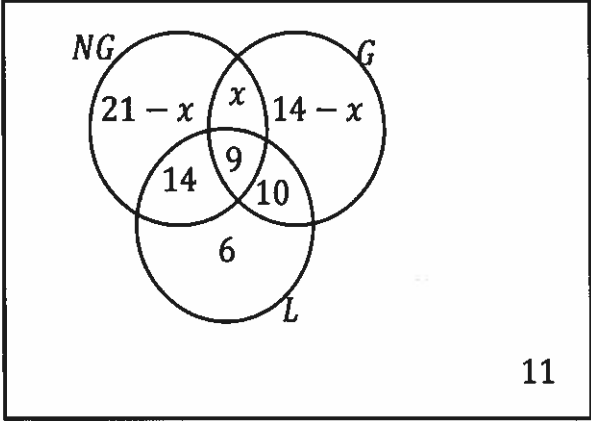
8.4	$D_x = -4x - 1$ <p>Perpendicular: $m_{\text{tangent}} \times m_{\text{straight line}} = -1$</p> $\therefore m_{\text{tangent}} = \frac{1}{2}$ $-4x - 1 = \frac{1}{2}$ $x = \frac{3}{8}$	$\checkmark D_x = -4x - 1$ $\checkmark m_{\text{tangent}} = \frac{1}{2}$ $\checkmark -4x - 1 = \frac{1}{2}$ $\checkmark x = \frac{3}{8} \quad (4)$
8.5	$p(x) = \frac{f(x)}{g(x)} = \frac{2(x-3)(x^2 + 2x + 4)}{(x^2 + 2x + 4)}$ $= 2(x-3)$ $= 2x - 6$ $p'(x) = 2$ $\therefore p'(3) = 2$	$\checkmark \text{factorisation}$ $\checkmark \text{simplifying } 2x - 6$ $\checkmark p'(x) = 2$ $\checkmark p'(3) = 2 \quad (4)$
[20]		

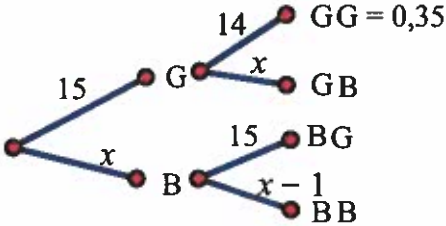
QUESTION 9/VRAAG 9

9.1	$\frac{dy}{dx} = 6x^2 + px + q$ <p>At F/by F: $\frac{dy}{dx} = 0$</p> $\therefore 6(2)^2 + 2p(2) + q = 0$ $24 + 4p + q = 0 \rightarrow \text{equation 1/ vergelyking 1}$ $-9 = 2(2)^3 + p(2)^2 + q(2) + 3$ $-9 = 19 + 4p + 2q$ $4p + 2q + 28 = 0 \rightarrow \text{equation/ vergelyking 2}$ $\text{Eq 1} - \text{Eq 2} = -q - 4 = 0$ $q = 4$	$\checkmark \frac{dy}{dx} = 6x^2 + px + q = 0$ $\checkmark \text{simplification}$ $\checkmark \text{Substitute /vervang}$ $(2; -9)$ $\checkmark q = 4$
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	<p>In Eq (2) $4p + 2(-4) + 28 = 0$</p> $4p - 8 + 28 = 0$ $\therefore p = -5$	$\checkmark \therefore p = -5$ (5)
9.2	$y = 2x^3 - 5x^2 - 4x + 3$ $y = 2x^3 - 5x^2 - 4x + 3 = 0$ $(x+1)(2x^2 - 7x + 3) = 0$ $(x+1)(2x-1)(x-3) = 0$ $B\left(\frac{1}{2}; 0\right) \text{ en/and } C(3;0)$	$\checkmark (x+1)(2x-1)(x-3) = 0$ $\checkmark B\left(\frac{1}{2}; 0\right)$ $\checkmark C(3;0)$ (3)
9.3	<p>By D/at D :</p> $D_x = 6x^2 - 10x - 4 = 0$ $(3x+1)(x-2) = 0$ $x = \frac{-1}{3} \text{ of/or } x = 2$ <p>Substitute/vervang $x = \frac{-1}{3}$</p> $2\left(\frac{-1}{3}\right)^3 - 5\left(\frac{-1}{3}\right)^2 - 4\left(\frac{-1}{3}\right) + 3 = \frac{100}{27} = 3\frac{19}{27}$ $\therefore D\left(\frac{-1}{3}; 3\frac{19}{27}\right)$	$\checkmark 6x^2 - 10x - 4 = 0$ \checkmark Substitute/vervang $x = \frac{-1}{3}$ $\checkmark \therefore D\left(\frac{-1}{3}; 3\frac{19}{27}\right)$ (3)
		[11]

QUESTION 10		
10.1	$B\left(x; 4 - \frac{x^2}{4}\right)$	✓ (1)
10.2	<p>Area $\triangle OBD$:</p> $A = \frac{1}{2} \cdot OD \cdot DB$ $= \frac{1}{2}(x)\left(4 - \frac{x^2}{4}\right)$ $= 2x - \frac{x^3}{8}$	$\checkmark \frac{1}{2}(x)\left(4 - \frac{x^2}{4}\right)$ $\checkmark 2x - \frac{x^3}{8} \quad (2)$
10.3	<p>For max/vir maks:</p> $\frac{dA}{dx} = 0$ $2 - \frac{3}{8}x^2 = 0$ $x^2 = \frac{16}{3}$ $x = \pm \sqrt{\frac{16}{3}}$ $x = \pm 2,3091$ $x = 2,31$	$\checkmark \checkmark 2 - \frac{3}{8}x^2 = 0$ $\checkmark x^2 = \frac{16}{3}$ $\checkmark x = 2,31 \quad (4)$
		[7]

QUESTION 11/VRAAG 11		
11.1.1	$80 - 69 = 11$	✓ 11 (1)
11.1.2		✓ $(21-x) \text{ \& } (14-x)$ ✓ 14 & 10 ✓ 6 & 11 (3)
11.1.3	$21 - x + x + 9 + 14 + 10 + 6 + 14 - x + 11 = 80$ $-x + 85 = 80$ $\therefore x = 5$	✓ equating/vergeljking=80 ✓ $\therefore x = 5$ (2)
11.1.4	$P(\text{at least 2}) = \frac{5+9+10+14}{80} = \frac{38}{80} = 0,475$ $P(\text{ten minste 2}) = \frac{5+9+10+14}{80} = \frac{38}{80} = 0,475$	✓ $\frac{5+9+10+14}{80}$ ✓ 0,475 (2)
11.2.1	$P(A \text{ or/of } B) = P(A) + P(B) - P(A \text{ en/ and } B)$ $= 0,95 + 0,98 - 0,94$ $= 0,99$	✓ $0,95 + 0,98 - 0,94$ ✓ 0,99 (2)
11.2.2	$P(\text{not detected/nie bespeur}) = 1 - 0,99$ $= 0,01$	✓ 0,01 (1)

<p>11.3</p>	<p>Let the number of boys be x,</p>  <p> $\frac{15}{15+x} \times \frac{14}{14+x} = \frac{35}{100}$ $35(15+x)(14+x) = 15 \times 14 \times 100$ $x^2 + 39x + 210 = 600$ $x^2 + 39x + 390 = 0$ $(x+39)(x-10) = 0$ $\therefore x = 10 \text{ i.e. there are 10 boys in the class.}$ </p>	<p>✓ probability equation</p> <p>✓ quadratic equation</p> <p>✓ factors</p> <p>✓ answer (4)</p>
		<p>[15]</p>

TOTAL:150