education

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NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P1

SEPTEMBER 2020

MARKS: 150

TIME: 3 hours

EMATHP1

This question paper consists of 10 pages and 1 information sheet.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 11 questions.
2. Answer ALL the questions.
3. Number the answers correctly according to the numbering system used in this question paper.
4. Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining the answers.
5. Answers only will NOT necessarily be awarded full marks.
6. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
7. If necessary, round off answers to TWO decimal places, unless stated otherwise.
8. Diagrams are NOT necessarily drawn to scale.
9. An information sheet with formulae is included at the end of the question paper.
10. Write neatly and legibly.

QUESTION 1

1.1 Solve for x:

1.1.1 $9x^2 - 7x - 3 = 0$ (Leave your answer correct to TWO decimal places.) (3)

1.1.2 $5x^2 - 10x > 0$ (3)

1.1.3 $4 - \sqrt{x + 5} = x + 3$ (6)

1.2 If $(x - 3)(y + 4) = 0$ determine x if:

1.2.1 $y = 4$ (1)

1.2.2 $y = -4$ (1)

1.3 Solve simultaneously for x and y:

$2y + x = 1$ and $x^2 + y^2 = y - x$ (6)

1.4 Consider: $5x^2 - kx + 16 = (x + 2).Q(x) + 10$ where k is a constant and $Q(x)$ is a polynomial in terms of x. Calculate k. (3)

[23]

QUESTION 2

2.1 Ann plans to start studying for her grade 12 final examination. On the first day she studies 1 hour (60 minutes) and plans to increase the study time with 15 minutes each day. As soon as Ann reaches 6 hours' study time, she will continue to study 6 hours each day thereafter.

2.1.1 Calculate the number of hours Ann will study on the 10^{th} day. (3)

2.1.2 Determine on which day Ann will study 6 hours for the first time. (2)

2.1.3 Calculate the total number of hours that Ann will study in the first 30 days. (4)

2.2 Prove that $x + y + z$ forms a geometric series if $\log x + \log y + \log z$ forms an arithmetic series. (4)

[13]

QUESTION 3

3.1 Consider the series: $64 + 32 + 16 + ...$

3.1.1 Determine the ninth term in the sequence. (3)

3.1.2 Determine the sum to infinite. (2)

3.2 A quadratic number pattern $T_n = an^2 + bn + c$ has a first term equal to 2. The general term of the first differences is given by $6n + 8$.

3.2.1 Show that a = 3. (3)

3.2.2 Determine the general term T_n. (3)

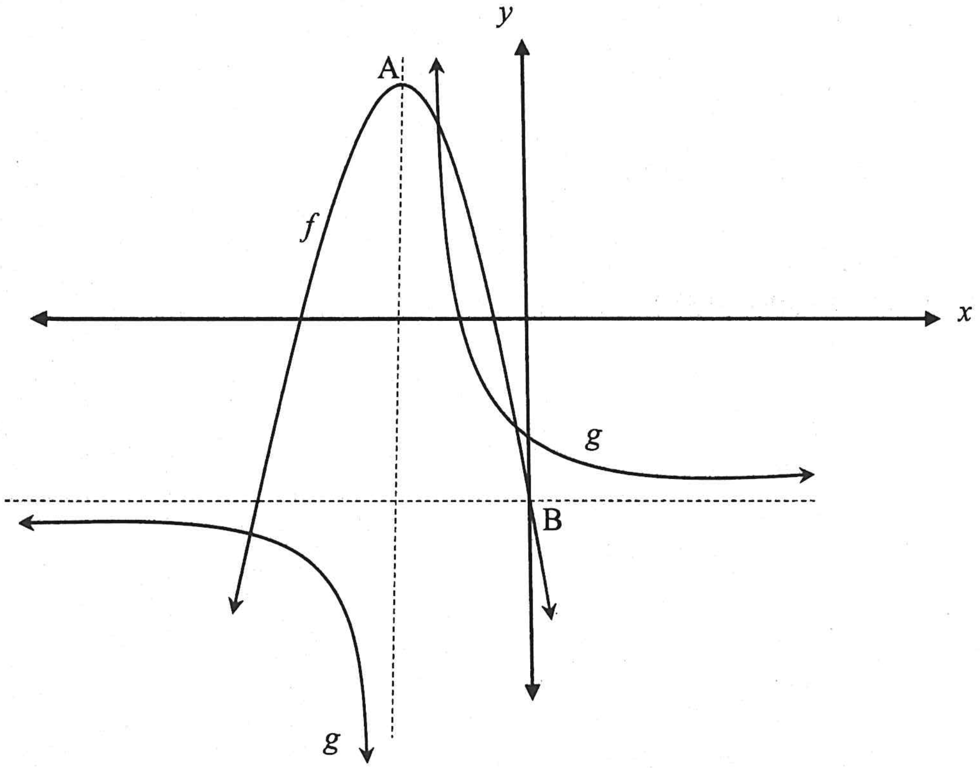
3.3 Given the series: $17p^8k^{15} + 20p^9k^{14} + 23p^{10}k^{13} + ... + 53p^{20}k^3$
Write the series in sigma notation. (4)

[15]

QUESTION 4

The graphs of $f(x) = -x^2 - 6x - 4$ and $g(x) = \dfrac{2}{x+p} + q$ are sketched below.

B is the y-intercept of f and A is the turning point of f. The vertical asymptote of g forms the axis of symmetry of f. The horizontal asymptote of g cuts the y-axis at B.



4.1 Determine the coordinates of A, the turning point of f. (3)

4.2 Determine the coordinates of B, the y-intercept of f. (1)

4.3 Determine the x-intercepts of f. (3)

4.4 Write down the equation of g. (2)

4.5 Determine the equation of the axis of symmetry of g that has a positive gradient. (2)

4.6 Determine the coordinates of the intersection of the axis of symmetry which is determined in QUESTION 4.5 and g, if $x > -3$. (5)

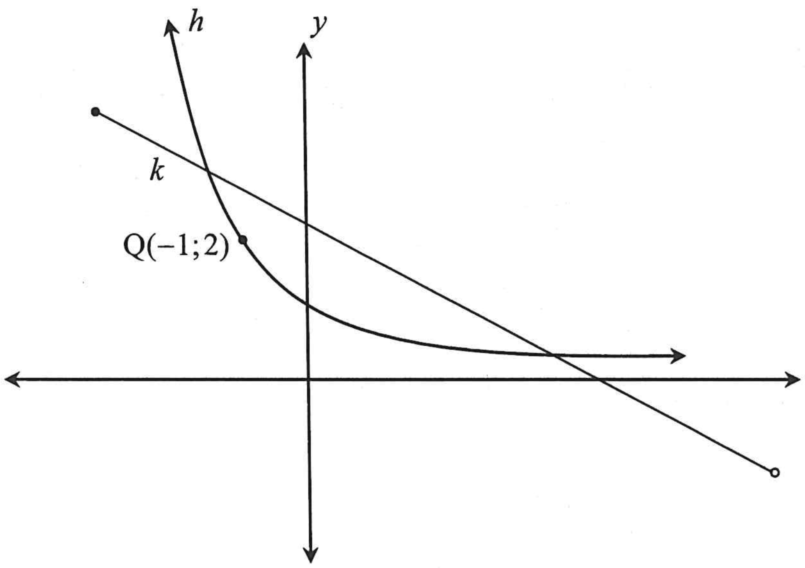
4.7 Determine the equation of m, the tangent to g, with the point where they touch, the intercept calculated in QUESTION 4.6. (3)

4.8 For which values of x will: $f(x).f'(x) \geq 0$ (2)

[21]

QUESTION 5

Given: $k(x) = -\frac{2}{3}x + 3$ for $-4 \leq x < 6$ and $h(x) = 2^{-x}$. Q(−1;2) is a point on h.



5.1 Determine the x-intercept of k. (2)

5.2 Determine the domain of k^{-1}. (2)

5.3 Determine the equation of h^{-1}. (2)

5.4 Give the coordinates of the x-intercept of h^{-1}. (2)

5.5 For which values of x is: $k^{-1}(x) < 0$? (2)

5.6 If $k(x) = q'(x)$, where q is a function defined for $-4 \leq x < 6$. Draw a neat sketch graph of q. Clearly show the x-values of the turning point(s) and end points. (3)

[13]

QUESTION 6

Patric takes out an annuity that he can live from after he retires in twenty years' time. He needs R3 000 000 in his annuity when he retires. The bank gives him an interest rate of 10% per annum compounded monthly.

6.1 Calculate his monthly instalment into the fund if he starts paying immediately and thereafter at the end of each month until his last payment in 20 years' time. (4)

6.2 After 20 years Patric retires, but decides not to let the R3 000 000 be paid out. Instead he decides to withdraw monthly amounts of R20 600 at the end of each month. He withdraws his first amount at the end of the fourth month. The interest that he earns over this period is 8% per year, compounded monthly. Determine how many months can he continue with his lifestyle. (7)

6.3 Calculate the amount of Patric's final withdrawal. (4)

[15]

QUESTION 7

7.1 Given: $f(x) = -x^2 + 7x + 9$

Determine $f'(x)$ from first principles. (5)

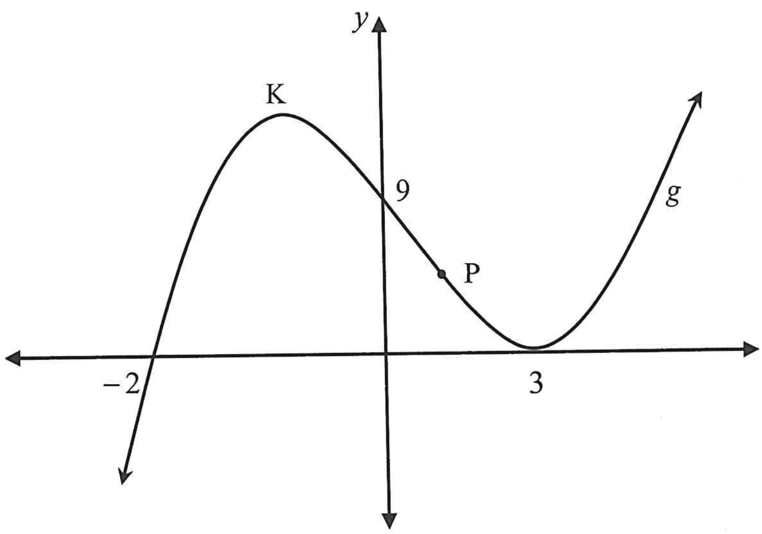
7.2 Determine $f'(x)$ if $f(x) = \frac{4}{x^2} + 3x^5$ (3)

7.3 Determine $\frac{dy}{dx}$ if: $\frac{y}{x-3} = 1 + x$ (3)

[11]

QUESTION 8

The graph of $g(x) = ax^3 + bx^2 + cx + d$ is sketched below. The graph of g intersects the x-axis at $x = -2$ and touches the x-axis at $x = 3$. K is a turning point of g. The graph g cuts the y-axis at (0; 9).



8.1 Show that $a = \frac{1}{2}$, $b = -2$, $c = -\frac{3}{2}$ and $d = 9$. (4)

8.2 Determine the x-coordinate of the turning point K. (4)

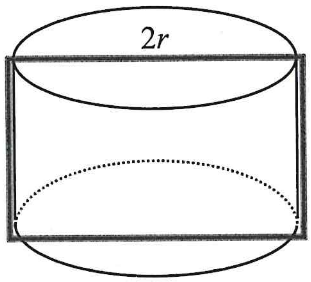
8.3 For which values of x is g concave up? (3)

8.4 Determine the coordinates of P if the gradient of the tangent to the graph at P is equal to $-\frac{7}{2}$. P touches g where g is concave up. (5)

[16]

QUESTION 9

Sam has a 16 m steel cable to wrap around a cylindrical tank to strengthen (reinforce) the tank as shown in the shaded part of the sketch.



9.1 Show that the height can be written as $h = 8 - 2r$ in terms of the radius. (2)

9.2 Write the volume of the tank in terms of r. (3)

9.3 What must the radius and the height of the tank be so that the volume of the tank will be a maximum? (5)
[10]

QUESTION 10

Tom and Jerry enter the Ironman Competition. The probabilities that they will complete the race have been determined to be 0,85 and 0,67 respectively. The probability that Tom and Jerry will complete the competition is independent of each other. Determine the probability (correct to TWO decimal places) that:

10.1 Both will complete the Ironman Competition. (2)

10.2 Only Tom will complete the competition. (2)

10.3 At least one of the two will complete the competition. (3)
[7]

QUESTION 11

The digits 0 to 9 are used to create a 5-digit-number for a lucky draw for the grade 12 fundraising. The digits may repeat. The numbers lie between 10 000 and 20 000. These numbers are written on pieces of paper and thrown into a bottle.

To win a prize, you have to draw a 5-digit-number that has at least one six and the digits may not repeat.

11.1 Determine the number of papers in the bottle. (2)

11.2 What is the probability that a person who selects a paper randomly from the bottle, will win a prize? Show all your calculations. (4)
[6]

TOTAL: 150

INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $\quad$ $A = P(1-ni)$ $\quad$ $A = P(1-i)^n$ $\quad$ $A = P(1+i)^n$

$T_n = a + (n-1)d$ $\quad$ $S_n = \frac{n}{2}(2a + (n-1)d)$

$T_n = ar^{n-1}$ $\quad$ $S_n = \frac{a(r^n - 1)}{r-1}$; $r \neq 1$ $\quad$ $S_\infty = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x[(1+i)^n - 1]}{i}$ $\quad$ $P = \frac{x[1-(1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad$ $M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad$ $y - y_1 = m(x - x_1)$ $\quad$ $m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad$ $m = \tan\theta$

$$(x-a)^2 + (y-b)^2 = r^2$$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $\quad$ $a^2 = b^2 + c^2 - 2bc.\cos A$

$$area\ \Delta ABC = \frac{1}{2}ab.\sin C$$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\quad$ $\sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\quad$ $\cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$ $\quad$ $\sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum x}{n}$ $\quad$ $\sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $\quad$ $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$\hat{y} = a + bx$ $\quad$ $b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$