Name of School

Grade 12

Mathematics P1
September 2020

MARKS: 150
TIME: 3 hours

This question paper consists of 10 pages and 1 information sheet.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 12 Questions.

2. Answer ALL the questions.

3. Clearly show ALL calculations, diagrams, graphs, etc. that you have used in determining your answers.

4. Answers only will not necessarily be awarded full marks.

5. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.

6. If necessary, round off answers to TWO decimal places, unless stated otherwise.

7. Diagrams are NOT necessarily drawn to scale.

8. An information sheet, with formulae, is included at the end of the question paper.

9. Number the answers correctly according to the numbering system used in this question paper.

10. Write legibly and present your work neatly.

QUESTION 1

1.1   Solve for x:

      1.1.1 $x^2 - 6x - 27 = 0$ (3)

      1.1.2 $2 + x = 4x^2$ (correct to TWO decimal places)   (4)

      1.1.3 $\left(\sqrt{x} - 2\right)^2 = 1$ (5)

      1.1.4 $x^2 + x - 6 \geq 0$ (3)

1.2   Solve for x and y simultaneously in the following equations:

      $y = 4x + 2$ and $(2 + x)(x + 1) = y + 12$ (4)

1.3   Given $10! = a! \times b! \times c!$ where $a \neq 0, b \neq 0$ and $c \neq 0$

      Determine the possible values for a, b and c.   (4)

                                                                                                             [23]

QUESTION 2

2.1   Consider the following sequence: 243 ; 81 ; 27 ; 9 ; ………

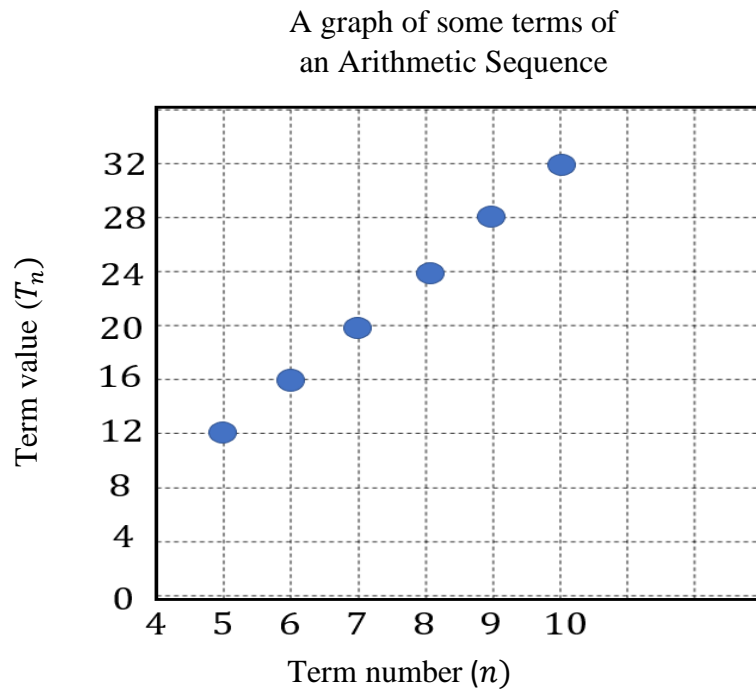   2.1.1   Write down the next term in the sequence.   (1)

      2.1.2   Does the series converge? Give a reason for your answer.   (2)

      2.1.3   Prove that the general term can be written as: $T_n = 3^{6-n}$ (3)

      2.1.4   Calculate S_∞ (2)

2.2   In the diagram below is a graph of some of the terms of an arithmetic sequence.

The graph indicates that $T_8 = 24$

A graph of some terms of
an Arithmetic Sequence

2.2.1   Determine the constant difference for this sequence.   (1)

2.2.2   Calculate what term in this sequence will be equal to 124.   (4)
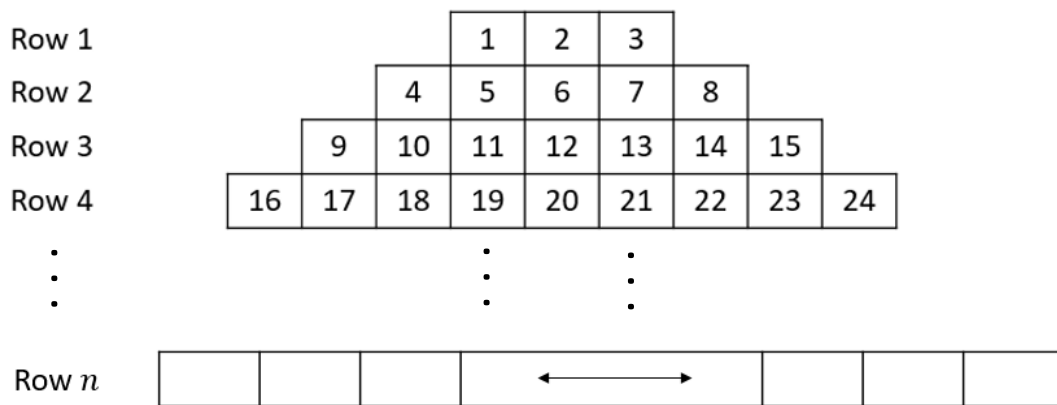
2.3   Given: $S_n = \dfrac{5\left(1 - 3^n\right)}{-2}$

2.3.1   Calculate S_4 (2)

2.3.2   Calculate T_5 (2)

[17]

QUESTION 3

In a storage room there are sets of drawers that are used for storage. These drawers are packed in rows and numbered as shown in the diagrams below.


Row 1				1	2	3			
Row 2			4	5	6	7	8		
Row 3		9	10	11	12	13	14	15	
Row 4	16	17	18	19	20	21	22	23	24

Row n

3.1 The numbers of the **last drawer** in each row form a sequence.

Determine the general term of this sequence. (4)

3.2 How many drawers are there in row 8? (2)

3.3 The manager bought 255 handles for the drawers.
Determine how many complete rows will have handles. (4)

[10]

QUESTION 4

The sketch shows the graphs of $f(x) = a(x - 2)^2 + q$ and $g(x) = x - 6$. The graph of f cuts the x-axis at A$(-2; 0)$ and B. Graphs f and g intersect at B and D. The coordinates of E is $(4; k)$.
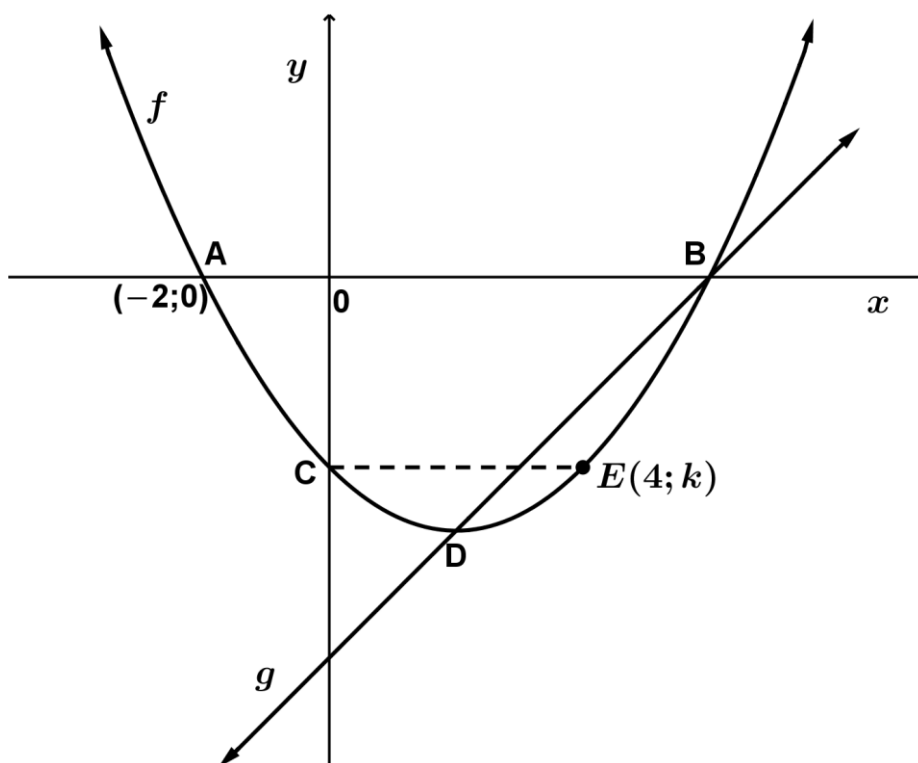
4.1   Write down the length of AB.   (2)

4.2   Determine the coordinates of D, the turning point of f.   (2)

4.3   Show that the equation of f can be written as $f(x) = \dfrac{x^2}{4} - x - 3$.   (4)

4.4   Determine the value of k if E is the reflection of C with respect to the axis of symmetry of f.   (1)

4.5   Determine the equation of the tangent to the curve at E if the tangent is parallel to g.   (2)

4.6   For which values of x will $f(x) - g(x) < 0$.   (2)

4.7   Determine the coordinates of the turning point of $h(x)$ if $h(x) = f(x + 2) + 1$.   (2)

                                                                                                                           [15]

QUESTION 5

Consider the function: $f(x) = \dfrac{-2}{x-1} - 1$

5.1 Write down the equations of the asymptotes of f. (2)

5.2 Calculate the intercepts with the axes. (3)

5.3 Draw a sketch graph of f, showing all asymptotes and intercepts with the axes. (3)

5.4 Determine the equation of the axis of symmetry of f with a negative gradient. (2)

[10]

QUESTION 6

The drawing below, represents the graph of $f(x) = \log_a x$.

$(4 ; -2)$ is a point on the graph.
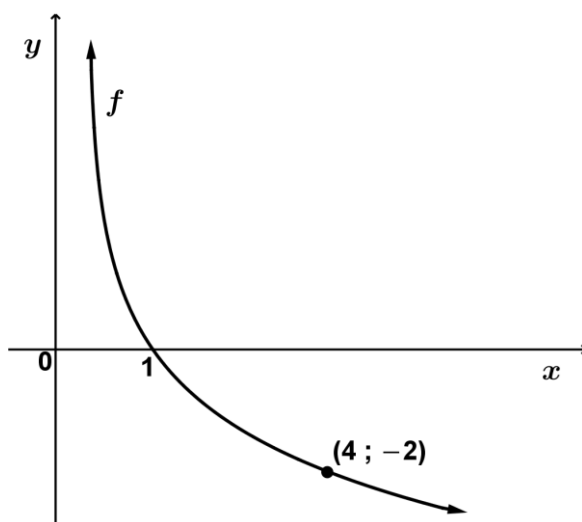
6.1 Show that $a = \dfrac{1}{2}$ (2)

6.2 Determine the equation of $f^{-1}(x)$ in the form $y = ...$ (2)

6.3 Write down the equation of h if h is the reflection of f in the x-axis. (2)

6.4 Determine the value(s) of x for which $\log_a(x+4) \geq 0$ (2)

[8]

QUESTION 7

7.1 Thandi decides to invest R45 000 into a savings account. She is charged r % interest per annum compounded half-yearly. After 5 years the investment matures and she receives R 66 611. Calculate the interest rate (r). (4)

7.2 On the 30 June 2013 and at the end of each month thereafter, Jonathan deposited R2 500 into a bank account that pays 6% per annum, compounded quarterly. He continues to deposit this amount until 30 June 2018.

How much money will Jonathan have in his account immediately after depositing the R2 500 on 30 June 2018? (4)

7.3 On 1 February 2018, Genevieve took a loan of R82 000 from a bank to pay for her studies. She will make her first payment of R3 200 on 31 July 2018 and continue to make payments of R3 200 until she settles the loan. The bank charges interest at 15% per annum, compounded monthly.

7.3.1 Calculate how much Genevieve will owe the bank on 1 July 2018. (3)

7.3.2 How many instalments of R3200 must she pay to settle the loan? (4)

[15]

QUESTION 8

8.1 From first principles, determine $f'(x)$ if $f(x) = 4x^2 - x$. (5)

8.2 Determine:

8.2.1 $D_x\left[x^2 - \dfrac{1}{2x^3} + \sqrt{x} \right]$ (4)
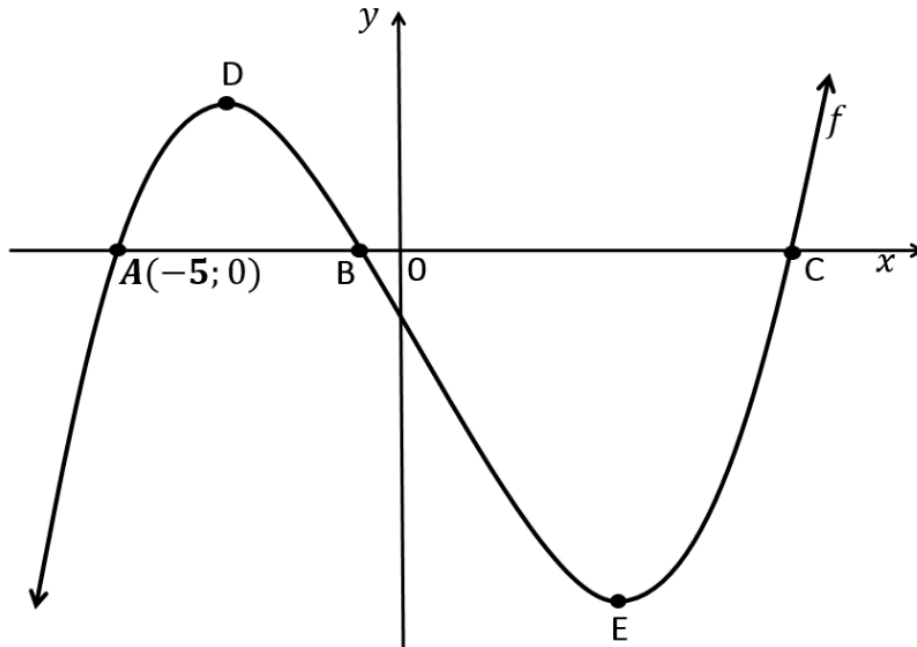
8.2.2 $\dfrac{dy}{dk}$ if $y = (k - 2p)^2$ (3)

8.3 The functions $f(x) = -x^2 + 9$ and $g(x) = \dfrac{p}{x} - 3$ have a common

tangent at the point where $x = 2$. Calculate the value of p. (4)

[16]

QUESTION 9

The function $f(x) = 2x^3 - 3x^2 - 72x - 35$ is indicated in the diagram below

$A(-5; 0)$, B and C are the x-intercepts and D and E are the turning points of f.

9.1   Calculate the coordinates of B and C (4)
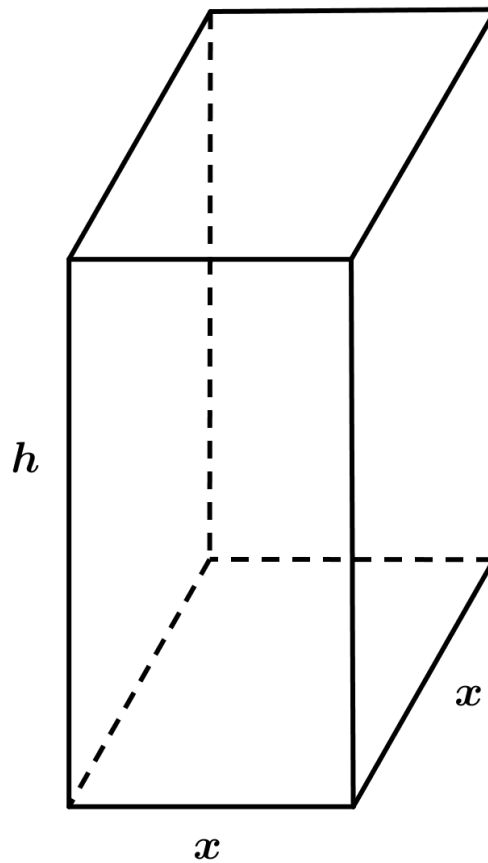
9.2   Calculate the coordinates of D and E (5)

9.3   For which values of x will the graph be concave down?   (3)

9.4   Determine the values of x if $f(x).f'(x) \geq 0$, where $x < 0$.   (2)

[14]

QUESTION 10

A piece of cardboard with area $1\,350\ cm^2$ is folded to form a closed

rectangular box with a square base. The side of the square is $x\ cm$ and the height of the box is
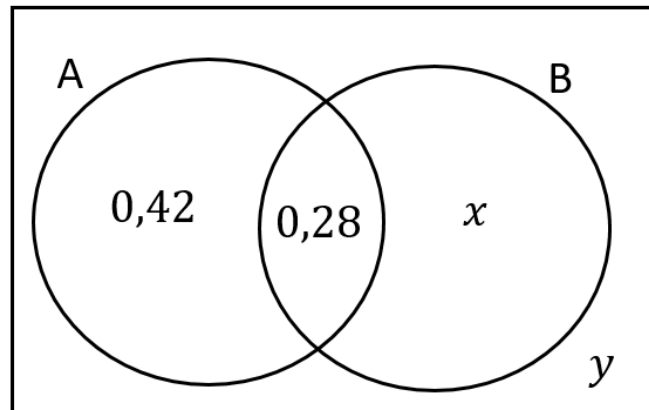
$h\ cm$.

Determine the value of x for which the volume will be a maximum.   (7)

[7]

QUESTION 11

11.1   The probabilities for two events, A and B, are shown in the diagram.

A and B are independent events.

11.1.1   Show that $x = 0,12$ (3)

11.1.2   Determine the value of y.   (1)

11.1.3   Calculate: P(A or B).   (2)

11.2   The probability that it will rain tomorrow is 70%.

If it rains the probability that I will arrive late at the office is 25%.

If it does not rain the probability that I will arrive late at the office is 10%.

Calculate the probability that I will be on time for work tomorrow.   (4)

                                                                                [10]

QUESTION 12

Consider the word **WINTER**.

12.1   How many arrangements can be made by using the letters in this word if the letters

may not be repeated?   (1)

12.2   How many arrangements can be made if the word must start with
a "T" and ends with a vowel if the letters may not be repeated?   (2)

12.3   Determine the probability of making a new word **<u>not</u>** starting with
a "T" and ending with a vowel.   (2)

                                                                                [5]
                                                            TOTAL: 150

INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1 + ni) \qquad A = P(1 - ni) \qquad A = P(1 - i)^n \qquad A = P(1 + i)^n$$

$$T_n = a + (n-1)d \qquad S_n = \frac{n}{2}(2a + (n-1)d)$$

$$T_n = ar^{n-1} \qquad S_n = \frac{a(r^n - 1)}{r - 1} \; ; r \neq 1 \qquad S_\infty = \frac{a}{1-r} \; ; -1 < r < 1$$

$$F = \frac{x[(1+i)^n - 1]}{i} \qquad P = \frac{x[1 - (1+i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c \qquad y - y_1 = m(x - x_1) \qquad m = \frac{y_2 - y_1}{x_2 - x_1} \qquad m = \tan\theta$$

$$(x - a)^2 + (y - b)^2 = r^2$$

In $\triangle ABC$: $\quad \dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C} \qquad a^2 = b^2 + c^2 - 2bc.\cos A \qquad area \triangle ABC = \dfrac{1}{2}ab.\sin C$

$$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta \qquad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$$

$$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta \qquad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases} \qquad \sin 2\alpha = 2\sin\alpha.\cos\alpha$$

$$\bar{x} = \frac{\sum fx}{n} \qquad\qquad \sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)} \qquad\qquad P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\hat{y} = a + bx \qquad\qquad b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$$