GAUTENG PROVINCE
EDUCATION
REPUBLIC OF SOUTH AFRICA

PREPARATORY EXAMINATION
VOORBEREIDENDE EKSAMEN

2017

MEMORANDUM /

NASIEN RIGLYNE

MATHEMATICS (SECOND PAPER) (10612)
WISKUNDE (TWEEDE VRAESTEL) (10612)

19 pages / *bladsye*

GAUTENG DEPARTMENT OF EDUCATION /

GAUTENGSE DEPARTEMENT VAN ONDERWYS

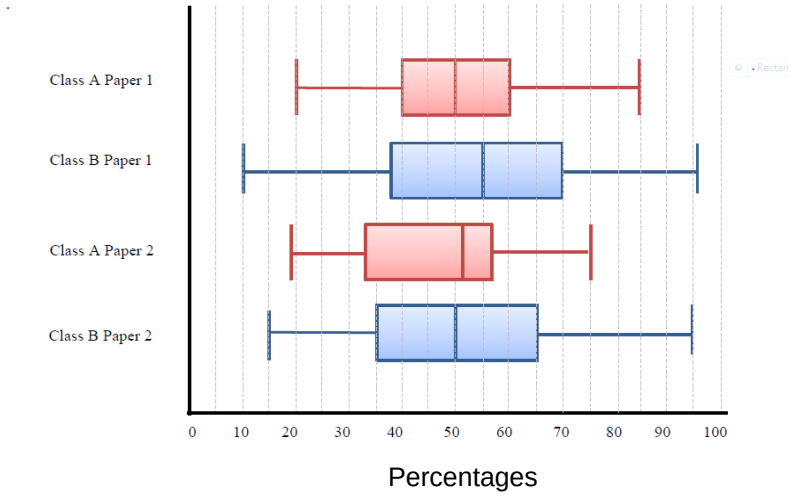
PREPARATORY EXAMINATION /

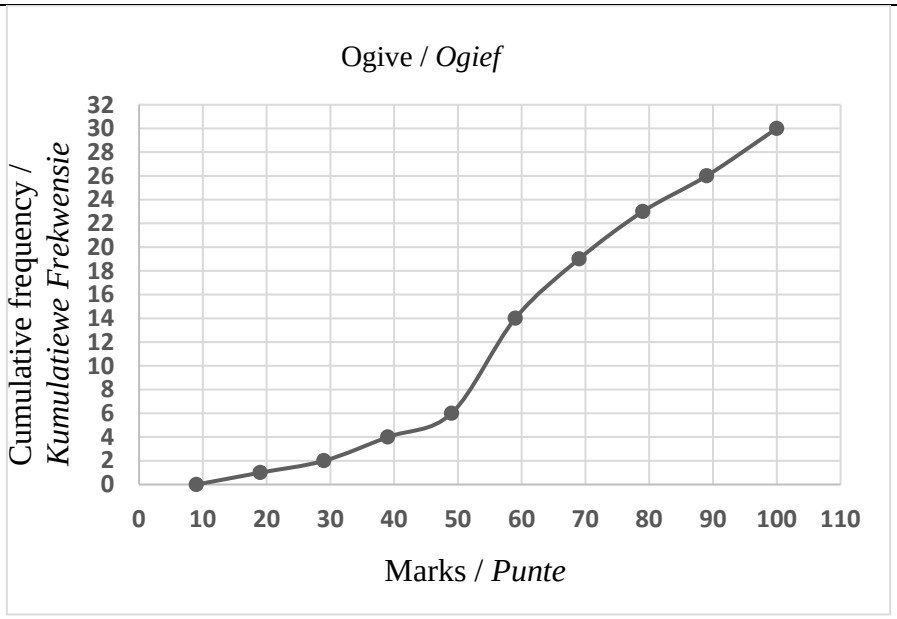
VOORBEREIDENDE EKSAMEN

MATHEMATICS / *WISKUNDE*
(Second Paper / Tweede *Vraestel*)

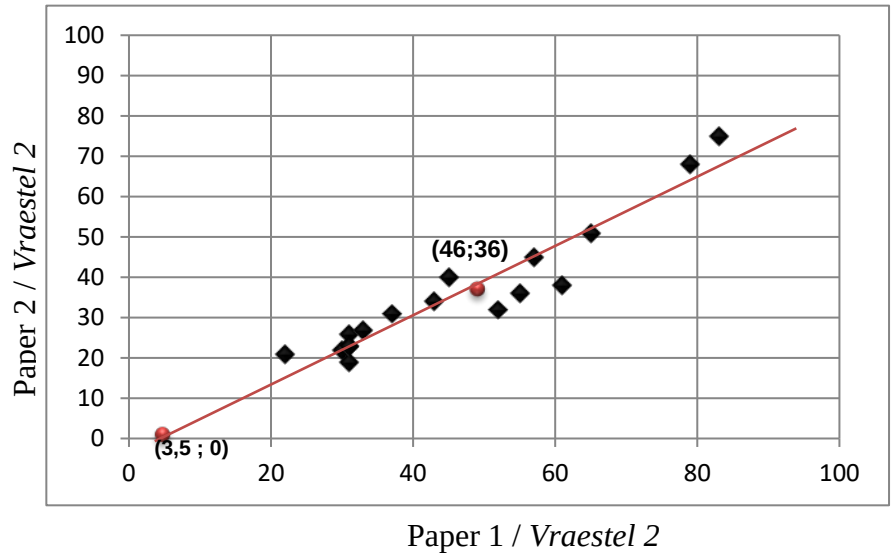
MEMORANDUM

QUESTION/*VRAAG* 1

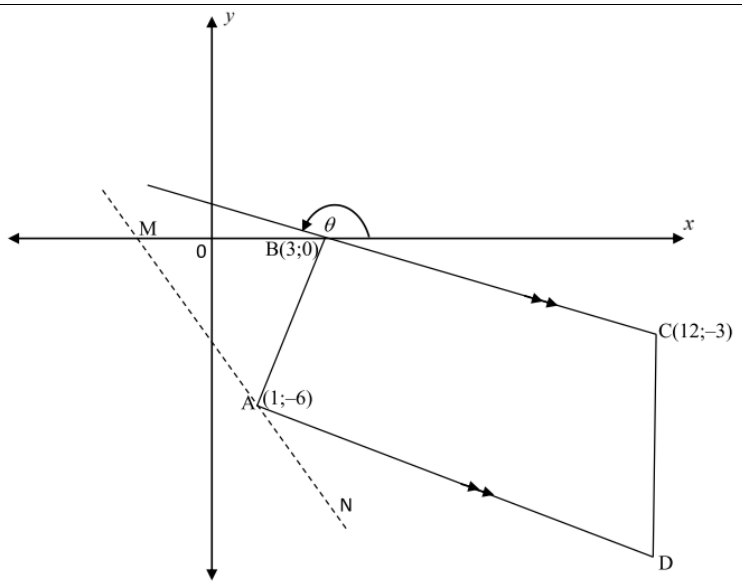
1.1		
1.1.1	median / *mediaan* = 55	✓ 55 (1)
1.1.2	IQR/*IKV* = 60 – 40 = 20	✓60 – 40 ✓20 (2)
1.1.3	15% – 65%	✓15% – 65% (1)
1.1.4	Skewed to the left / *Skeef na links* OR Negatively skewed / *Negatief skeef*	✓answer/*antwoord* (1)

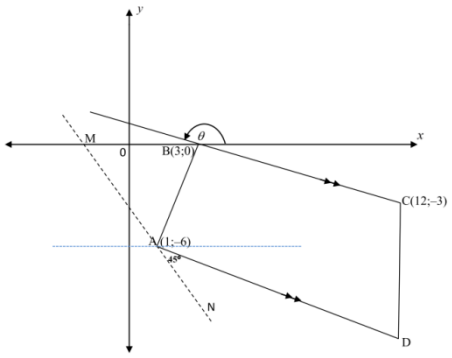
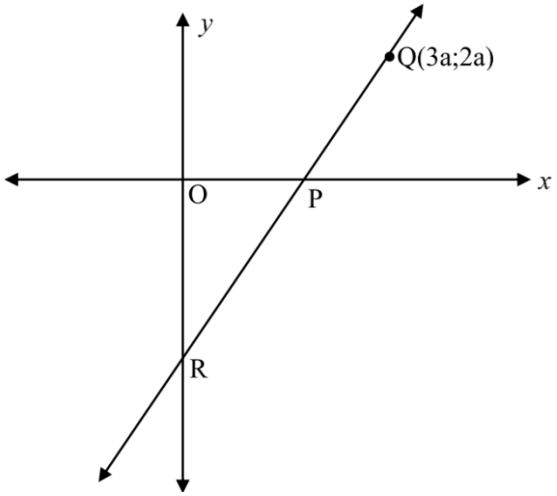
1.2	<table><tr><th>**Class interval /** ***Klasinterval***</th><th>**Cumulative Frequency /** ***Kumulatiewe Frekwensie***</th></tr><tr><td>$10 \leq x \leq 19$</td><td>1</td></tr><tr><td>$20 \leq x \leq 29$</td><td>2</td></tr><tr><td>$30 \leq x \leq 39$</td><td>4</td></tr><tr><td>$40 \leq x \leq 49$</td><td>6</td></tr><tr><td>$50 \leq x \leq 59$</td><td>14</td></tr><tr><td>$60 \leq x \leq 69$</td><td>19</td></tr><tr><td>$70 \leq x \leq 79$</td><td>23</td></tr><tr><td>$80 \leq x \leq 89$</td><td>26</td></tr><tr><td>$90 \leq x \leq 100$</td><td>30</td></tr></table>	
1.2.1		✓ top points of interval / *hoogte punt van interval* ✓shape / *vorm* ✓grounding point / *laagste punt* (3)
1.2.2	$\frac{19}{30} \times 100$ = 63%	✓ 63% (accuracy / akkuraatheid) (1)
1.2.3	26 – 23 = 3 learners/*leerders*	✓ 3 (1)
1.2.4	Mark between 30 – 39/*Enige punt tussen 30 – 39*	✓30 – 39 (1)
1.2.5	90 – 19 = 71	✓71 (1)
		[12]

QUESTION/*VRAAG* 2

2.1	$\hat{y} = 0{,}84x - 2{,}92$	✓a = 0,84x ✓b = – 2,92 ✓ equation / *vergelyking* (3)
2.2	$r = 0{,}95$	✓ $r = 0{,}95$ (1)
2.3		line through the point: / *lyn deur die punte:* ✓(46;36) average point / *gemiddelde punt* ✓ (3,5 ; 0) x -intercept / x *-afsnit* (2)
2.4	$y = 0{,}84(98) - 2{,}92$ $y = 79{,}4$ **OR / OF** calculator / *optel masjien* Yes / agree The correlation suggests a positive association between variables, therefore predictions using the least squares regression line is valid, reliable and usually accurate. *Ja / stem saam* *Die korrelasie dui op 'n positiewe assosiasie tussen veranderlikes, daarom is voorspellings met behulp van die kleinste kwadrate regressielyn geldig, betroubaar en gewoonlik akkuraat.*	✓ yes / *ja* ✓ reason / *rede* (2)
		[8]

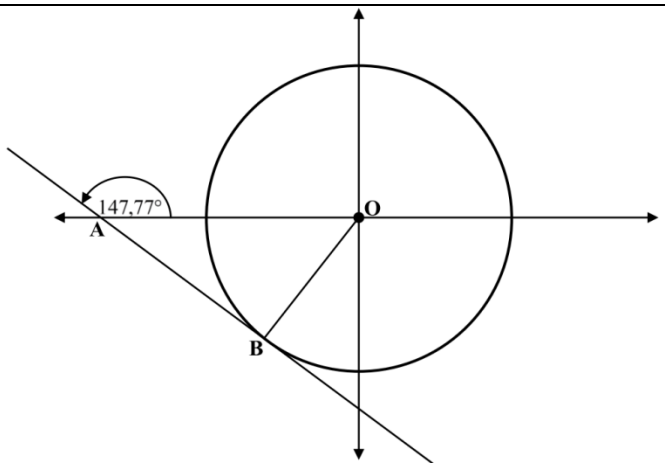
QUESTION/*VRAAG* 3

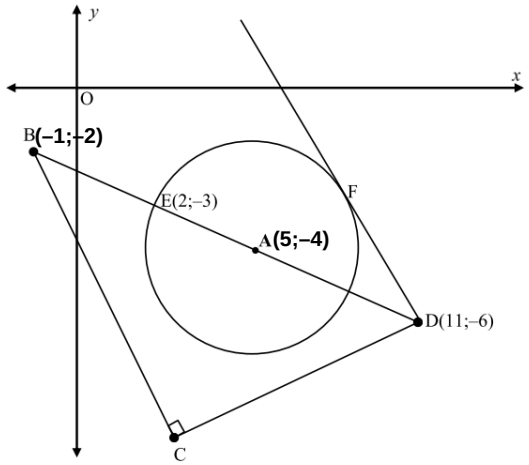
3.1		
3.1.1	$m_{BC} = \frac{-3-0}{12-3}$ $= -\frac{1}{3}$ $\tan\theta = m_{BC}$ $= -\frac{1}{3}$ $\theta = 180° - 18{,}43°$ $= 161{,}57°$	✓ $m_{BC} = -\frac{1}{3}$ ✓ $\tan\theta = -\frac{1}{3}$ ✓ $18{,}43°$ ✓ $\theta = 161{,}57°$ (4)
3.1.2	$m_{AD} = m_{BC}$ AD parallel BC $= -\frac{1}{3}$ $m_{AB} = \frac{-6-0}{1-3}$ $= 3$ $m_{AB}.m_{AD} = 3.\left(-\frac{1}{3}\right)$ $= -1$ $\therefore$ AB $\perp$ AD	✓ $m_{AD} = -\frac{1}{3}$ ✓ $m_{AB} = 3$ ✓ $m_{AB}.m_{AD} = 3.\left(-\frac{1}{3}\right)$ (3)

3.1.3	 Inclination of the line / *Inklinasie van die lyn* $\alpha = 161{,}6° - 45°$ $= 116{,}6°$ $\tan\alpha = m_{line}$ $\tan 116{,}6° = -2$ $y - y_1 = m(x - x_1)$ **OR/*OF*** $y = mx + q$ $y + 6 = -2(x - 1)$ $-6 = -2(1) + q$ $y + 6 = -2x + 2$ $q = -4$ $y = -2x - 4$	✓ $\alpha = 116{,}6°$ ✓ $m = -2$ ✓substitute / *vervang* A(1;–6) ✓ $y = -2x - 4$ (4)
3.2		
3.2.1	$y - y_1 = m(x - x_1)$ $y - 2a = 2(x - 3a)$ $y = 2x - 6a + 2a$ $y = 2x - 4a$	✓substitute / *vervang* (3a;2a) and / *en* $m = 2$ ✓ $y = 2x - 4a$ (2)
3.2.2	P$(2a;0)$ Q$(0;-4a)$ Area / *Oppervlakte* $\Delta POQ = \frac{1}{2}$b.h $= \frac{1}{2}(2a)(4a)$ $= 4a^2$	✓ P$(2a;0)$ ✓ Q$(0;-4a)$ ✓correct substitution / *korrekte vervanging* ✓ $4a^2$ (4)

3.2.3	$m_{DE} = m_{EQ}$ $\frac{-2+14}{3+3} = \frac{2a+2}{3a-3}$ $2(3a-3) = 2a+2$ $6a-6 = 2a+2$ $4a = 8$ $a = 2$ **OR / *OF*** $m_{DQ} = m_{DE}$ $\frac{2a+14}{3a+3} = \frac{-2+14}{3+3}$ $2(3a+3) = 2a+14$ $6a+6 = 2a+14$ $4a = 8$ $a = 2$ $m_{DQ} = m_{EQ}$ $\frac{2a+14}{3a+3} = \frac{2a+2}{3a-3}$ $(2a+14)(3a-3) = (2a+2)(3a+3)$ $6a^2 + 36a - 42 = 4a^2 + 12a + 6$ $2a^2 + 24a - 48 = 0$ $a^2 + 12a - 24 = 0$ $(a+12)(a-2) = 0$ $a = -12$ or / of $a = 2$ invalid / *ongeldig*	If learner assumed that D and E lie on the straight line QPR do not award any marks. / *Indien leerder aanvaar het dat D en E op die reguit lyn QPR lê, moet daar geen punte toegeken word nie.* **OR/*OF***	✓ $\frac{-2+14}{3+3} = \frac{2a+2}{3a-3}$ ✓simplification / *vereenvoudiging* ✓ $a = 2$ (3) ✓ $\frac{2a+14}{3a+3} = \frac{-2+14}{3+3}$ ✓ simplification / *vereenvoudiging* ✓ $a = 2$ (3) ✓ $\frac{2a+14}{3a+3} = \frac{2a+2}{3a-3}$ ✓ simplification / *vereenvoudiging* ✓ $a = 2$ chosen / *gekies* (3) **[20]**

QUESTION/*VRAAG* 4

4.1		
4.1.1	$x^2 + y^2 = 64$ **OR / *OF*** $x^2 + y^2 = 8^2$	✓ $x^2 + y^2 = 64$ **OR / *OF*** $x^2 + y^2 = 8^2$ (1)
4.1.2	$O\hat{A}B = 32{,}23°$ (∠s on a str line / $\angle^e$ *op reguit lyn*) $O\hat{B}A = 90°$ (tan ⊥ radius **OR** tan ⊥ diameter / *raaklyn* ⊥ *radius*) $\sin 32{,}23° = \frac{OB}{OA}$ $\sin 32{,}23° = \frac{8}{OA}$ $OA = \frac{8}{\sin 32{,}23°}$ $= 15$ $\therefore$ A(–15;0)	✓ $O\hat{A}B = 32{,}23°$ ✓ $\sin 32{,}23° = \frac{8}{OA}$ ✓ OA = 15 ✓ A(–15;0) (4)

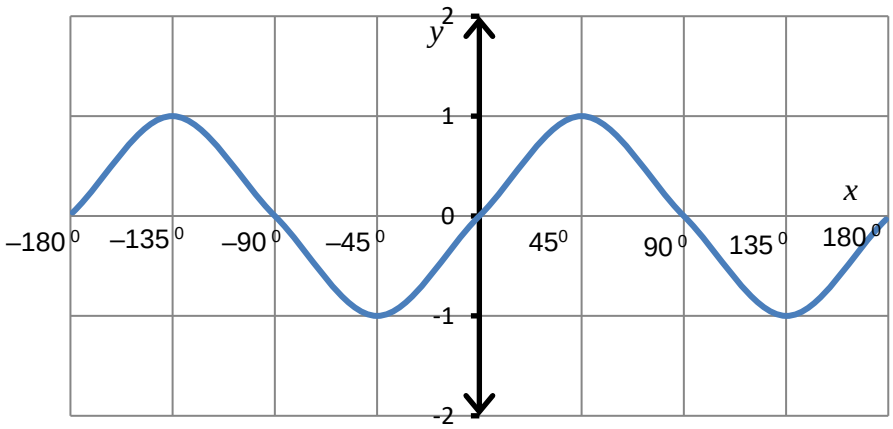
4.2		
4.2.1	$x^2-10x+y^2+8y+31=0$ $(x-5)^2+(y+4)^2=-31+25+16$ $(x-5)^2+(y+4)^2=-31+25+16$ $(x-5)^2+(y+4)^2=10$	✓ $(x-5)^2+(y+4)^2$ ✓ $-31+25+16$ ✓ $(x-5)^2+(y+4)^2=10$ (3)
4.2.2	$A(5;-4)$	✓ $A(5;-4)$ (1)
4.2.3	$r^2=10$ $r=\sqrt{10}$	✓ $r=\sqrt{10}$ (1)
4.2.4	$\frac{x_B+5}{2}=2$ and/*en* $\frac{y_B-4}{2}=-3$ $x_B=-1$ $y_B=-2$ $\therefore B(-1;-2)$	✓ $\frac{x_B+5}{2}=2$ ✓ $\frac{y_B-4}{2}=-3$ ✓ $B(-1;-2)$ (3)
4.2.5	$k=5-\sqrt{10}$ or / *of* $k=5+\sqrt{10}$ Any one of the two can be written first and then be allocated the two marks. / *Enigeen van die twee kan eerste geskryf word en dan die twee punte toegeken word.*	✓✓ $k=5+\sqrt{10}$ ✓ $k=5-\sqrt{10}$ (3)
4.2.6	$r^2=AF^2=10$ $AD=\sqrt{(11-5)^2+(-6+4)^2}$ $=\sqrt{36+4}$ $=\sqrt{40}$ $=2\sqrt{10}$ $A\hat{F}D=90°$ (radius ⊥ tangent / *raaklyn ⊥ radius*) $FD^2=AD^2-AF^2$ Pythagoras $=\left(\sqrt{40}\right)^2-\left(\sqrt{10}\right)^2$ $=30$ $FD=\sqrt{30}$	✓ $AF^2=10$ ✓ substitute in distance formula / *vervanging in afstandsformule* ✓ $AD=\sqrt{40}$ or $2\sqrt{10}$ ✓ $FD=\sqrt{30}$ (4)
		[20]

QUESTION/*VRAAG* 5

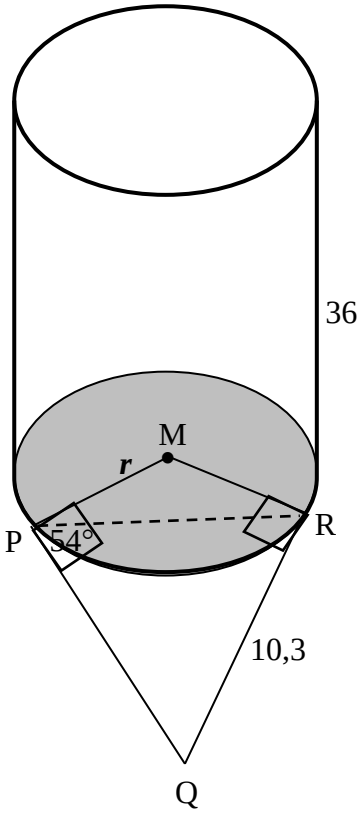
5.1	$\tan\theta = -\frac{1}{4}$ $\hat{O}_1 = 14{,}04^\circ$ $\theta = 360^\circ - 14{,}04^\circ$ $= 345{,}96^\circ$	✓ $\tan\theta = -\frac{1}{4}$ ✓ $\hat{O}_1 = 14{,}04^\circ$ ✓ $345{,}96^\circ$ (3)
5. 2	$\frac{2\cos(90^\circ - x)}{\sin(180^\circ - 2x)} \times \frac{\cos(60^\circ - x)\cos x - \sin(60^\circ - x)\sin x}{\tan(-x)}$ $= \frac{2\sin x}{\sin 2x} \times \frac{\cos(60^\circ - x + x)}{-\tan x}$ $= \frac{2\sin x}{2\sin x\cos x} \times \frac{\cos 60^\circ}{-\frac{\sin x}{\cos x}}$ $= \frac{1}{\cos x} \times \frac{1}{2} \times -\frac{\cos x}{\sin x}$ $= -\frac{1}{2\sin x}$	✓ $2\sin x$ ✓ $\cos(60^\circ - x + x)$ ✓ $\sin 2x$ ✓ $-\tan x$ ✓ $2\sin x\cos x$ ✓ $\frac{\sin x}{\cos x}$ ✓ $-\frac{1}{2\sin x}$ (7)
5.3.1	$\sin 108^\circ = \sin(90^\circ + 18^\circ)$ $= \cos 18^\circ$ $= k$	✓ reduction/ *herleiding* ✓ k (2)
5.3.2	$\cos(-36)^\circ = \cos 36^\circ$ $= \cos 2(18^\circ)$ $= 2\cos^2 18^\circ - 1$ $= 2k^2 - 1$	✓ $\cos 36^\circ$ ✓ $2\cos^2 18^\circ - 1$ ✓ $2k^2 - 1$ (3)
5.4	$2\sin x\cos x + 2\sin x + \cos^2 x + \cos x = 0$ $2\sin x(\cos x + 1) + \cos x(\cos x + 1) = 0$ $(\cos x + 1)(2\sin x + \cos x) = 0$ $\cos x = -1$ or / *of* $2\sin x = -\cos x$ $x = 180^\circ + k.360^\circ$ or / *of* $2\tan x = -1$ $\tan x = -\frac{1}{2}$ $x = -26{,}57^\circ + k.180^\circ \quad k \in Z$ **OR / *OF*** ref$\angle = 26{,}57^\circ$ $x = 153{,}43^\circ + k.180^\circ \quad k \in Z$ $x = \{-26{,}57^\circ ; 153{,}43^\circ ; 180^\circ\}$	✓ common factor/ *gemeenskaplike faktor* ✓ $\cos x = -1$ ✓ $\tan x = -\frac{1}{2}$ ✓ $153{,}4^\circ$ ✓ $-26{,}57^\circ$ ✓ 180° (6)

5.5.1	$\tan\theta = \frac{p}{1}$ $p^2 + y^2 = r^2$ $p^2 + 1 = r^2$ $r = \sqrt{p^2+1}$ $\sin 2\theta = 2\sin\theta\cos\theta$ $= 2\left(\frac{p}{\sqrt{p^2+1}}\right)\left(\frac{1}{\sqrt{p^2+1}}\right)$ $= \frac{2p}{p^2+1}$	✓ $r = \sqrt{p^2+1}$ ✓ $2\left(\frac{p}{\sqrt{p^2+1}}\right)\left(\frac{1}{\sqrt{p^2+1}}\right)$ (2)
5.5.2	$\frac{(p+1)^2}{p^2+1}$ $= \frac{p^2+2p+1}{p^2+1}$ $= \frac{2p}{p^2+1} + \frac{p^2+1}{p^2+1}$ $= \sin 2\theta + 1$ Maximum of / *Maksimum van* $\sin 2\theta$ is 1 Maximum of / *Maksimum van* $\frac{(p+1)^2}{p^2+1}$ will be 2	✓ $\frac{p^2+2p+1}{p^2+1}$ ✓ $\frac{2p}{p^2+1} + \frac{p^2+1}{p^2+1}$ **OR/*OF*** $\sin 2\theta + 1$ ✓ 2 (3) **[26]**

QUESTION/*VRAAG* 6

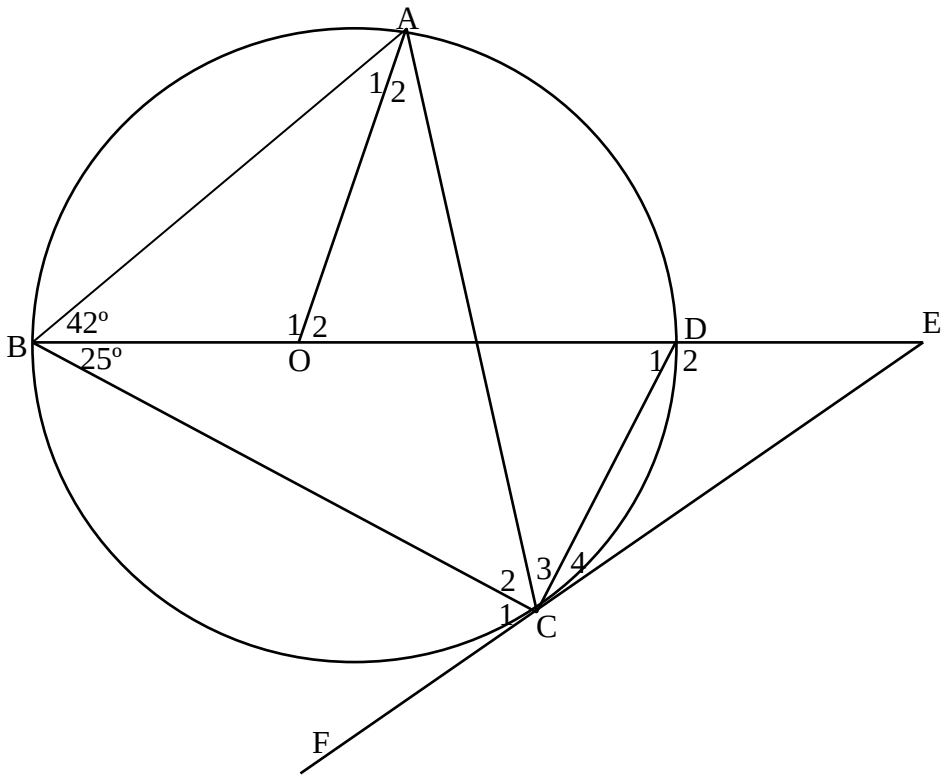
6.1	$a = 1$ $b = 2$	✓ $a = 1$ ✓✓ $b = 2$ (3)
6.2		✓ *x*-intercepts/ *-afsnit* ✓ T/P ✓shape/*vorm* (3)
6.3	$x \in (45° ; 135°)$ **OR/*OF*** $45° < x < 135°$	✓ critical values / *kritiese waardes* ✓ correct interval / korrekte interval (2) **[8]**

QUESTION/*VRAAG* 7



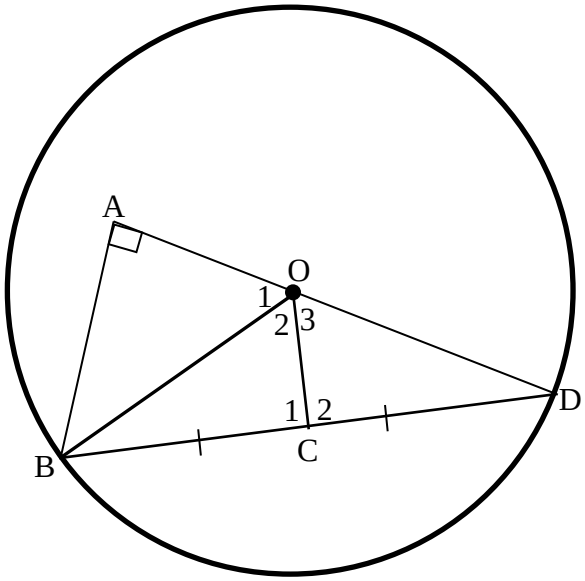
7.1	PQ = PR (tangents from the same point / *raaklyne vanuit dies. punt*) $P\hat{R}Q = 54^o$ ($\angle$'s opp. equal sides / $\angle^e$ *teenoor gelyke sye*) $\hat{Q} = 180^o - (54^o + 54^o)$ ($\angle$'s of a Δ / $\angle^e$ *van* Δ) $= 72^o$	✓ $P\hat{R}Q = 54^o$ ✓ answer / *antwoord* (2)
7.2	$PR^2 = (10,3)^2 + (10,3)^2 - 2(10,3)(10,3)\cos 72^o$ $= 12,11$	✓substitution into correct cos rule / *vervanging na korrekte cos reël* ✓answer / *antwoord* (2)
7.3	MPQR is a cyclic quad. / *MPQR is 'n koordevierhoek* (opp. $\angle$'s sup) $\therefore \hat{M} = 108^o$ $(12,11)^2 = r^2 + r^2 - 2(r)(r)\cos 108^o$ $146,6521 = 2r^2 - 2r^2\cos 108^o$ $= 2r^2(1 - \cos 108^o)$ $r^2 = \frac{146,6521}{2(1-\cos 108^o)}$ $= 56,02$ Volume of/*van* silo = $\pi r^2 h$ $= \pi(56,02)(36)$ $= 6335,71 m^3$	✓ $\hat{M} = 108^o$ ✓ substitution into correct cos rule / *vervanging na korrekte cos reël* ✓answer / *antwoord* ✓substitution into correct volume formula/*vervanging na korrekte volume formule* ✓answer / *antwoord* (5) **[9]**

QUESTION/*VRAAG* 8



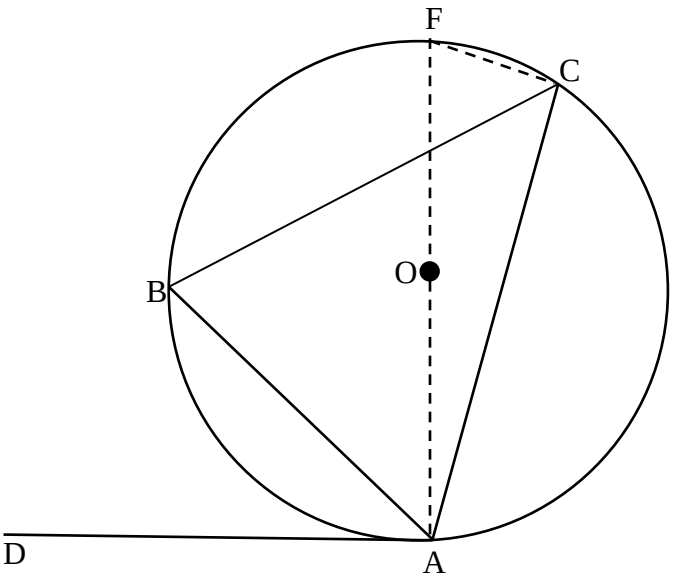
8.1	...centre / ...*middelpunt*		✓ answer / *antwoord* (1)
8.2.1	$B\hat{C}D = 90^o$	[∠ in semi *Θ* **OR** diameter subtends right angle **OR** *∠ in semi Θ **OF** deursnee onderspan rete hoek **OF*** $\angle \text{ in } \frac{1}{2}\Theta$ / *∠in halwe Θ **OF*** $\angle \text{ in } \frac{1}{2}\Theta$]	✓ $B\hat{C}D = 90^o$ ✓R (2)
8.2.2	$\hat{A}_1 = 42^o$	[∠'s opposite equal radii / ∠e teenoor gelyke radii]	✓ $\hat{A}_1 = 42^o$ ✓R (2)
8.2.3	$\hat{O}_2 = 84^o$	[∠ at centre = 2× ∠ at circumference / *Midpts∠ = 2 × Omtrek∠*]	✓ $\hat{O}_2 = 84^o$ ✓R (2)
8.2.4	$\hat{C}_4 = 25^o$	[tan chord theorem / *∠ tussen raaklyn en koord]*	✓ $\hat{C}_4 = 25^o$ ✓R (2) **[9]**

QUESTION/*VRAAG* 9



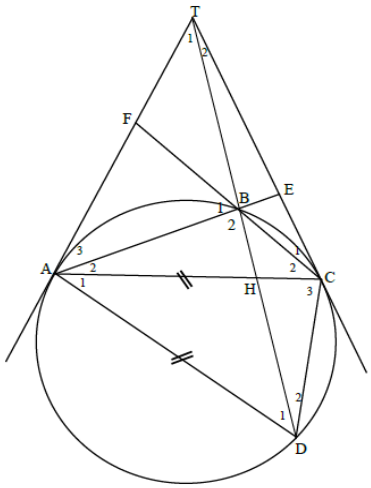
9.1	$DA.OD = OD(OD + OA)$ $= OD^2 + OD.OA$	✓ $OD(OD + OA)$ (1)
9.2	In ΔDA and ΔDCO $\hat{D} = \hat{D}$ [common / *gemeenskaplik*] $\hat{C}_2 = 90^o$ [line from centre to midpt of chord / *Midpt. Θ ; Midpt. koord*] / *lyn van middel tot midpunt van koord* $\hat{C}_2 = \hat{A}$ $\hat{B} = \hat{O}_3$ [3rd $\angle^s$ of Δ / *3e $\angle^e$ van Δ*] $\therefore \Delta DAB \,\|\|\| \,\Delta DCO$ $[\angle\angle\angle]$ $\therefore \frac{DA}{DC} = \frac{AB}{CO} = \frac{DB}{DO}$ $DA.DO = DC.DB$ $OD^2 + OD.OA = DC.2DC$ $\therefore OD^2 + OD.OA = 2DC^2$	✓ S $\hat{D} = \hat{D}$ ✓ S $\hat{C}_2 = 90^o$ ✓R ✓ S $\hat{C}_2 = \hat{A}$ ✓ S $\hat{B} = \hat{O}_3$ ✓ $\frac{DA}{DC} = \frac{AB}{CO} = \frac{DB}{DO}$ ✓ $OD^2 + OD.OA = DC.2DC$ (7) **[8]**

QUESTION/*VRAAG* 10



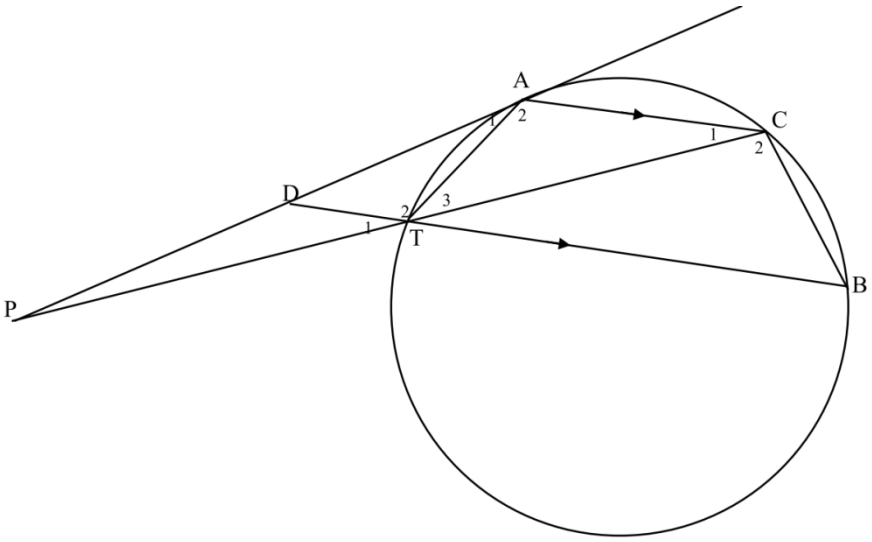
NB No construction: Breakdown 0/5

10. 1	***Construction***: Draw diameter AOF and join FC ***Konstruksie:*** *Teken middellyn AOF en verbind FC*		✓ Construction **on sketch**/ *Konstruksie* ***op skets*** ✓ S ✓ R
	$D\hat{A}F = 90^o$	[tan ⊥ radius / tan ⊥ diameter *raaklyn ⊥ radius*]	✓ R
	$F\hat{C}A = 90^o$	[∠ in semi Θ **OR** diameter subtends right angle **OR** $\angle$ in $\frac{1}{2}\Theta$ / ∠*in halwe* Θ ***OF*** $\angle$ in $\frac{1}{2}\Theta$]	✓ R
	$F\hat{A}B = F\hat{C}B$ $D\hat{A}B = B\hat{C}A$	[∠s in the same seg / ∠^{e} *in dies.* Θ *segm.*]	(5)



10.2.1	$\hat{D} = \hat{C}_3$	[∠s opp equal sides / $\angle^{e}$ *teenoor gelyke sye*]	✓ S/R
	$\hat{D} = \hat{B}_1$	[exterior angle of a cyclic quad/ *buite ∠ van kvh*]	✓ S ✓ R
	$\hat{C}_3 = \hat{B}_2$	[∠s in the same segment / $\angle^{e}$ *in dies. Θ segm.*]	✓ S R
	$\hat{B}_1 = \hat{B}_2$		(5)
10.2.2	$\hat{C}_1 + \hat{C}_2 = \hat{D}$	[tan chord theorem /*∠tussen raaklyn en koord*]	✓ S/R
	but $\hat{D} = \hat{B}_2$	[proved]	✓ S
	$\therefore \hat{B}_2 = \hat{C}_1 + \hat{C}_2$		✓ S
	$\therefore$ BECH is a cyclic quad is 'n koordevierh	[exterior ∠ = int opp∠ **OR** converse of ext ∠ of cyclic quad / *buite ∠ van vierhoek = teenoorst. binne ∠*]	✓ R (4)
10.2.3	$\hat{B}_2 = \hat{A}_3 + \hat{T}_1$	[exterior ∠ of a triangle/ *buite ∠ van Δ*]	✓ S/R
	but $\hat{B}_2 = \hat{C}_1 + \hat{C}_2$	[proved/ *bewys*]	
	and AT = CT	[tangents from the same point / *raaklyne vanuit dies. punt*]	✓ S/R ✓ S
	$\therefore \hat{C}_1 + \hat{C}_2 = \hat{A}_2 + \hat{A}_3$	[∠s opp equal sides / $\angle^{e}$ *teenoor gelyke sye*]	✓ S
	$\hat{A}_2 + \hat{A}_3 = \hat{A}_3 + \hat{T}_1$		
	$\therefore \hat{A}_2 = \hat{T}_1$	[∠ between line & chord **OR** converse of tan chord theorem / *∠ tussen lyn en koord = ∠ in teenoorst. Θ segm.*]	✓ R
	$\therefore$ CA is a tangent to circle ABT		(5) **[19]**

QUESTION/*VRAAG* 11



11.1	In ΔPAT and ΔPCA		
	$\hat{P} = \hat{P}$	[common / *gemeenskaplik*]	
	$\hat{A}_1 = \hat{C}_1$	[tan chord theorem / $\angle$ *tussen raaklyn en koord*]	✓ S/R
	$\hat{T}_1 + \hat{T}_2 = \hat{A}_1 + \hat{A}_2$	[$\angle$ sum in Δ **OR** sum of $\angle$s in Δ **OR** Int $\angle$s Δ / $\angle^e$ *van* Δ]	✓ S/R
	$\therefore \Delta PAT \text{ ||| } \Delta PCA$	[$\angle\angle\angle$ **OR/** ***OF*** equiangular / *gelykhoeking* /$\angle\angle\angle$]	✓ R (3)

11.2.1	$\frac{PA}{PC} = \frac{AT}{CA} = \frac{PT}{PA}$ $PA^2 = PT.PC$ $6^2 = x(x+5)$ $36 = x^2 + 5x$ $x^2 + 5x - 36 = 0$ $(x+9)(x-4) = 0$ $\therefore x = -9$ or / *of* $x = 4$ $\therefore PT = 4$	$\Delta PAT \text{ ||| } \Delta PCA$	✓ S/R ✓ S ✓ substitution / *vervanging* ✓ factors / *faktore* (4)
11.2.2	In ΔAPC $\frac{PD}{PA} = \frac{PT}{PC}$ $\frac{PD}{6} = \frac{4}{9}$ $PD = \frac{(6)(4)}{9}$ $= \frac{8}{3} / 2{,}67$	[line || one side of Δ **OR/*OF*** prop theorem; DT || AC / *lyn* || *een sy van* Δ]	✓ S ✓ R ✓ $\frac{PD}{6} = \frac{4}{9}$ ✓ answer /*antwoord* (4) **[11]**

TOTAL/*TOTAAL* [150]