GAUTENG DEPARTMENT OF EDUCATION

PREPARATORY EXAMINATION

2017

10612

MATHEMATICS

SECOND PAPER

TIME: **3 hours**

MARKS: **150**

17 pages and 1 information sheet

MATHEMATICS: Paper 2
1061E

10612E

X10

3

P.T.O.

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(Second Paper)

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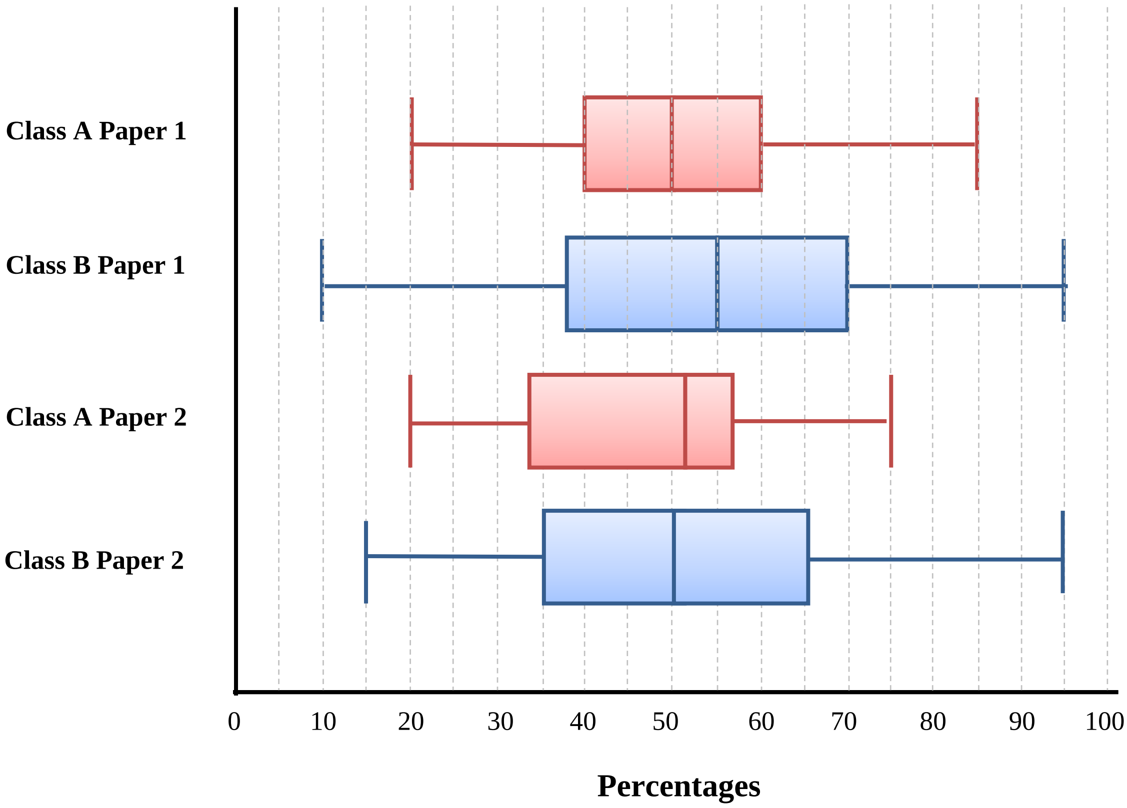
INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 11 questions.
2. Answer ALL the questions in the ANSWER BOOK provided.
3. Clearly show ALL calculations, diagrams, graphs et cetera that you used to determine the answers.
4. Answers only will NOT necessarily be awarded full marks.
5. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
6. If necessary, round-off answers to TWO decimal places, unless stated otherwise.
7. Diagrams are NOT necessarily drawn to scale.
8. An INFORMATION SHEET with formulae is included at the end of the question paper.
9. Write neatly and legibly.

QUESTION 1

1.1 The box and whisker plot below summarises the results obtained in Paper 1 and Paper 2 by Class A and Class B in a certain school in the 2016 Preparatory Examination.



1.1.1 Write down the median mark for **Class B Paper 1**. (1)

1.1.2 Write down the interquartile range for **Class A Paper 1**. (2)

1.1.3 Write down the range of percentages that were achieved by 75% of the learners in **Class B Paper 2**. (1)

1.1.4 Comment on the skewness of the data for **Class A Paper 2**. (1)

1.2 The table below represents **cumulative frequencies** of exam percentages. The marks are for a group of 30 learners.

Class interval	**Cumulative Frequency**
$10 \leq x \leq 19$	1
$20 \leq x \leq 29$	2
$30 \leq x \leq 39$	4
$40 \leq x \leq 49$	6
$50 \leq x \leq 59$	14
$60 \leq x \leq 69$	19
$70 \leq x \leq 79$	23
$80 \leq x \leq 89$	26
$90 \leq x \leq 100$	30

1.2.1 Sketch the ogive (cumulative frequency graph) from the table given above. (3)

1.2.2 What percentage of learners achieved lower than 70%? (1)

1.2.3 How many learners achieved marks from 80 – 89%? (1)

1.2.4 Write down a possible mark for a learner who achieved the third lowest mark. (1)

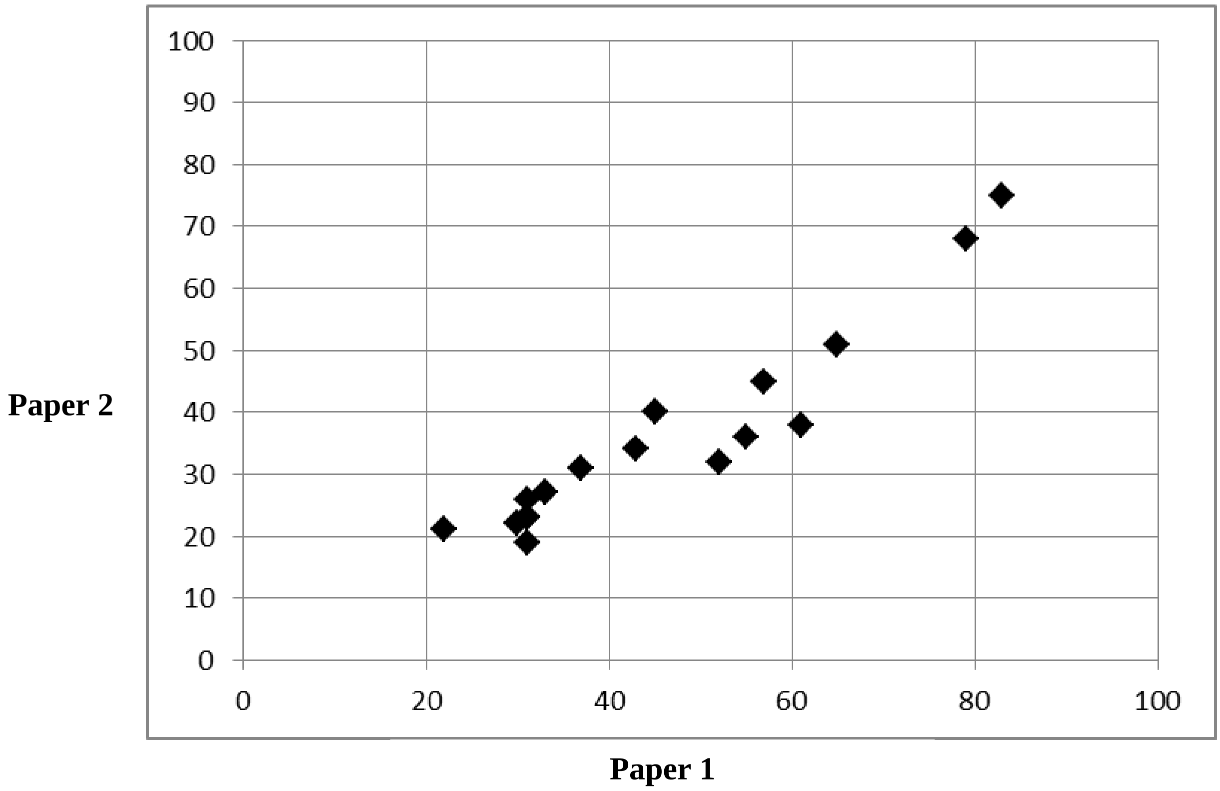
1.2.5 Use the graph to write down the SMALLEST possible range of marks achieved by the learners. (1)

[12]

QUESTION 2

The marks obtained as a percentage by 17 learners in Paper 1 and Paper 2 in the 2016 Preparatory Examination is given in the table below. The same information is represented in the scatter plot.

Paper 1	22	83	55	52	65	31	61	31	31	57	30	33	45	31	43	37	79
Paper 2	21	75	36	32	51	19	38	23	26	45	22	27	40	23	34	31	68



2.1 Calculate the equation of the least squares regression line for the data. (3)

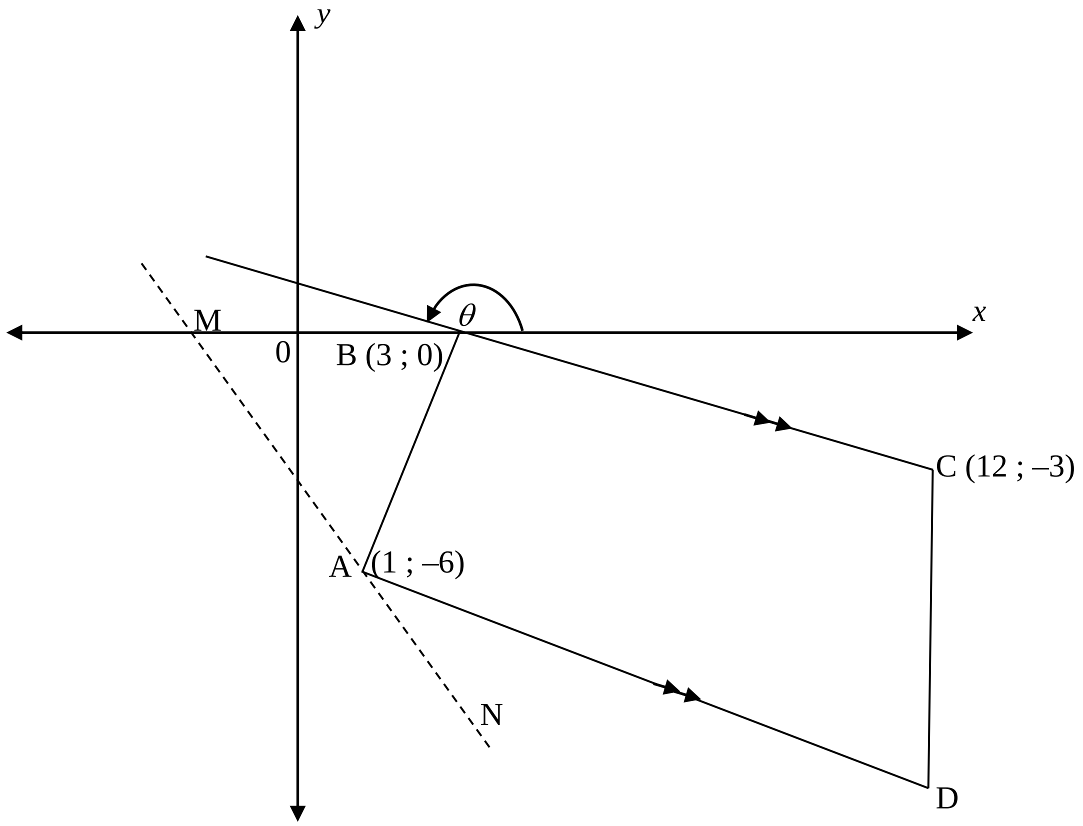
2.2 Calculate the correlation coefficient of the data. (1)

2.3 Sketch the least squares regression line on the scatter plot provided in the ANSWER BOOK. (2)

2.4 A learner achieved 98 % in Paper 1 and 79.4% in Paper 2.
Are these marks valid and reliable? Substantiate your answer. (2)
[8]

QUESTION 3

3.1 In the diagram below, $A(1\,;-6)$, $B(3\,;\,0)$, $C(12\,;-3)$ and D are the vertices of a trapezium having $AD \parallel BC$. The inclination of line BC is θ.

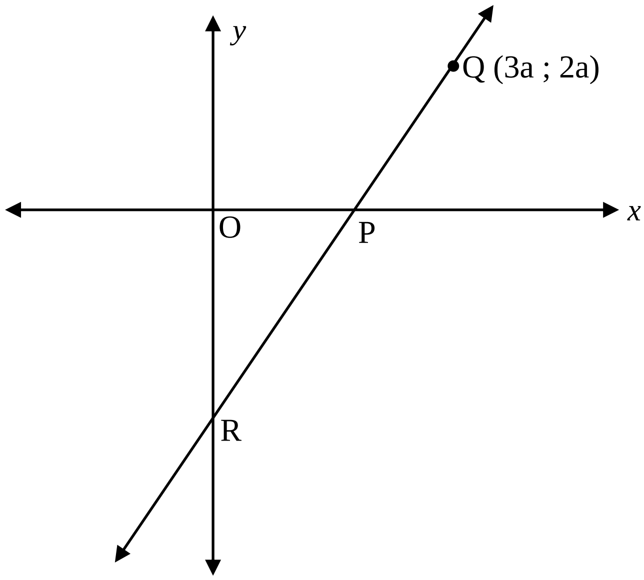


3.1.1 Calculate the size of θ. (4)

3.1.2 Prove that $AD \perp AB$. (3)

3.1.3 A straight line, MAN, only passes through vertex A of trapezium ABCD. An angle of 45° is formed between line MAN and line AD. Determine the equation of line MAN. (4)

3.2 A straight line with a gradient of 2, cuts the x-axis at P, the y-axis at R and passes through Q (3a ; 2a)



3.2.1 Write down the equation of QR, in terms of a. (2)

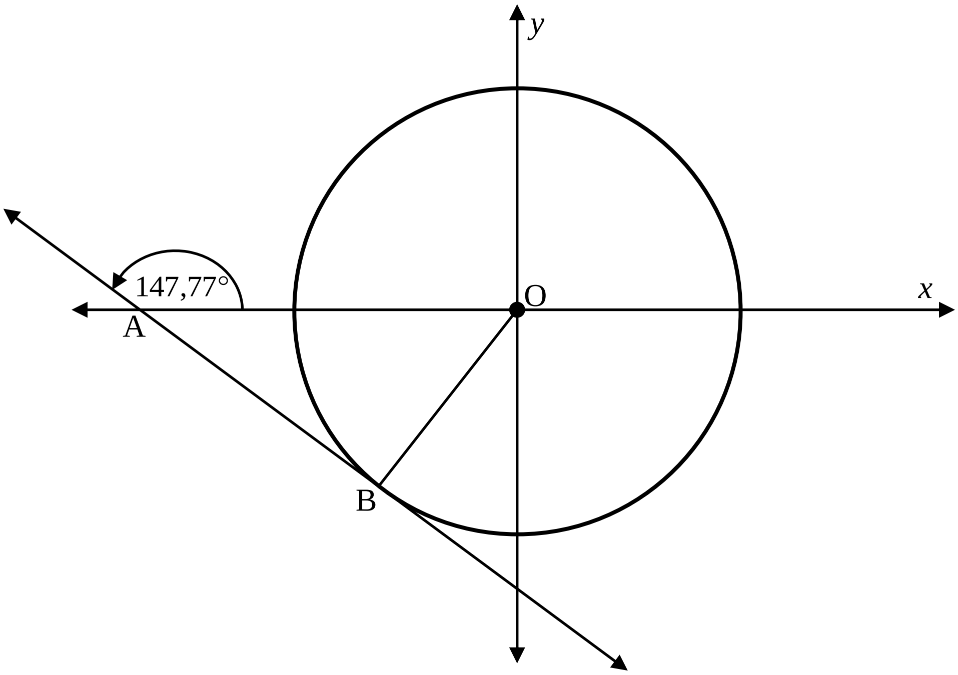
3.2.2 Hence, determine the area of ΔPOR in terms of a. (4)

3.2.3 Calculate the value of a, if the points D (–3 ; –14), Q and E (3 ; –2) are collinear. (3)

[20]

QUESTION 4

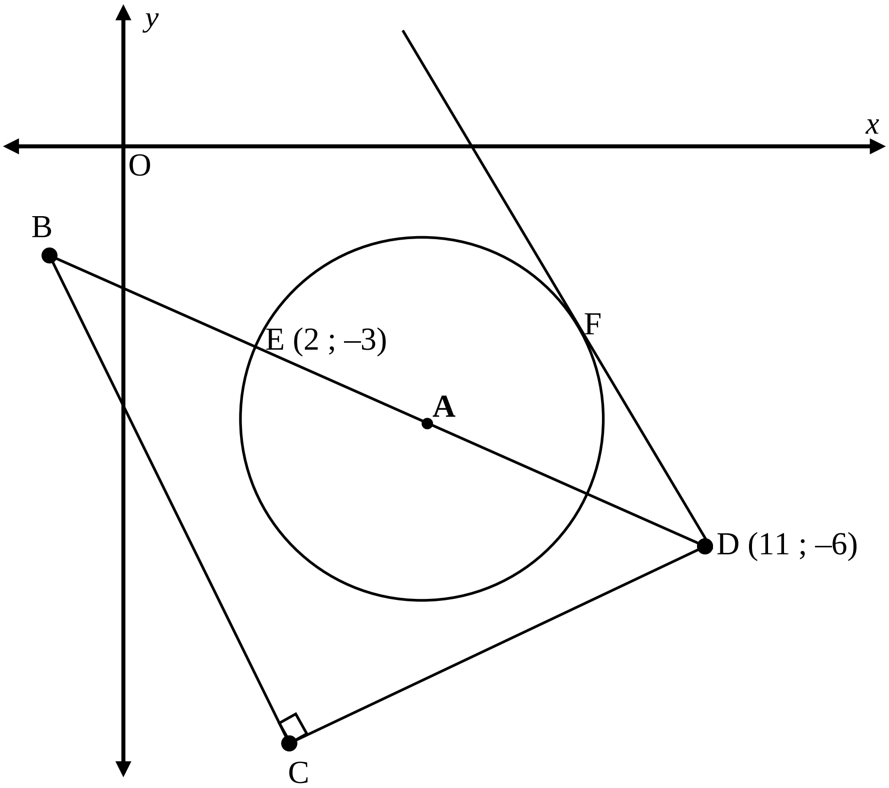
4.1 The figure below shows circle O with a radius of 8 units. Tangent AB touches the circle at B and cuts the negative x-axis at A such that the inclination of the tangent is 147,77°.



4.1.1 Write down the equation of the circle. (1)

4.1.2 Calculate the coordinates of A. (4)

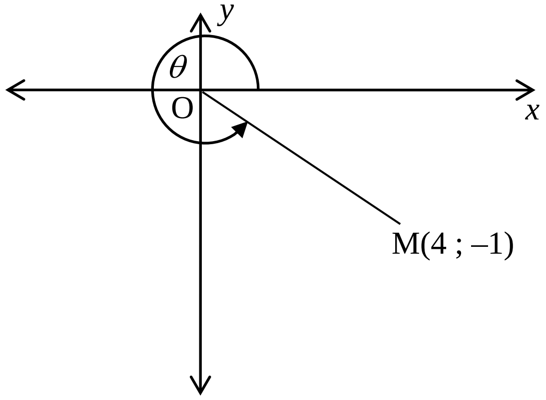
4.2 A is the centre of a circle having equation $x^2 - 10x + y^2 + 8y + 31 = 0$.
E (2 ; –3), a point on the circle, is also the midpoint of AB.
BEA is produced to D (11 ; –6). C is a point such that $BC \perp DC$.



4.2.1	Write the equation of the circle in the form $(x-a)^2 + (y-b)^2 = r^2$.	(3)
4.2.2	Write down the coordinates of A, the centre of the circle.	(1)
4.2.3	Write down the length of the radius of the circle in surd form.	(1)
4.2.4	Calculate the coordinates of B.	(3)
4.2.5	If $x = k$ is a tangent to circle A, determine the value(s) of k. (Leave your answer in surd form.)	(3)
4.2.6	DF is a tangent to the circle at F. Calculate the length of DF in surd form.	(4)
		[20]

QUESTION 5

In the diagram below, M (4 ; –1) is given and $X\hat{O}M = \theta$.



5.1 Calculate the size of θ. (3)

5.2 Simplify, the following expression to a single trigonometric function, **without the use of a calculator**:

$$\frac{2\cos(90^\circ - x)}{\sin(180^\circ - 2x)} \times \frac{\cos(60^\circ - x)\cos x - \sin(60^\circ - x)\sin x}{\tan(-x)}$$ (7)

5.3 If $\cos 18^\circ = k$, express the following in terms of k, **without the use of a calculator.**

5.3.1 $\sin 108^\circ$ (2)

5.3.2 $\cos(-36^\circ)$ (3)

5.4 Solve for x if:
$2\sin x\cos x + 2\sin x + \cos^2 x + \cos x = 0$ for $x \in [-180;180^\circ]$ (6)

5.5 Given that $\tan\theta = p$ in any right-angled triangle:

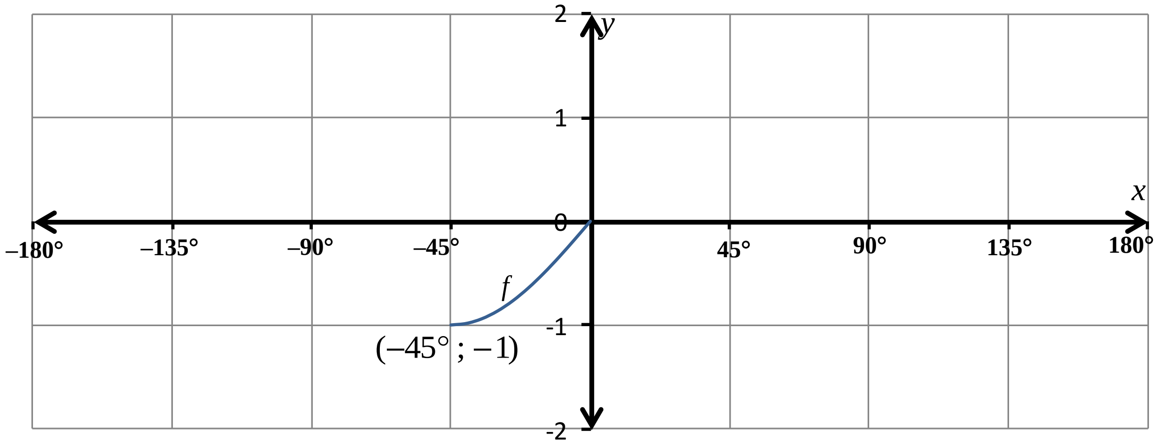
5.5.1 Show that $\sin 2\theta = \dfrac{2p}{p^2+1}$. (2)

5.5.2 Hence, or otherwise, calculate the maximum value of $\dfrac{(p+1)^2}{p^2+1}$. (3)

[26]

QUESTION 6

The sketch below shows a part of the graph of f where $f(x) = a\sin bx$. A turning point of f is at $(-45°\,;-1)$.



6.1 Write down the values of a and b. (3)

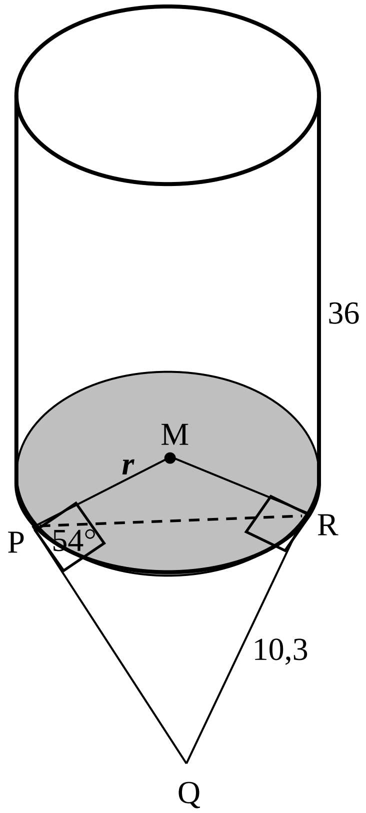
6.2 Use the grid provided in your ANSWER BOOK to complete the graph of f for $x \in [-180°; 180°]$. (3)

6.3 Hence, determine the values of x for which $f^{/}(x) < 0$ in the interval $x \in [0°; 180°]$. (2)

[8]

QUESTION 7

The diagram below represents a right cylindrical silo. M is the centre of the circular base.
PQ and RQ are tangents to the base at P and R. M, P, Q and R lie in the same horizontal plane.
The vertical height of the cylinder is 36 metres.
QR = 10,3 m and $R\hat{P}Q = 54°$.



7.1 Determine the size of $\hat{Q}$. (2)

7.2 Calculate the length of PR. (2)

7.3 Calculate the volume of the cylindrical section of the silo. (5)
[9]

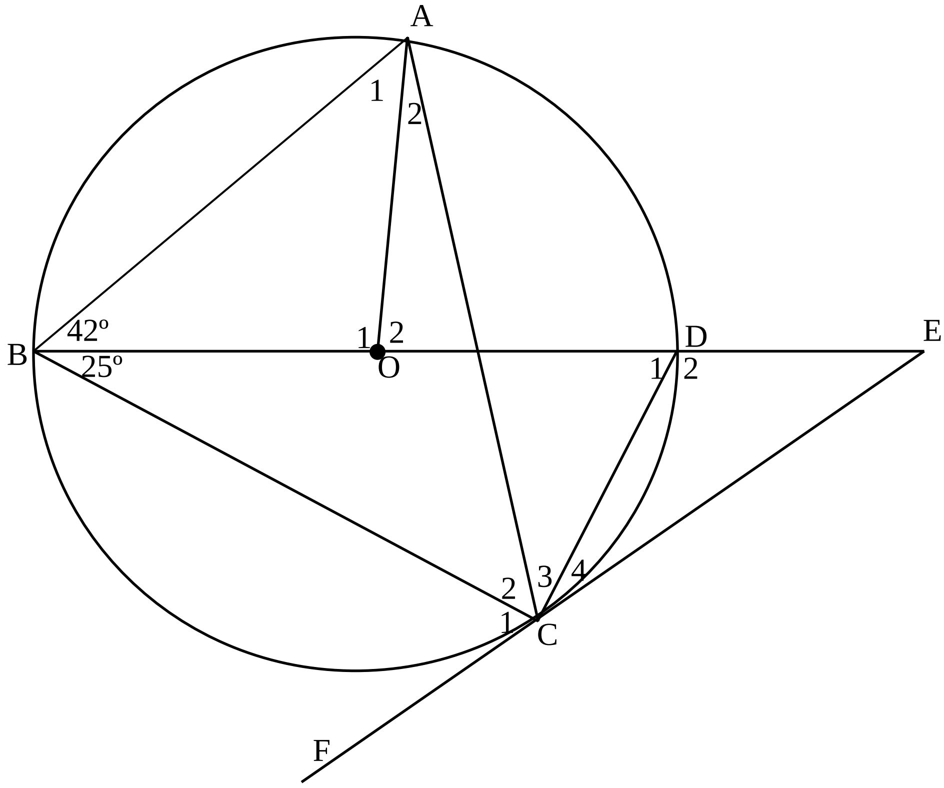
GIVE REASONS FOR ALL STATEMENTS AND CALCULATIONS IN QUESTIONS 8, 9, 10 AND 11.

QUESTION 8

8.1 Complete the statement so that it is TRUE.

The angle which an arc of a circle subtends at the … of a circle is twice the angle it subtends at the circumference of the circle. (1)

8.2 In the diagram below, the circle with centre O passes through A, B, C and D such that BOD is a diameter. BD is extended to E such that FCE is a tangent to the circle at C. $A\hat{B}E = 42^0$ and $D\hat{B}C = 25^0$.



Calculate:

8.2.1 $B\hat{C}D$ (2)

8.2.2 $\hat{A}_1$ (2)

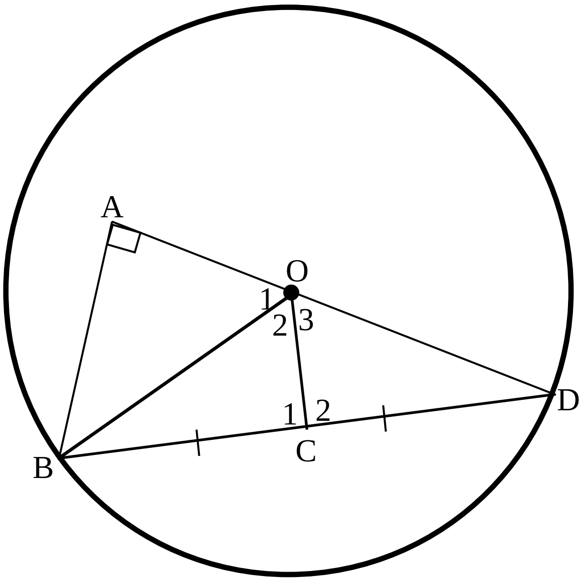
8.2.3 $\hat{O}_2$ (2)

8.2.4 $\hat{C}_4$ (2)

[9]

QUESTION 9

In the diagram below, O is the centre of the circle. C is the midpoint of chord BD. Point A lies within the circle such that $BA \perp AOD$.



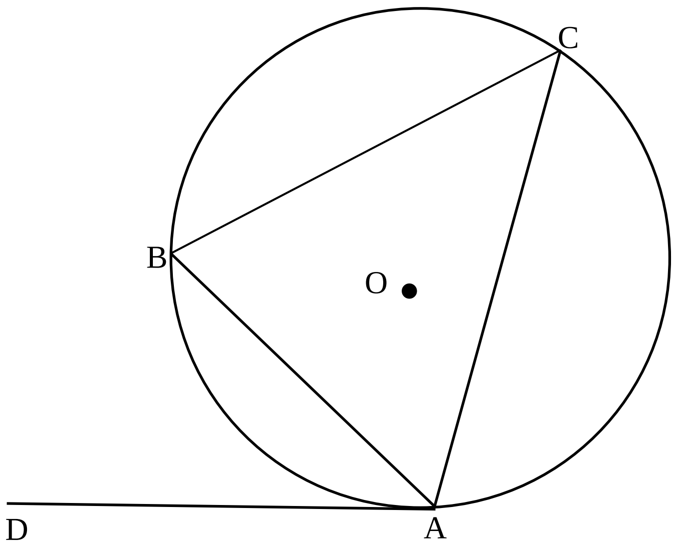
9.1 Show that $DA.OD = OD^2 + OD.OA$ (1)

9.2 Prove that $2DC^2 = OD^2 + OD.OA$ (7)

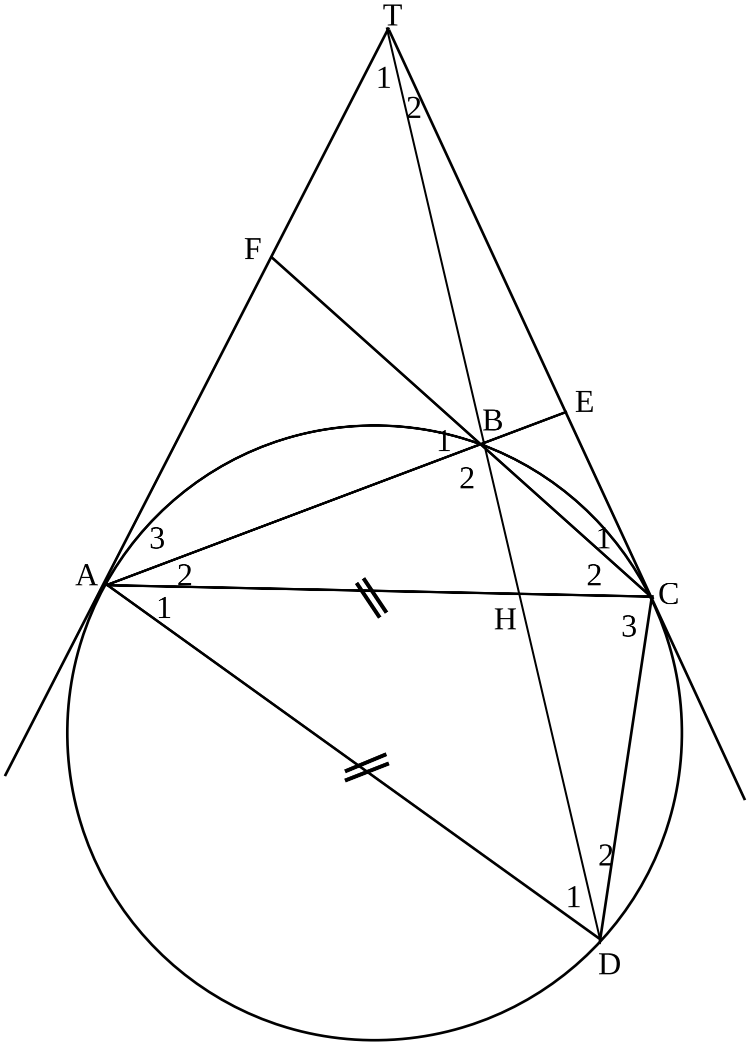
[8]

QUESTION 10

10.1 Use the given diagram to prove the theorem which states that if DA is a tangent to the circle centre O, then $D\hat{A}B = B\hat{C}A$. (5)



10.2 In the diagram below, ABCD is a cyclic quadrilateral with AC = AD. Tangents AT and CT touch the circle at A and C respectively. FBC, ABE, AHC and DBT are straight lines.



Prove:

10.2.1 $\hat{B}_1 = \hat{B}_2$. (5)

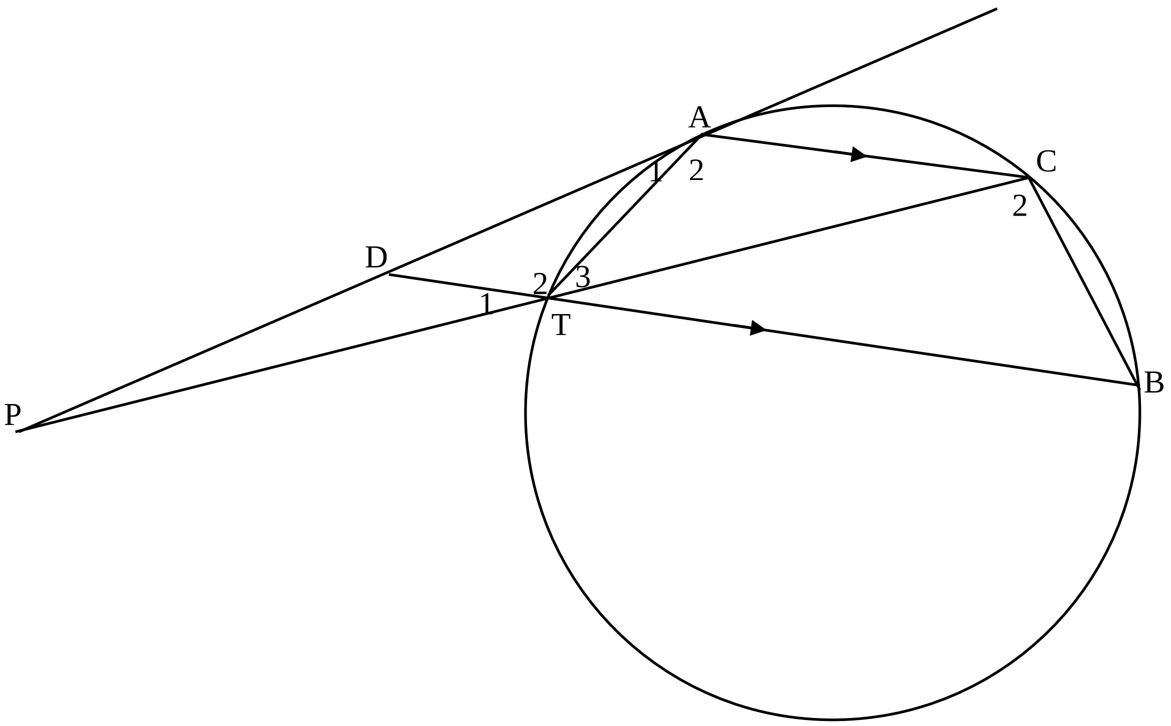
10.2.2 BECH is a cyclic quadrilateral. (4)

10.2.3 CA is a tangent to the circle passing through points A, B and T. (5)

[19]

QUESTION 11

In the diagram below, ACBT is a cyclic quadrilateral having AC || TB. CT is produced to P such that tangent PA meets the circle at A. BT produced meets PA at D.



11.1 Prove that $\Delta PAT \parallel\!\mid \Delta PCA$ (3)

11.2 If PA = 6, TC = 5 and PT = x,

11.2.1 show that PT = 4. (4)

11.2.2 calculate the length of PD. (4)
[11]

TOTAL: 150

END

INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $\quad A = P(1-ni)$ $\quad A = P(1-i)^n$ $\quad A = P(1+i)^n$

$T_n = a + (n-1)d$ $\quad S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$T_n = ar^{n-1}$ $\quad S_n = \frac{a\left(r^n - 1\right)}{r-1}$; $r \neq 1$ $\quad S_\infty = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x\left[(1+i)^n - 1\right]}{i}$ $\quad P = \frac{x[1-(1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad \mathrm{M}\left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad y - y_1 = m(x - x_1)$ $\quad m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad m = \tan\theta$

$$(x-a)^2 + (y-b)^2 = r^2$$

InΔABC: $\quad \frac{a}{\sin \mathrm{A}} = \frac{b}{\sin \mathrm{B}} = \frac{c}{\sin \mathrm{C}}$

$a^2 = b^2 + c^2 - 2bc.\cos A$

$area\ \Delta\mathrm{ABC} = \frac{1}{2}\ ab.\sin\mathrm{C}$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\quad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\quad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$$

$$\sin 2\alpha = 2\sin\alpha.\cos\alpha$$

$\bar{x} = \frac{\sum x}{n}$ $\quad \sigma^2 = \frac{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}{n}$

$\mathrm{P(A)} = \frac{n(\mathrm{A})}{n(\mathrm{S})}$ $\quad$ P(A or B) = P(A) + P(B) – P(A and B)

$\hat{y} = a + bx$ $\quad b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$