CAPE WINELANDS

EDUCATION DISTRICT

SCHOOL NAME HERE

GRADE 12

MATHEMATICS
PAPER 2

SEPTEMBER 2017

MARKS: 150

TIME: 3 hours

This paper consists of 10 pages, 1 Answer book and an information sheet.

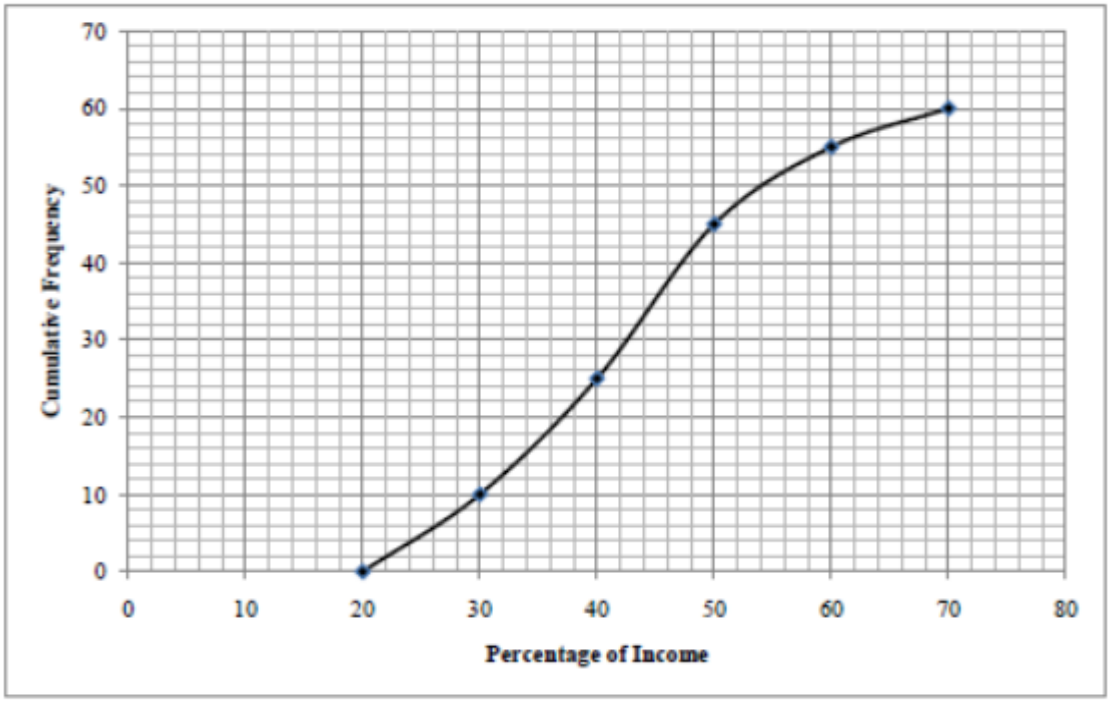
INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 10 questions.
2. Answer ALL the questions in the SPECIAL ANSWER BOOK provided. Write your name and class in the space provided and hand it in.
3. Number the answers correctly according to the numbering system used in this question paper.
4. Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining your answers.
5. Answers only will NOT necessarily be awarded full marks.
6. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
7. If necessary, round off answers to TWO decimal places, unless stated otherwise.
8. Diagrams are NOT necessarily drawn to scale.
10. An information sheet with formulae is included at the end of the question paper.
11. Write neatly and legibly.

QUESTION 1

The following ogive (cumulative frequency graph) shows what percentage of their income is spent by Chinese people on their children's education



1.1 Complete the cumulative frequency table (both columns) for the data in the ANSWER BOOK. (3)

1.2 Use the graph to estimate:

1.2.1 The number of people who spent more than 50% of their income on education (2)

1.2.2 The median of the data (2)

1.3 Write down the modal class of the data. (1)

[8]

QUESTION 2

The data below shows the marks obtained by ten Grade 12 learners from two different Mathematics classes.

Class A	16	36	20	38	40	30	35	22	40	24
Class B	45	70	44	56	60	48	75	60	63	38

2.1 Make use of the grid provided in the ANSWER BOOK to draw a scatter plot for the data. (2)

2.2 Calculate the equation for the least squares regression line for this data. (3)

2.3 Draw the least squares regression line for the data on the scatter plot drawn in QUESTION 2.1 (2)

2.4 Describe the relationship between the classes. (2)

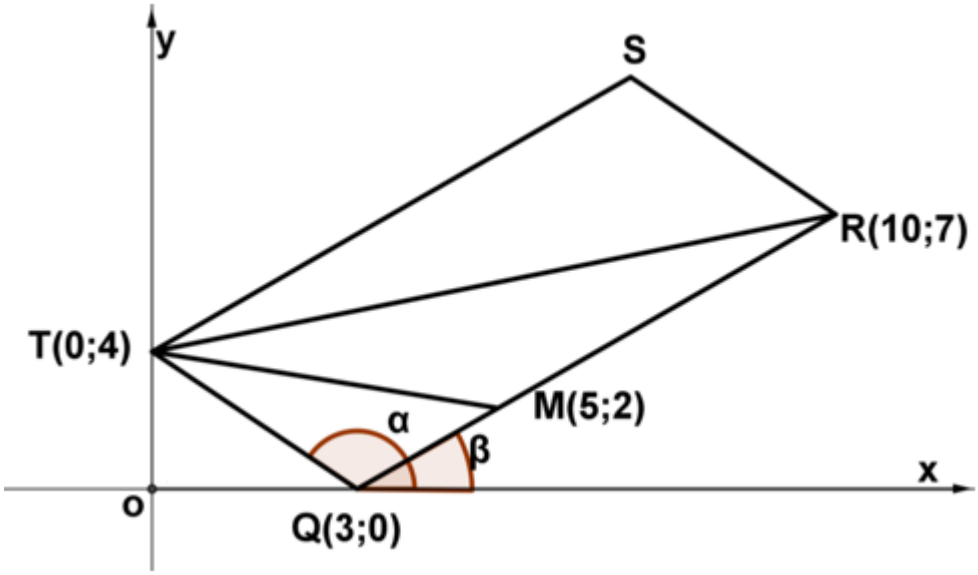
2.5 Calculate the mean and standard deviation for Class B (4)

2.6 How many scores in Class B fall within one standard deviation of the mean? (2)

[15]

QUESTION 3

In the diagram, Q (3; 0), R (10; 7), S and T (0; 4) are vertices of parallelogram QRST. From T a straight line is drawn to meet QR at M (5;2). The angles of inclination of TQ and RQ are α and β respectively.



3.1 Calculate the gradient of TQ (2)

3.2 Calculate the length of RQ. Leave your answer in surd form. (2)

3.3 F (k ; -8) is a point on the Cartesian plane such that T, Q and F are collinear. Calculate the value of k. (4)

3.4 Calculate the coordinates of S. (2)

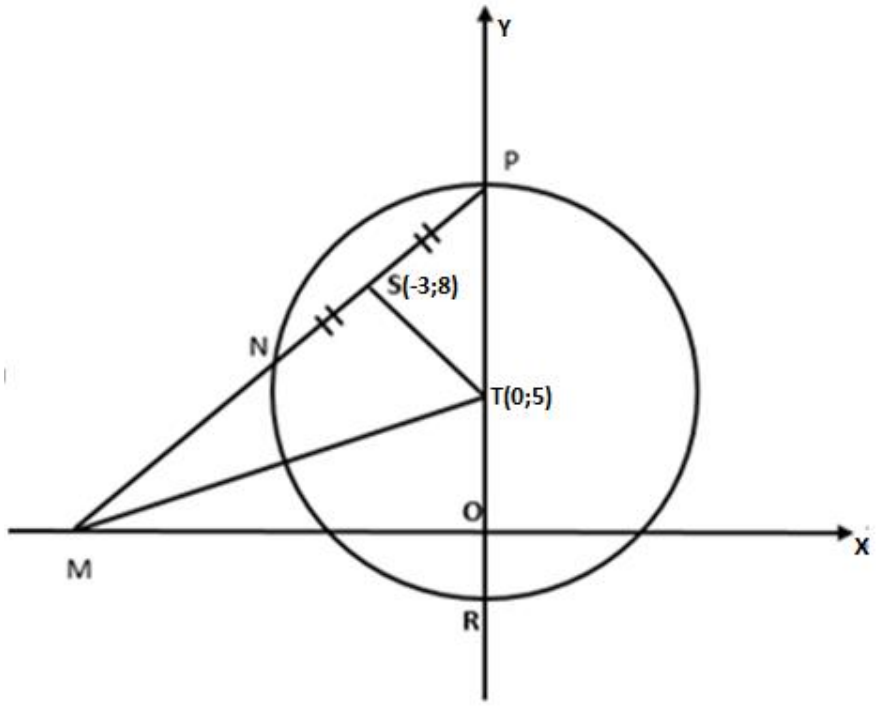
3.5 Calculate the size of $T\hat{S}R$ (6)

3.6 Calculate, in the simplest form, the ratio of: $\frac{MQ}{RQ}$ (3)

[19]

QUESTION 4

In the diagram, the circle with centre T (0; 5), cuts the *y*-axis at P and R. The line through P and S (-3; 8) cuts the circle at N and the *x*-axis at M. NS = PS. MT is drawn.



4.1 Give a reason why TS ⊥ NP. (1)

4.2 Determine the equation of the line PM in the form $y = mx + c$ (5)

4.3 Determine the equations of the tangents to the circle that are parallel to the *x*-axis. (4)

4.4 Determine the length of MT (4)

4.5 Another circle is drawn through the points S, T and M. Determine, with reasons, the equation of this circle in the form $(x - a)^2 + (y - b)^2 = r^2$ (5)

[19]

QUESTION 5

NO CALCULATOR IS ALLOWED IN THIS QUESTION

5.1 If $4\tan A = 3$ and $3\sin B - 1 = 0$, use a sketch and determine the value of (5)

$5\sin A - 6\cos^2 B$, where $180° \leq A \leq 360°$ and $0° \leq B \leq 90°$.

5.2 If $\cos 24° = p$, determine the following in terms of *p*:

5.2.1 $\cos 336°$ (2)

5.2.2 $\cos 48°$ (3)

5.3 Prove that: $\tan A = \frac{1-\cos 2A}{\sin 2A}$ (4)

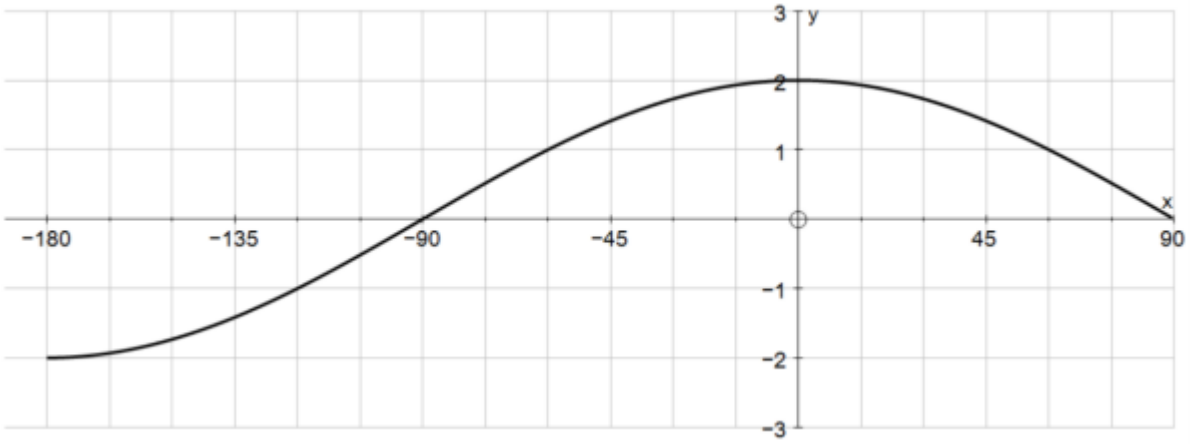
5.4 Simplify without using a calculator:

$$\sin 20° \cdot \cos 320° + \cos(-20°) \cdot \sin 400°$$ (5)

5.5 Determine the general solution of $2\cos^2\theta + 5\sin\theta + 1 = 0$ (7)
[26]

QUESTION 6

Below is the sketch of the graph of $f(x) = a\cos x$ for $x \in [-180°; 90°]$.



6.1 Write down the value of a. (1)

6.2 Sketch the graph of $g(x) = \sin 2x$ on the given axes for $x \in [-180°; 90°]$ in the ANSWERBOOK (3)

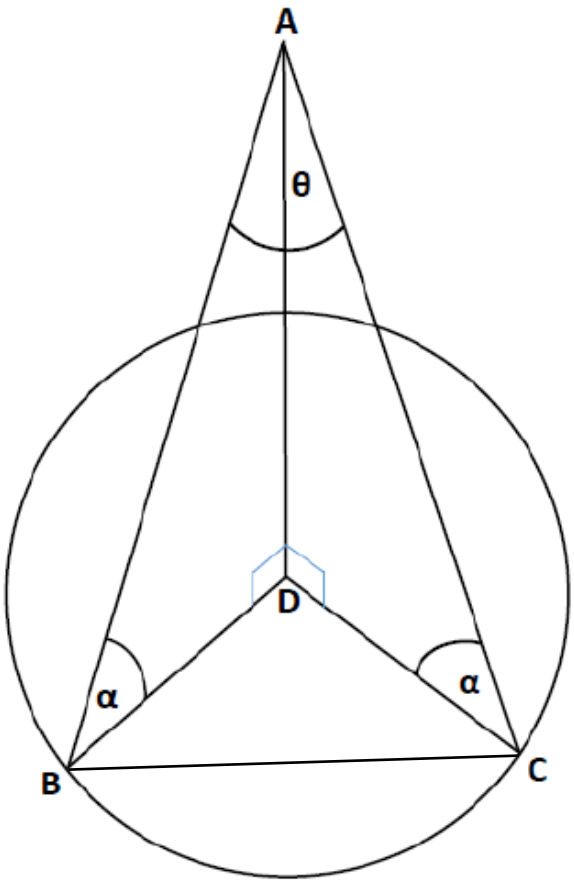
6.3 What is the period of g? (1)

6.4 Use your sketch and determine the value(s) of x for which $f(x) - g(x) = 0$ (2)

6.5 Calculate the maximum value of $3 - 10\sin x\cos x$ (3)
[10]

QUESTION 7

In the diagram B, C and D lie in the same horizontal plane. **D is the centre** of the circle with B and C on the circumference. α is the angle of elevation of A from B and the angle of elevation of A from C. $B\widehat{A}C = \theta$. The radius of the circle is r.



7.1 Write down the value of AB in terms of r and α (1)

7.2 Hence, show that: $BC = \frac{r\left(\sqrt{2(1-cos\theta)}\right)}{cos\alpha}$ (4)

7.3 If $BC = 100m,\ r = 50m$ and $\theta = 30°$, calculate the size of α (3)

[8]

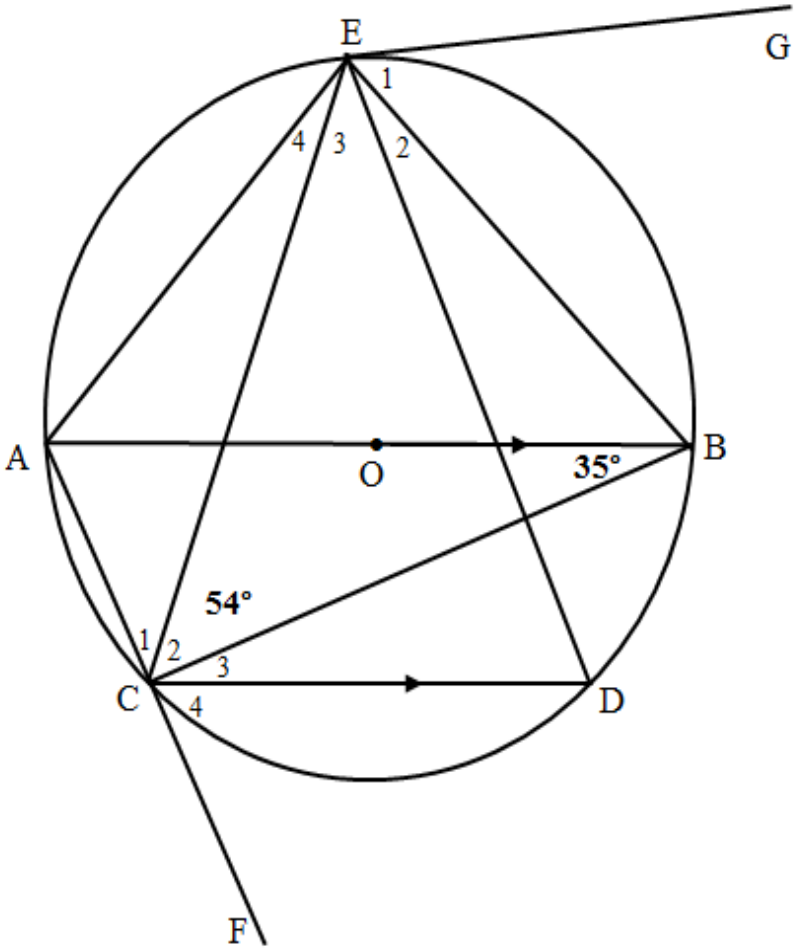
QUESTION 8

8.1 Complete the following theorems:

8.1.1 The angle in a semi-circle is equal to … (1)

8.1.2 The opposite angles of a cyclic quadrilateral are … (1)

8.2 O is the centre of the circle in the diagram with chord CD parallel to diameter AB. AC is produced to F and EG is a tangent to the circle. $A\hat{B}C = 35°$ and $\hat{C}_2 = 54°$



Calculate, WITH REASONS, the size of the following angles:

8.2.1 $\hat{E}_1$ (2)

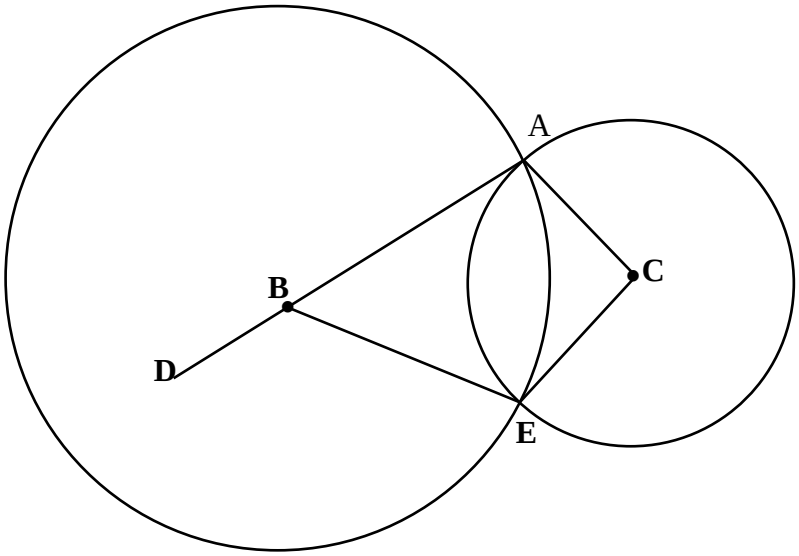
8.2.2 $\hat{C}_1$ (2)

8.2.3 $\hat{C}_3$ (2)

8.2.4 $A\hat{E}D$ (2)

8.2.5 $\hat{E}_3$ (3)

8.3 In the diagram below, two circles with centres at B and C intersect at A and E respectively. AB is produced to a point D. AB and BE are tangents to the smaller circle at A and E respectively.

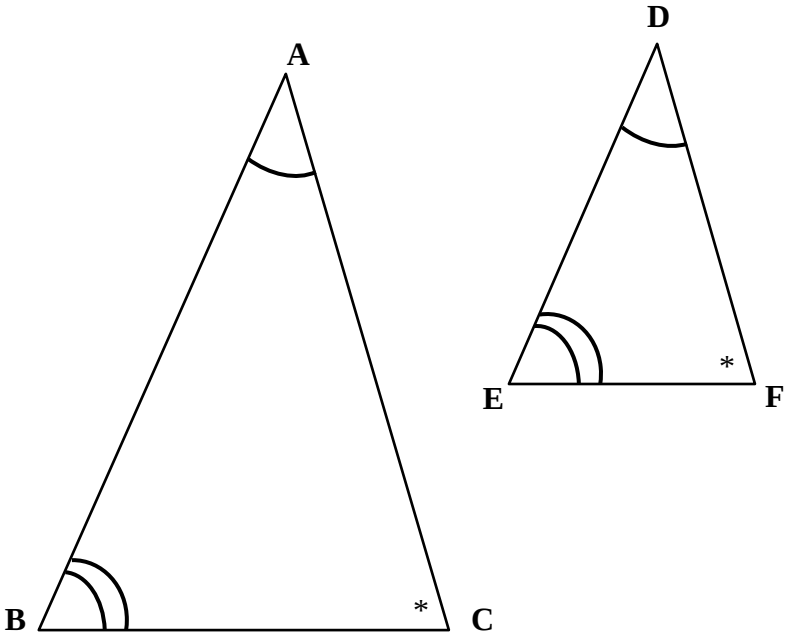


Prove that $D\hat{B}E = A\hat{C}E$. (5)

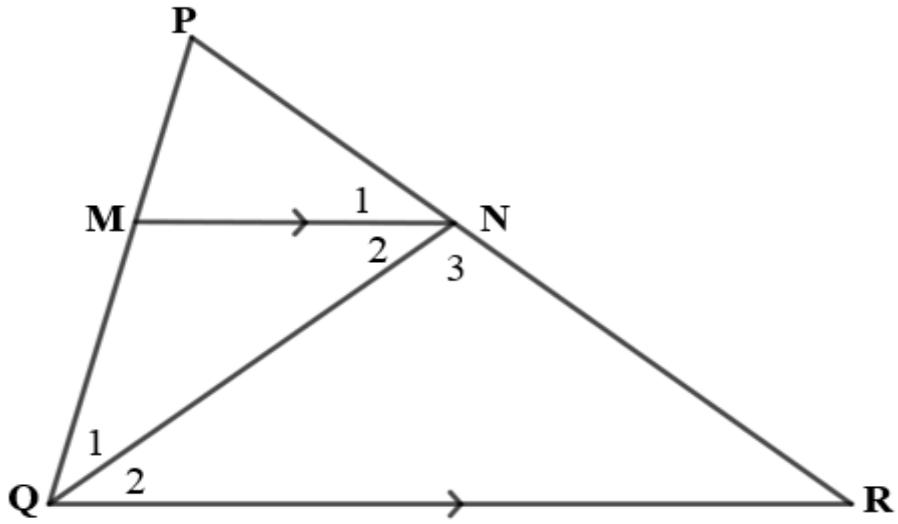
[18]

QUESTION 9

9.1 In the given diagram $\hat{A} = \hat{D}$, $\hat{B} = \hat{E}$ and $\hat{C} = \hat{F}$. Use the diagram in the ANSWER BOOK to prove the theorem that $\frac{AB}{DE} = \frac{AC}{DF}$ (7)



9.2 In the diagram below, ΔPQR has MN // QR. PQ = 5 cm, QR = 7,5 cm and QN bisects $P\widehat{Q}R$.



9.2.1 Prove that ΔQMN is isosceles. (3)

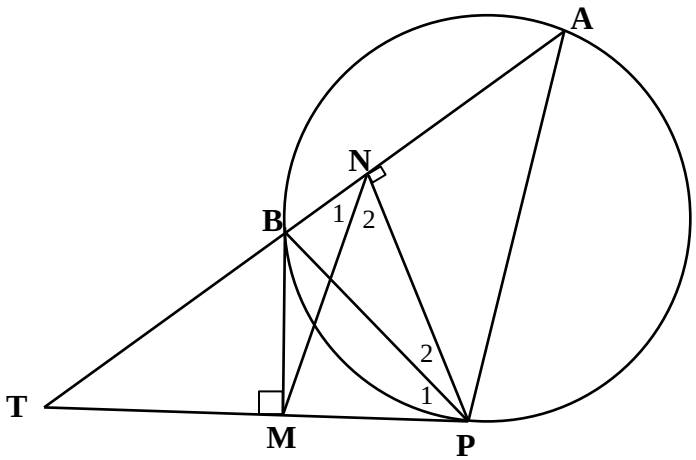
9.2.2 Prove that ΔPMN /// ΔPQR (4)

9.2.3 Hence, determine the value of $\frac{NP}{NR}$ (5)

[19]

QUESTION 10

In the diagram below, A, B and P are points on a circle. N is a point on AB such that AB ⊥ PN. AB produced meets the tangent PT at T. M is a point on PT such that BM ⊥ PT



10.1 Prove that BMPN is a cyclic quadrilateral. (2)

10.2 Prove that $\frac{TN}{AN} = \frac{TM}{MP}$ (6)

[8]

TOTAL: 150

INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1 + ni)$ $A = P(1 - ni)$ $A = P(1 - i)^n$ $A = P(1 + i)^n$

$T_n = a + (n-1)d$ $S_n = \frac{n}{2}[2a + (n-1)d]$

$T_n = ar^{n-1}$ $S_n = \frac{a(r^n - 1)}{r - 1}\ ; r \neq 1$ $S_\infty = \frac{a}{1 - r}\ ; -1 < r < 1$

$F = \frac{x[(1+i)^n - 1]}{i}$ $P = \frac{x[1 - (1+i)^{-n}]}{i}$

$f'(x) = \lim_{h \to 0}\left(\frac{f(x+h) - f(x)}{h}\right)$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $M(\frac{x_1 + x_2}{2}\ ;\ \frac{y_1 + y_2}{2})$

$y = mx + c$ $y - y_1 = m(x - x_1)$ $m = \frac{y_2 - y_1}{x_2 - x_1}$ $m = tan\theta$

$(x-a)^2 + (y-b)^2 = r^2$

In ΔABC: $\frac{a}{sinA} = \frac{b}{sinB} = \frac{c}{sinC}$

$a^2 = b^2 + c^2 - 2bc.cosA$

$Area\ \Delta ABC\ = \frac{1}{2}ab.sin$

$\sin(\alpha + \beta) = sin\alpha.cos\beta + cos\alpha.sin\beta$ $\sin(\alpha - \beta) = sin\alpha.cos\beta - cos\alpha.sin\beta$

$\cos(\alpha + \beta) = cos\alpha.cos\beta - sin\alpha.sin\beta$ $\cos(\alpha - \beta) = cos\alpha.cos\beta + sin\alpha.sin\beta$

$$cos2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$$

$sin2\alpha = 2sin\alpha.cos\alpha$

$\bar{x} = \frac{\sum fx}{n}$ $\sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $P(A\ or\ B) = P(A) + P(B) - P(A\ and\ B)$

$\hat{y} = a + bx$ $b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$