Province of the
EASTERN CAPE
EDUCATION

NATIONAL SENIOR CERTIFICATE
NASIONALE SENIOR SERTIFIKAAT

GRADE/*GRAAD* 12

SEPTEMBER 2020

MATHEMATICS P2/*WISKUNDE V2*
MARKING GUIDELINE/*NASIENRIGLYN*

MARKS/*PUNTE*: 150

This marking guideline consists of 13 pages.
Hierdie nasienriglyn bestaan uit 13 bladsye.

QUESTION 1/*VRAAG 1*

1.1	$a = -4{,}1536$ $b = 0{,}958$ $y = -4{,}1536 + 0{,}958x$ Answer Only: Full Marks	✓ $a = -4{,}1536$ ✓ $b = 0{,}958$ ✓ $y = -4{,}1536 + 0{,}958x$	(3)
1.2	$r = 0{,}98$	✓ $r = 0{,}98$	(1)
1.3	Very strong positive correlation/ *Baie sterk positiewe korrelasie*	✓answer / *antwoord*	(1)
1.4	$y = -4{,}1536 + 0{,}958(51)$ $y = 45\%$ Answer Only: Full Marks	✓substitution / *vervanging* ✓answer / *antwoord*	(2)
1.5	$\bar{x} = 60{,}8$ Standard deviation / *Standaardafwyking* = 17, 51 $(60{,}8 - 17{,}51\ ;\ 60{,}8 + 17{,}51)$ $(43{,}29\ ;\ 78{,}31)$ 6 learners / *leerders*	✓ Standard deviation/ *Standaardafwyking* = 17, 51 ✓$(43{,}29\ ;\ 78{,}31)$ ✓6 learners / *leerders*	(3)
			[10]

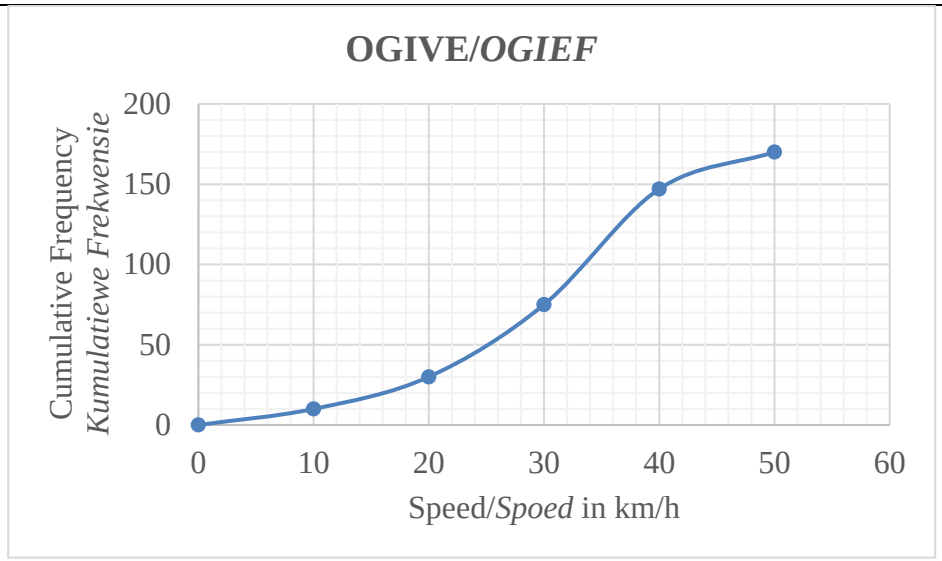
QUESTION 2/*VRAAG 2*

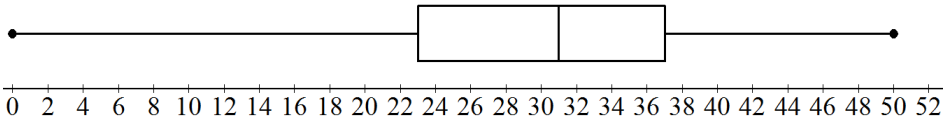
2.1

Speed/*Spoed* (km/h)	**Frequency *Frekwensie* (f)**	**Cumulative Frequency *Kumulatiewe Frekwensie***
$0 < x \leq 10$	10	10
$10 < x \leq 20$	**20**	30
$20 < x \leq 30$	45	**75**
$30 < x \leq 40$	72	**147**
$40 < x \leq 50$	**23**	170

✓freq column / *frek. kolom*

✓cum freq column *kum frek kolom* (2)

2.2		✓starting pt / *beginpunt* ✓end point *eindpunt* ✓shape *vorm*	(3)

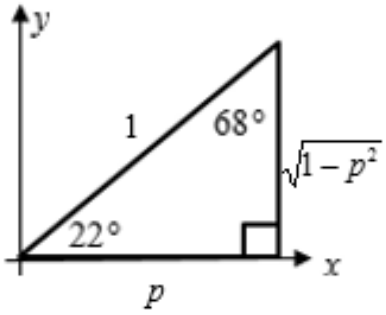
2.3	$Q_1 = 23$ (accept / *aanvaar* 22 – 24) Median / *Mediaan* = 31 (accept / *aanvaar* 30 – 32)	✓ Q_1 ✓ Median *Mediaan*	(2)
2.4		✓ for / *vir* $Q_3 = 37$ (accept / *aanvaar* 36 – 38) ✓ correct shape / *korrekte vorm*	(2)
2.5	170 – 110 = 60 cyclists / *fietsryers* (accept / *aanvaar* 59 – 61)	✓ answer / *antwoord*	(1)
			[10]

QUESTION 3 / *VRAAG 3*

3.1	$m_{QR} = \frac{-2-(-4)}{0-6} = -\frac{1}{3}$	✓substitution / *vervanging* ✓answer / *antwoord*	(2)
3.2	$m_{PQ} = 3$ $m_{PQ} \times m_{QR} = 3 \times \frac{-1}{3} = -1$ $\therefore \hat{PQR} = 90°$	✓ $m_{PQ} = 3$ ✓ $m_{PQ} \times m_{QR} = 3 \times \frac{-1}{3} = -1$	(2)
3.3	Sub/*Verv*: $y = -x + 2$ into/*in* $3x - y - 2 = 0$ $\therefore 3x - (-x + 2) - 2 = 0$ $3x + x - 2 - 2 = 0$ $4x = 4$ $x = 1$ $y = 1$ $\therefore P(1;1)$	✓ substitution / *vervanging* ✓ x – coordinate / x – *koördinaat* ✓ y – coordinate / y – *koördinaat*	(3)
3.4	$QR = \sqrt{(0-6)^2 + (-2-(-4))^2}$ $QR = 2\sqrt{10}$ **OR/OF** $\sqrt{40}$ **OR/OF** 6, 32 Answer Only: Full Marks	✓substitution in correct f *vervanging in korrekte* f ✓answer / *antwoord*	(2)

3.5	PR is the diameter (angle subtended by diameter = $90°$) / *PR is die middellyn (hoek onderspan deur middellyn = $90°$)* Midpoint of / *Middelpunt van* PR $\left(\frac{7}{2};-\frac{3}{2}\right)$ $PR=\sqrt{(1-6)^2+(1+4)^2}$ $PR=\sqrt{50}$ $r=\frac{\sqrt{50}}{2}$ $\left(x-\frac{7}{2}\right)^2+\left(x+\frac{3}{2}\right)^2=\left(\frac{\sqrt{50}}{2}\right)^2$ **OR/OF** $(x-3,5)^2+(x+1,5)^2=\left(\frac{\sqrt{50}}{2}\right)^2$	✓ for the statement PR is the diameter / *vir stelling PR is die middellyn* ✓✓ Midpoint of PR *Middelpunt van* PR ✓ for the radius / *vir die radius* ✓equation / *vergelyking*	(5)
3.6	$\tan P\hat{N}X=-1$ $\therefore P\hat{N}X=135°$ $\tan P\hat{M}X=3$ $\therefore P\hat{M}X=71,57°$ $\theta=135°-71,57°=63,43°$	✓ $\tan P\hat{N}X=-1$ ✓ $\therefore P\hat{N}X=135°$ ✓ $\tan P\hat{M}X=3$ ✓ $\therefore P\hat{M}X=71,57°$ ✓ answer / *antwoord*	(5)
3.7	$A=\frac{1}{2}\times PQ\times QR$ $A_{\Delta PQR}=\frac{1}{2}\times\sqrt{10}\times\sqrt{40}$ $A_{\Delta PQR}=10$ square units / *vierkante eenhede* **OR/OF** $A_{\Delta PQR}=\frac{1}{2}\times PQ\times PR\times\sin 63,43°$ $A_{\Delta PQR}=\frac{1}{2}\times\sqrt{10}\times\sqrt{50}\times\sin 63,43°$ $A_{\Delta PQR}=10$ square units / *vierkante eenhede*	✓formula / *formule* ✓$\sqrt{10}$ ✓answer / *antwoord*	(3)
			[22]

QUESTION 4 / *VRAAG 4*			
4.1	$x^2-6x+y^2-4y+9=0$ $x^2-6x+9+y^2-4y+4=-9+9+4$ $(x-3)^2+(y-2)^2=4$ $C(3;2)$ and / *en* $r=2$	✓completing square *voltooiing van vierkant* ✓standard form / *standaardvorm* ✓3 ✓2	(4)
4.2	$m_{\tan}=-2$ $m_{BV}=\frac{1}{2}$ $y-2=\frac{1}{2}(x-3)$ $y=\frac{1}{2}x+\frac{1}{2}$	✓ $m_{BV}=\frac{1}{2}$ ✓substitution / *vervanging* ✓answer / *antwoord*	(3)
4.3	$y=4$	✓answer / *antwoord*	(1)
4.4	TA = 4 units / *eenhede* TB = TA (tangents from the same point) (*raaklyne vanaf dieselfde punt*) TB = 4 units / *eenhede*	✓length of TA / *lengte van* TA ✓S ✓R ✓answer / *antwoord*	(4)
4.5	$T(-1;4)$ $y=-2x+k$ $4=-2(-1)+k$ $k=2$	✓substitution / *vervanging* ✓answer / *antwoord*	(2)
4.6	$\tan S\hat{T}A=-2$ $S\hat{T}A=116,57°$ $\therefore A\hat{C}B=116,57°$ (ext.angle of a c.q.) (*buitehoek van koordevierhoek*) **OR/OF** Draw/*Trek*: CE || OX ; then/*dan* $\tan V\hat{C}E=\frac{1}{2}$ $\therefore V\hat{C}E=26,57°$ $\therefore A\hat{C}B=180°-(90°-26,57°)=116,57°$ ($\angle$ s on straight line / $\angle$*e op reguitlyn*)	✓ $\tan S\hat{T}A=-2$ ✓ $S\hat{T}A=116,57°$ ✓answer / *antwoord* ✓reason / *rede* **OR/OF** ✓ $\tan V\hat{C}E=\frac{1}{2}$ ✓ $\therefore V\hat{C}E=26,57°$ ✓answer / *antwoord* ✓reason / *rede*	(4)
			[18]

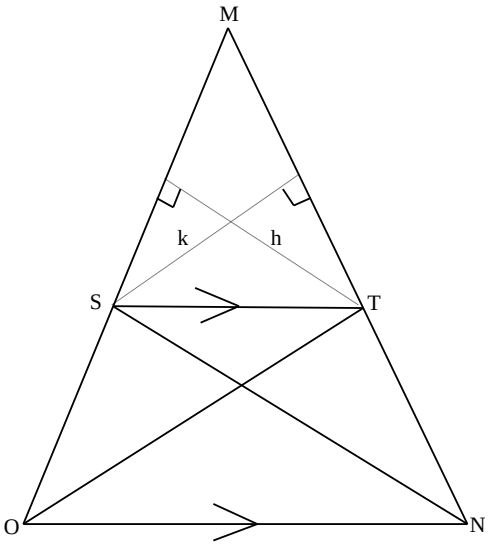
QUESTION 5/*VRAAG 5*			
5.1.1	$\cos 158°$ $= -\cos 22°$ $= -p$  	✓ $-\cos 22°$ ✓ $-p$	(2)
5.1.2	$\sin 112°$ $= \sin(90° + 22°)$ $= \cos 22°$ $= p$	✓ $\cos 22°$ ✓ p	(2)
5.1.3	$\sin 38°$ $= \sin(60° - 22°)$ $= \sin 60° \cos 22° - \cos 60° \sin 22°$ $= \frac{\sqrt{3}}{2} p - \frac{1}{2}\sqrt{1-p^2}$	✓ $\sin(60° - 22°)$ ✓ expansion / *uitbreiding* ✓ $\frac{\sqrt{3}}{2} p$ ✓ $\frac{1}{2}\sqrt{1-p^2}$	(4)
5.2	$\sin P = \sin 2P$ $\sin P - \sin 2P = 0$ $\sin P - 2\sin P \cos P = 0$ $\sin P(1 - 2\cos P) = 0$ $\sin P = 0$ or/*of* $\cos P = \frac{1}{2}$ $P \in [0°; 60°; 180°; 300°; 360°]$ **OR/OF** $P = 2P + 360°k$ or/*of* $P = 180° - 2P + 360°k$, $P \in \mathbb{Z}$ $P = -360°k$ or/*of* $3P = 180° + 360°k$ $P = 60° + 120°k$ $P \in [0°; 60°; 180°; 300°; 360°]$	✓ standard form / *standaardvorm* ✓ expansion / *uitbreiding* ✓ factorisation / *faktorisering* ✓ all correct values of P *alle korrekte waardes van P* ✓ $P = 2P + 360°k$ ✓ $P = 180° - 2P + 360°k$ ✓ $P = 60° + 120°k$ ✓ all correct values of P *alle korrekte waardes van P*	(4)
5.3	$A + B + C = 180°$ $A + B = 180° - C$ $\cos(A + B) = \cos(180° - C)$ $\cos(A + B) = -\cos C$	✓ $A + B = 180° - C$ ✓ $\cos(A + B) = \cos(180° - C)$	(2)

5.4	$\frac{\cos^2 x-\cos x-\sin^2 x}{2\sin x\cos x+\sin x}=\frac{1}{\tan x}-\frac{1}{\sin x}$ LHS / *LK* $=\frac{\cos^2 x-\cos x-\sin^2 x}{2\sin x\cos x+\sin x}$ $=\frac{\cos^2 x-\cos x-\left(1-\cos^2 x\right)}{\sin x(2\cos x+1)}$ $=\frac{2\cos^2 x-\cos x-1}{\sin x(2\cos x+1)}$ $=\frac{(2\cos x+1)(\cos x-1)}{\sin x(2\cos x+1)}$ $=\frac{\cos x}{\sin x}-\frac{1}{\sin x}=\frac{1}{\tan x}-\frac{1}{\sin x}$ = RHS / *RK*	✓ $1-\cos^2 x$ ✓ $\sin x(2\cos x+1)$ ✓ $2\cos^2 x-\cos x-1$ ✓ $(2\cos x+1)\ (\cos x-1)$ ✓ $\frac{\cos x}{\sin x}-\frac{1}{\sin x}$	(5)
5.5	$4+7\cos\theta+\cos 2\theta=0$ $4+7\cos\theta+2\cos^2-1=0$ $2\cos^2\theta+7\cos\theta+3=0$ $(2\cos\theta+1)\ (\cos\theta+3)=0$ $\cos\theta=-\frac{1}{2}$ or/*of* $\cos\theta=-3$ (N/A) $\theta=120°+360°.k$ or/*of* $\theta=240°+360°.k$, $x\in\mathbb{Z}$	✓ $2\cos^2-1$ ✓ standard form / *standaardvorm* ✓ factors / *faktore* ✓ $\cos\theta=-\frac{1}{2}$ or/*of* $\cos\theta=-3$ ✓ $\theta=120°+360°.k$ ✓ $\theta=240°+360°.k$	(6)
			[25]

QUESTION 6 / *VRAAG 6*			
6.1	$b = \frac{1}{2}$	✓answer / *antwoord*	(1)
6.2	A$(30°;1)$	✓$30°$ ✓1	(2)
6.3	$g(90°) = \cos(90° - 30°)$ $= \cos 60°$ $= \frac{1}{2}$ Q$\left(90°;\frac{1}{2}\right)$	✓$90°$ ✓$\frac{1}{2}$	(2)
6.4	$x = 160°$	✓$x = 160°$	(1)
6.5	$-1 \le y \le 3 \quad y \in R$ **OR/OF** $y \in [-1;3]$	✓✓answer / *antwoord*	(2)
			[8]
QUESTION 7 / *VRAAG 7*			
7.1	$\hat{LNM} = 180° - 2p$ (angles opp. = sides) (*hoeke teenoor gelyke sye*)	✓answer / *antwoord* ✓reason / *rede*	(2)
7.2	$\frac{LM}{\sin(180° - 2p)} = \frac{d}{\sin p}$ $\frac{LM}{\sin 2p} = \frac{d}{\sin p}$ $LM = \frac{d \sin 2p}{\sin p}$	✓ for applying the sine rule *gebruik van sinusreël* ✓$\sin 2P$	(2)
7.3	$\tan q = \frac{h}{LM}$ $h = LM \tan q$ $h = \frac{d \sin 2p}{\sin p}.\tan q$ $h = \frac{2d \sin p \cos p \tan q}{\sin p}$ $h = 2d \cos p \tan q$	✓$\tan q = \frac{h}{LM}$ ✓$h = \frac{d \sin 2p}{\sin p}.\tan q$ ✓$h = \frac{2d \sin p \cos p \tan q}{\sin p}$	(3)
			[7]

QUESTION 8 / *VRAAG 8*			
8.1	bisects the chord / *halveer die koord*	✓ answer/*antwoord*	(1)
8.2	$EB = 8 - y$ In ΔAEB: $10^2 = x^2 + (8-y)^2$ ……(1) Eqn of the circle / *Verg. van die sirkel*: $x^2 + y^2 = 64$ $x^2 = 64 - y^2$ ……..(2) Subst./*Verv.* (2) into/*in* (1) $100 = 64 - y^2 + 64 - 16y + y^2$ $100 = 128 - 16y$ $16y = 28$ $y = \frac{7}{4}$ $\therefore OE = \frac{7}{4}$	✓ for/*vir* EB ✓Pythagoras in ΔAEB ✓equation of the circle *vergelyking van sirkel* ✓ substitution *vervanging* ✓answer / *antwoord*	(5)
8.3	**Double** the size of **the angle** subtended by the same arc. **Dubbel** die grootte van **die hoek** wat deur dieselfde boog onderspan word.	✓answer / *antwoord*	(1)
8.4.1	$\hat{O}_2 = 2\hat{B}_2$ ($\angle$ at centre = 2 $\times\angle$ at the circumf) (*Middelpunts* $\angle$ = 2 x *Omtrekshoek*)	✓statement / *stelling* ✓reason / *rede*	(2)
8.4.2	$\hat{C}_3 = \hat{D}_1 + \hat{D}_2$ ($\angle$ s opp = sides)/ ($\angle$*e teenoor = sye*)	✓statement / *stelling* ✓reason / *rede*	(2)
8.4.3	$\hat{B}_1 + \hat{B}_2 = 180° - (\hat{D}_1 + \hat{D}_2)$ (opp. $\angle$*s* of a cyclic quad) (*teenoorst.* $\angle$*e van 'n koordevierhoek*)	✓statement / *stelling* ✓reason / *rede*	(2)
8.4.4	$\hat{D}_1 = \hat{C}_1$ ($\angle$ s in the same segment) ($\angle$*e in dieselfde segment*)	✓statement / *stelling* ✓reason / *rede*	(2)
			[15]

QUESTION 9 / *VRAAG 9*			
9.1	$P\hat{C}Q = 80°$ ($\angle$ s opp = sides)/(*$\angle$e teenoor = sye*) $P\hat{C}B = 100°$ ($\angle$ s on a straight line) (*$\angle$e op 'n reguitlyn*) $\therefore$ BC is not a diameter (angle between the tangent and BC is not equal to $90°$) *BC is nie 'n middellyn nie.* (*hoek tussen die raaklyn en BC is nie gelyk aan* $90°$ *nie*)	✓statement / *stelling* ✓reason / *rede* ✓statement / *stelling* ✓reason / *rede* ✓conclusion / *gevolgtrekking*	(5)
9.2	$\hat{P}_1 = \hat{B}$ (alt $\angle$ s, PQ || AB) / (*verw.$\angle$e*, PQ || AB) $\hat{B} = \hat{C}_3$ ($\angle$ s opp = sides; radii) (*$\angle$e teenoor = sye: radiusse*) $\hat{C}_3 = \hat{C}_1$ (vert. opp. angles) / (*regoorst. hoeke*) $\therefore \hat{P}_1 = \hat{C}_1$ $\therefore$ PQ = QC (sides opp = angles) (*sye teenoor = hoeke*)	✓statement / *stelling* ✓reason / *rede* ✓statement / *stelling* ✓reason / *rede* ✓statement / *stelling* ✓statement and reason *stelling en rede*	(6)
9.3	$\hat{A} = \hat{E}_2$ (ext. $\angle$ of a cq) / (*buite$\angle$ van koordev.*) $\hat{D} = 180° - \hat{E}_2$ (co–interior $\angle$ s; BE || CD) (*ko-binne $\angle$e* : BE || CD) $\hat{D} + \hat{A} = 180°$ $\therefore$ ACDF is a cq (opp $\angle$ s supplementary) / *is 'n koordev* (*teenoorst. $\angle$e is suppl.*) **OR/OF** $\hat{D} = \hat{E}_1$ (corres. $\angle$ s; BE || CD) (*ooreenk. $\angle$e* : BE || CD) $\hat{E}_2 = 180° - \hat{E}_1$ ($\angle$ s on a straight line) (*$\angle$e op 'n reguitlyn*) $\hat{A} = 180° - \hat{E}_1$ (opp $\angle$ s of a cq) (*teenoorst. $\angle$e is suppl.*) $\hat{D} + \hat{A} = 180°$ $\therefore$ ACDF is a cyclic quad./ *is 'n koordevierhoek* (opp $\angle$ s of a quad. supplementary) (*teenoorst. $\angle$e van koordev. is supplementêr*)	✓statement / *stelling* ✓reason / *rede* ✓statement and reason *stelling en rede* ✓statement and reason *stelling en rede* ✓reason / *rede* ✓statement / *stelling* ✓reason / *rede* ✓statement and reason *stelling en rede* ✓statement and reason *stelling en rede* ✓reason / *rede*	(5)
			[16]

QUESTION 10 / *VRAAG 10*			
10.1	 RTP/*TE BEWYS*: $\frac{MS}{SO}=\frac{MT}{TN}$ Construction: Join SN, and OT, and construct perpendicular heights / *Konstruksie*: *Verbind SN en OT, trek loodregte hoogtes* Proof / *Bewys*: $\frac{\text{area }\Delta MST}{\text{area }\Delta OST}=\frac{\frac{1}{2}\times MS\times h}{\frac{1}{2}\times SO\times h}=\frac{MS}{SO}$ $\frac{\text{area }\Delta MST}{\text{area }\Delta TNS}=\frac{\frac{1}{2}\times MT\times k}{\frac{1}{2}\times TN\times k}=\frac{MT}{TN}$ But / *Maar* ΔMST is common / *gemeen* And / *En* area of ΔOST = area of ΔTNS (same base, same height) *area van* ΔOST = *area van* ΔTNS (*dies. basis en dies. hoogte*) $\therefore \frac{MS}{SO}=\frac{MT}{TN}$	*area / oppervlakte* ✓area of the two triangles *area van twee driehoeke* ✓$\frac{MS}{SO}$ ✓area of the two triangles *area van twee driehoeke* ✓$\frac{MT}{TN}$ ✓ statement and reason *stelling en rede*	(5)

10.2.1	In ΔAPS and/*en* ΔBRS $\hat{P}_4 = \hat{R}_1$ (tan – chord theorem) (*raaklyn-koord Stelling*) $\hat{A} = \hat{B}_2 = 90°$ (given) / (*gegee*) $\Delta APS \text{ ||| } \Delta BRS$ (AAA) / ($\angle\angle\angle$)	✓statement and reason *stelling en rede* ✓statement / *stelling* ✓3rd $\angle$ / 3de $\angle$ **OR/OF** reason for similarity *rede vir gelykvormigheid*	(3)
10.2.2	$\frac{AP}{BR} = \frac{PS}{RS} = \frac{AS}{BS}$ (similar triangles) (*gelykvormige driehoeke*) $\therefore AP.RS = BR.PS$	✓ for the statement *vir die stelling*	(1)
10.2.3	$\hat{P}_2 = 90°$ ($\angle$s in a semi – circle) ($\angle e$ *in 'n semi-sirkel*) Let/*Laat*: $\hat{P}_4 = x$ $\therefore \hat{S}_1 = 90 - x$ ($\angle$s of APS) / ($\angle e$ *van* APS) $\therefore \hat{Q} = 90 - x$ (ext $\angle$ of a cq) / (*buite*$\angle$ *van kv*) $\therefore \hat{R}_2 = x$ ($\angle$s of QPR) / ($\angle e$ *van* QPR) $\therefore \hat{P}_4 = \hat{R}_2$	✓$\hat{P}_2 = 90°$ ($\angle$ in a semi – circle) / ($\angle e$ *in 'n semi-sirkel)* ✓$\hat{S}_1 = 90 - x$ ✓$\hat{Q} = 90 - x$ ✓$\hat{R}_2 = x$	(4)
10.2.4	In ΔASP and/*en* ΔPQR $\hat{A} = \hat{P}_2$ (proven / *bewys*) $\hat{P}_4 = \hat{R}_2$ (proven / *bewys*) $\Delta ASP \text{ ||| } \Delta PQR$ (AAA) / ($\angle\angle\angle$) $\frac{AS}{PQ} = \frac{SP}{QR} = \frac{AP}{PR}$ (similar triangles) (*gelykvormige driehoeke*) $\therefore AP \cdot QR = SP \cdot PR$ $\therefore \frac{AP}{PS} = \frac{PR}{RQ}$ But / *Maar*: $\frac{AP}{PS} = \frac{BR}{RS}$ (from / *vanaf* 10.2) $\therefore \frac{PR}{RQ} = \frac{BR}{RS}$ $\therefore BR.RQ = RS.RP$	✓statement and reason *stelling en rede* ✓statement / *stelling* ✓reason for similarity *rede vir gelykvormigheid* ✓$\therefore \frac{AP}{PS} = \frac{PR}{RQ}$ ✓ $\frac{AP}{PS} = \frac{BR}{RS}$ ✓$\therefore \frac{PR}{RQ} = \frac{BR}{RS}$	(6)
			[19]
		TOTAL/*TOTAAL*:	**150**