education

Department of Education
FREE STATE PROVINCE

PREPARATORY EXAMINATION/
VOORBEREIDENDE EKSAMEN

GRADE/*GRAAD* 12

MATHEMATICS P2/
WISKUNDE V2

SEPTEMBER 2020

MARKS/*PUNTE*: 150

MARKING GUIDELINES/
NASIENRIGLYNE

This marking guidelines consists of 13 pages./
Hierdie nasienriglyne bestaan uit 13 bladsye.

NOTE:

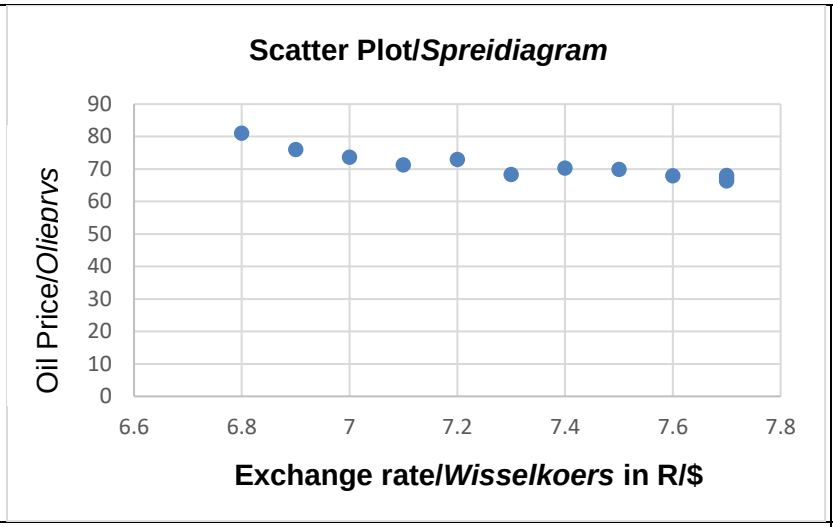
- If a candidate answers a question TWICE, only mark the FIRST attempt.
- If a candidate has crossed out an attempt of a question and not redone the question, mark the crossed out version.
- Consistent accuracy applies in ALL aspects of the marking memorandum. Stop marking at the second calculation error.
- Assuming answers/values in order to solve a problem is NOT acceptable.

NOTA:

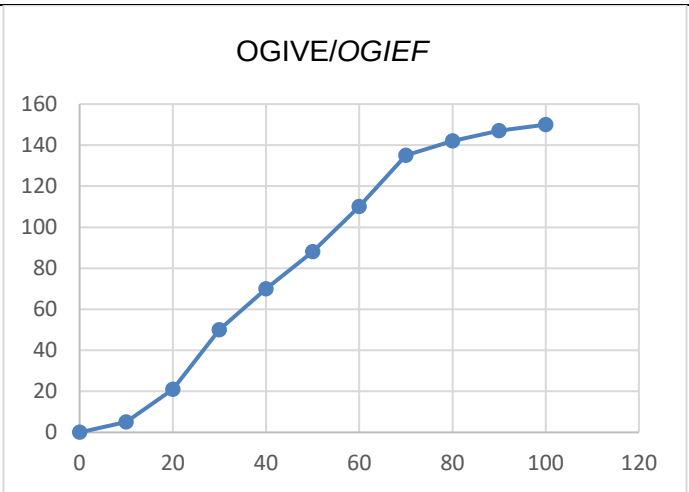
- *As 'n kandidaat 'n vraag TWEE KEER beantwoord, sien slegs die EERSTE poging na.*
- *As 'n kandidaat 'n antwoord van 'n vraag doodtrek en nie oordoen nie, sien die doodgetrekte poging na.*
- *Volgehoue akkuraatheid word in ALLE aspekte van die nasienriglyne toegepas. Hou op nasien by die tweede berekeningsfout.*
- *Om antwoorde/waardes te aanvaar om 'n probleem op te los, word NIE toegelaat NIE.*

GEOMETRY/*MEETKUNDE*	
S	**A mark for a correct statement** **(A statement mark is independent of a reason)**
	'n Punt vir 'n korrekte bewering ***('n Punt vir 'n bewering is onafhanklik van die rede)***
R	**A mark for the correct reason** **(A reason mark may only be awarded if the statement is correct)**
	'n Punt vir 'n korrekte rede ***('n Punt word slegs vir die rede toegeken as die bewering korrek is)***
S/R	**Award a mark if statement AND reason are both correct**
	Ken 'n punt toe as die bewering EN die rede beide korrek is

QUESTION/*VRAAG* 1

1.1		✓ 4 points correctly plotted ✓ 9 points correctly plotted ✓ All points (3)
1.2	$y = 158{,}67 - 11{,}96x$	✓✓✓ Correct equation (3)
1.3	$r = -0{,}91$	✓ value of r (1)
1.4	Exchange rate increase,oil price decrease/*Wisselkoers verhoog, olieprys verlaag* **OR/*OF*** Strong Negative correlation/*Sterk negatiewe korrelasie*	✓ ✓ reason (2)
1.5	y =71,05	✓ y =71,05 (1)
1.6	Standard deviation/*Standaard afwyking* : $\sigma = 4{,}09$	✓ $\sigma = 4{,}09$ (1)
1.7	71,05+2(4,09) = 79,23 (With calculator 79,22) December	✓ 79,23 ✓ December/ *Desember* (2)
		[13]

QUESTION/*VRAAG* 2

2.1		✓grouded ✓Cf ✓upper limit ✓ curve (4)
2.2.1	(8;144) 144 below 85 % (accept/*aanvaar* 144-146)	✓(8;144) ✓ 144 (2)
2.2.2	$Q_2 = 42{,}77$ (accept/*aanvaar* 41-43)	✓$Q_2 = 42{,}77$ (1)
		[7]

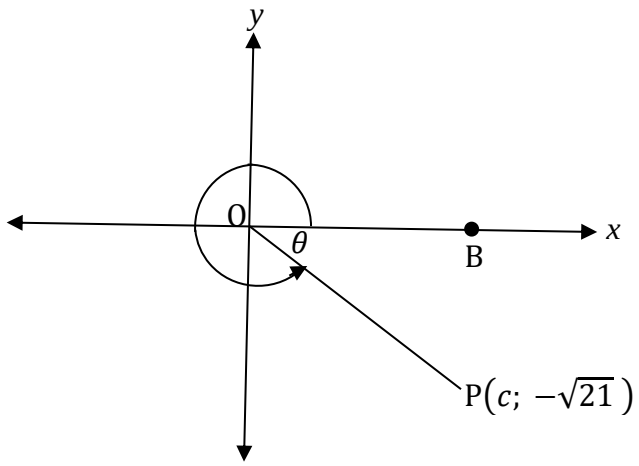
QUESTION/*VRAAG* 3

3.1	$M_{AD} = \frac{7-(-3)}{1-(-4)}$ $= 2$	✓ Substitution into the correct formula ✓ Answer (2)
3.2	AD// BC $M_{AD} = M_{BC} = 2$ $-8 = 2(2) + c$ $\therefore y = 2x - 4$	✓ $M_{BC} = 2$ ✓ sub ✓ Answer (3)
3.3	At F $y = 0$ $0 = 2x - 4$ $\therefore x = 2$ $\therefore F(2;0)$	✓ $0 = 2x - 4$ ✓ $x = 2$ (2)
3.4	$B(x;y) - 7\ \ C(x+2;y-5)$ $A(1;7) \rightarrow B^{/}(3;2)$ OR $x_B = -2 + (1+4) = 3$ $y_B = -8 + (7+3) = 2$	✓✓ $B^{/}(3;2)$ (2)
3.5	$M_{BC} = \tan\theta = 2$ $\theta = 63{,}43°$ $M_{DC} = \frac{-8-(-3)}{-2-(-4)} = \frac{-5}{2}$ $\tan\beta = \frac{-5}{2}$ $\beta = 180° - 68{,}20° = 111{,}80°$ $\alpha = 111.80° - 63{,}43° = 48{,}37°$	✓ $\theta = 63{,}43°$ ✓ $\frac{-5}{2}$ ✓$\beta = 111{,}80°$ ✓ $\alpha = 48{,}37$ (4)
3.6	DC= $\sqrt{(-4+2)^2 + (-3+8)^2}$ $= \sqrt{29}$ CF= $\sqrt{(-2-2)^2 + (-8-0)^2}$ $= \sqrt{80}$ Area/*Oppvl* $\Delta DCF = \frac{1}{2}$DC.CF $\sin\alpha$ $= \frac{1}{2}\sqrt{29}.\sqrt{80}\sin 48{,}37°$ $= 18$ units/*eenhede*	✓ sub correct formula ✓ $\sqrt{29}$ ✓ sub correct formula ✓ $\sqrt{80}$ ✓ sub correct formula ✓18 *units* (6)
		[19]

QUESTION/*VRAAG* 4

4.1	$x^2 + y^2 + 2x + 6y + 2 = 0$ $x^2 + 2x + 1 + y^2 + 6y + 9 = -2 + 10$ $(x + 1)^2 + (y + 3)^2 = 8$ $M(-1; -3)$	✓ 8 ✓✓$M(-1; -3)$ (3)
4.2	$r = \sqrt{8}$	✓ $r = \sqrt{8}$ (1)
4.3	$x^2 + (x - 2)^2 + 2x + 6(x - 2) + 2 = 0$ $2x^2 + 4x - 6 = 0$ $x^2 + 2x + 3 = 0$ $(x + 3)(x - 1) = 0$ $x = -3$ or $x \neq 1$ $y = -3 - 2 = -5$ $\therefore D(-3; -5)$	✓substitution ✓$x^2 + 2x + 3 = 0$ ✓$(x + 3)(x - 1) = 0$ ✓✓ $D(-3; -5)$ (5)
4.4	Angle between radius/diameter and tangent/ *Hoek tussen radius/deursnee en raaklyn*	✓ R (1)
4.5	$MB^2 = MD^2 + DB^2$ Pyth $= \left(\sqrt{8}\right)^2 + 4\left(\sqrt{2}\right)^2$ $= 40$ MB= $\sqrt{40}$ radius of Circle/*radius van sirkel* C_2	✓ S ✓ $\left(\sqrt{8}\right)^2 + 4\left(\sqrt{2}\right)^2$ ✓ 40 ✓ MB= $\sqrt{40}$ (4)
4.6	$(x + 1)^2 + (y + 3)^2 = 40$	✓Equation ofC_2 (1)
4.7	Distance from/*Afstand van* $\left(2\sqrt{5}; 0\right)$ to centre/*tot middelpunt* $= \sqrt{\left(2\sqrt{5} + 1\right)^2 + (0 + 3)^2}$ $= 6{,}24$ $6{,}24 < 6{,}32\ \left(\sqrt{40}\right)$ Distance to centre < radius of circle/*Afstand tot middelpunt < radius van die sirkel* $\therefore \left(2\sqrt{5}; 0\right)$ lies inside/*lê binne*	✓ correct sub ✓ 6,24 ✓$6{,}24 < 6{,}32\ \left(\sqrt{40}\right)$ ✓ lies inside (4)
4.8	$M_{MDP} = \frac{-3-(-5)}{-1-(-3)} = \frac{2}{2} = 1$ $\therefore$ M tangent $= -1$	✓ $M_{MDP} = 1$ ✓ M tangent $= -1$ (2)
		[21]

QUESTION/*VRAAG* 5

5.1.1	 $c^2+\left(\sqrt{21}\right)^2=5^2$ $c^2=25-21$ $c^2=4$ $\therefore c=2$	✓ Subst. into pyth ✓ Answer (2)
5.1.2 a)	$\cos\theta=\frac{2}{5}$	✓ Answer (1)
b)	$\frac{-\sqrt{21}}{2}+\left(\frac{-\sqrt{21}}{5}\right)^2$ $-5\sqrt{21}+42/50$	✓ $\frac{-\sqrt{21}}{2}+\left(\frac{-\sqrt{21}}{5}\right)^2$ ✓ Answer (2)
c)	$2\sin\theta\cos\theta$ $=2\left(\frac{-\sqrt{21}}{5}\right)\left(\frac{2}{5}\right)$ $=\frac{-4\sqrt{21}}{25}$	✓ $2\left(\frac{-\sqrt{21}}{5}\right)\left(\frac{2}{5}\right)$ ✓ Answer (2)
5.2	$\frac{(-\sin x).\tan x.\cos(360°-30°)}{(-\sin x)^2}$ $=\frac{-\sin\ x.\tan x(\cos30°)}{\sin^2 x.}$ $=\frac{-\sqrt{3}}{2}\tan x\div\sin x$ $=\frac{-\sqrt{3}}{2\cos x}$	✓ $-\sin x$ ✓ $(-\sin x)^2$ ✓ $\frac{-\sqrt{3}}{2}$ ✓✓ $\frac{-\sqrt{3}}{2\cos x}$ (5)
		[**12**]

QUESTION/*VRAAG* 6

6.1	$LHS = \dfrac{2\sin^2 x}{2\dfrac{\sin x}{\cos x} - 2\sin x\cos x}$ $= \dfrac{2\sin^2 x}{2\sin x\left(\dfrac{1}{\cos x} - \cos x\right)}$ $= \dfrac{\sin x}{\dfrac{1-\cos^2 x}{\cos x}}$ $\quad \therefore LHS = RHS$ $= \sin x \times \dfrac{\cos x}{\sin^2 x}$ $= \dfrac{\cos x}{\sin x}$	✓ $\dfrac{\sin x}{\cos x}$ ✓ $2sin\,xcos\,x$ ✓ $2\sin x\left(\frac{1}{\cos x} - \cos x\right)$ ✓ $\dfrac{\sin x}{\dfrac{1-\cos^2 x}{\cos x}}$ ✓ $sin^2 x$ (5)
6.2	$Sin^2 20° + Sin^2 40° + Sin^2 80°$ $= Sin^2 20° + [Sin(60° - 20°)]^2 + [Sin(60° + 20°)]^2$ $= Sin^2 20° + [Sin60°Cos20° - Cos60°Sin20°]^2$ $\quad + [Sin60°Cos20° + Cos60°Sin20°]^2$ $= Sin^2 20° + \left[\dfrac{\sqrt{3}}{2}Cos20° - \dfrac{1}{2}Sin20°\right]^2$ $\quad + \left[\dfrac{\sqrt{3}}{2}Cos20° + \dfrac{1}{2}Sin20°\right]^2$ $= \dfrac{3}{2}Sin^2 20° + \dfrac{3}{2}Cos^2 20°$ $= \dfrac{3}{2}(Sin^2 20° + Cos^2 20°)$ $= \dfrac{3}{2}(1)$ $= \dfrac{3}{2}$	✓✓compound ✓sub of special angle ✓simplification ✓common factor ✓square identity ✓answer (7)
		[12]

QUESTION/*VRAAG* 7

7.1	In Δ ABC $Sin\theta = \frac{h}{AC}$ $\therefore AC = \frac{h}{Sin\theta}$	✓ trig ratio ✓ answer (2)
7.2	In Δ ABD $Sin\alpha = \frac{h}{AD}$ $\therefore AD = \frac{h}{Sin\alpha}$	✓ trig ratio ✓ answer (2)
7.3	$CD^2 = AC^2 + AD^2 - 2AC.AD\cos\beta$ $= \left(\frac{h}{Sin\theta}\right)^2 + \left(\frac{h}{Sin\alpha}\right)^2 - 2.\frac{h}{Sin\theta}.\frac{h}{Sin\alpha}Cos\beta$ $= \frac{h^2}{Sin^2\theta} + \frac{h^2}{Sin^2\alpha} - \frac{2h^2Cos\beta}{Sin\theta Sin\alpha}$	✓ cosine rule ✓ substitution ✓ answer (3)
		[7]

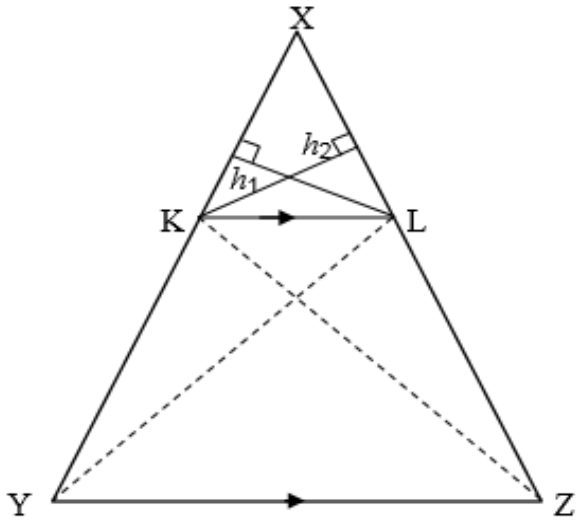
QUESTION/*VRAAG* 8

8.1	$\cos\frac{x}{2} = \sin(x - 30°)$ $= \cos[(90° - (x - 30°))]$ $= \cos(120° - x)$ $\therefore \frac{x}{2} = 120° - x + 360°.k$ $x = 240° - 2x + 720°.k$ $x = 80° + 240°.k$ OR $\frac{x}{2} = 360° - (120° - x) + 360°.k$ $\frac{x}{2} = 360° - 120° + x + 360°k$ $\frac{x}{2} = 240° + x + 360°k$ $x = 480° + 2x + 720k$ $-x = 480° + 720k$ $\therefore x = -480° - 720°k$ where/*waar* $k \in z$ $x_A = -160°$ and/*en* $x_B = 80°$	✓ $\cos(120° - x)$ ✓ $80° + 240°.k$ ✓ $240° + x + 360°k$ ✓ $-480° - 720°k$ ✓ $x_A = -160°$ ✓ $x_B = 80°$ (6)
8.2	$-150° < x < 30°$	✓ ✓ Critical values ✓ Notation (3)
		[9]

QUESTION/*VRAAG* 9

9.1	$AB^2 = AO^2 + BO^2 - 2(AO)(BO)\cos\hat{O}_1$ $\left(\frac{3}{2}x\right)^2 = x^2 + x^2 - 2x^2\cos\hat{O}_1$ $\frac{9}{4}x^2 - 2x^2 = -2x^2\cos\hat{O}_1$ $\cos\hat{O}_1 = \frac{1}{4}x^2 \div 2x^2$ $= \frac{1}{8}$ $\therefore \hat{O}_1 = 97°$	✓cosine rule ✓ substitution ✓ simplification ✓$\frac{1}{8}$ ✓$\hat{O}_1 = 97°$ (5)
9.2	$\hat{C} = 48{,}5°$ angle at centre/*hoek by middelpunt* = twice angle at circ/*twee keer hoek by sirk* $\hat{B}_1 + \hat{B}_2 = 83°$ int angles of/*binne hoeke van* Δ $\frac{AC}{\sin 83°} = \frac{15}{\sin 48{,}5°}$ $\therefore AC = 19{,}88$	✓ S ✓ S ✓ sine rule ✓ $AC = 19{,}88$ (4)
9.3.1	$\hat{Q}_1 = 32°$ tan chord theorem/*tan koordstelling*	✓✓ S/R (2)
9.3.2	$\hat{Q}_1 = \hat{P}_4 = 32°$ angle opp = sides/*hoeke teenoor = sye*	✓✓ S/R (2)
9.3.3	$\hat{M}_1 = \hat{Q}_1 + \hat{P}_4$ ext angle of Δ = op pint angles/ *Verl hoek van* Δ *= opp hoeke* $= 32° + 32°$ $\therefore \hat{M}_1 = 64°$	✓ S ✓ answer (2)
9.3.4	$\hat{S} = 180° - 122°$ sum of angles of Δ = 180°/ *Som van hoeke van* Δ *= 180°* $\therefore \hat{S} = 58°$ But/*Maar* $\hat{S} = \hat{R} = 58°$ angle sub by same chord/ *hoek verv by dieselfde koord*	✓✓ S/R ✓✓ S/R (4)
		[19]

QUESTION/*VRAAG* 10

10.1	In the same proportion/ *In dieselfde verhouding*	✓ same proportion (1)
		
10.2	Const: Join KZ & LY & draw h_1 from K ⊥ XL & h_2/ *Konst: Verbind KZ & LY & skets h_1 van K ⊥ XL & h_2* Proof/*Bewys* $\frac{\text{Area}/Oppvl\ \Delta XKL}{\text{Area}/Oppvl\ \Delta LYK} = \frac{\frac{1}{2}XK \times h_1}{\frac{1}{2}KY \times h_2} = \frac{XK}{KY}$ $\frac{\text{Area}/Oppvl\ \Delta XKL}{\text{Area}/Oppvl\ \Delta KLZ} = \frac{\frac{1}{2}XL \times h_2}{\frac{1}{2}LZ \times h_2} = \frac{XL}{LZ}$ Area of $\Delta XKL = Area\ \Delta XKL$ common/ *Oppvl van $\Delta XKL = Oppvl\ \Delta XKL$ algemeen* But Area $\Delta LKY = Area\ \Delta KLZ$ same base & height; LK// YZ/ *Maar Oppvl $\Delta LKY = Oppvl\ \Delta KLZ$ selfde basis & hoogte; LK// YZ* $\frac{\text{Area}/Oppvl\ \Delta XKL}{\text{Area}/Oppvl\ \Delta LYK} = \frac{\text{Area}/Oppvl\ \Delta XKL}{\text{Area}/Oppvl\ \Delta KLZ}$ $\therefore \frac{XK}{KY} = \frac{XL}{LZ}$	✓ const ✓ $\frac{Area\ \Delta XKL}{Area\ \Delta LYK} = \frac{XK}{KY}$ ✓ $\frac{Area\Delta XKL}{Area\Delta KLZ} = \frac{XL}{LZ}$ ✓ S ✓ R ✓ S (6)

10.3.1	$\frac{ED}{DC} = \frac{AT}{AC} = \frac{2}{3}$ line // to one side of △/ DE $= \frac{2}{3} \times 9$ *lyn // aan die eenkant van ⊿* $= 6$ and/*en* BD $= 6$ given/*gegee* $\therefore D$ is the midpoint of BE/*is the middelpunt van BE*	✓✓ S/R ✓ answer (3)
10.3.2	BF= FT conv of midpoint theorem FD= $\frac{1}{2}$TE midpoint theorem/*middelpunt bewys* $\therefore TE = 4$ cm	✓ S/R ✓ R ✓ $TE = 4$ (3)
10.3.3 (a)	$\frac{Area\ of\ \triangle ADC}{Area\ of \triangle ABD}$ $= \frac{1}{2} \times b_1 \times h \div \frac{1}{2} \times b_2 \times h$ same height/*selfde hoogte* $= \frac{3y}{2y}$ $= \frac{3}{2}$	✓✓ S/R ✓ $\frac{3}{2}$ (3)
(b)	$\frac{\frac{1}{2} \times TC \times EC \times \sin C}{\frac{1}{2} \times AC \times BC \times \sin C}$ $= \frac{(x)(y)}{(3x)(5y)}$ $= \frac{1}{15}$	✓✓ $\frac{\frac{1}{2} \times TC \times EC \times \sin C}{\frac{1}{2} \times AC \times BC \times \sin C}$ ✓ $\frac{1}{15}$ (3)
		[19]

QUESTION/*VRAAG* 11

11.1	In △TPS and △ QSR $\frac{PS}{QS}=\frac{1,5}{4}=\frac{3}{8}$ $\frac{TP}{SR}=\frac{4,5}{12}=\frac{3}{8}$ $\frac{TS}{QR}=\frac{3,6}{9,6}=\frac{3}{8}$ ∴ △TPS/// △ QSR sides of triangles in proportion *sye van die driehoeke is in verhouding* ∴ $P\hat{T}S=\hat{R}$ ∴ TP is a tangent converse of tan chord theorem/ *is 'n raaklyn teenoorg. van tan koordstelling*	✓$\frac{PS}{QS}=\frac{1,5}{4}$ ✓$\frac{TP}{SR}=\frac{4,5}{12}$ ✓$\frac{TS}{QR}=\frac{3,6}{9,6}$ ✓✓ S/R ✓ $P\hat{T}S=\hat{R}$ ✓R (7)
11.2	$\hat{P}=Q\hat{S}R$ △s /// QS|| TP corr angles =/*korr hoeke* $\frac{TQ}{9,6}=\frac{1,5}{12}$ proportional theorem/*verhoudingstelling* ∴ $TQ=1,2$	✓ S ✓ R ✓✓ S/R ✓ $TQ=1,2$ (5)
		[12]

TOTAL/*TOTAAL*: 150