LIMPOPO

PROVINCIAL GOVERNMENT

REPUBLIC OF SOUTH AFRICA

DEPARTMENT OF

EDUCATION

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS PAPER 2

SEPTEMBER 2020

EMATHP2

MARKS: 150

TIME: 3 HOURS

This question paper consists of 13 pages and an information sheet.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

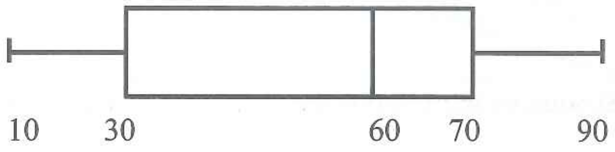
1. This question paper consists of 10 questions.
2. Answer ALL the questions.
3. Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining your answers.
4. ANSWERS ONLY will not necessarily be awarded full marks.
5. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
6. If necessary, round answers off to TWO decimal places, unless stated otherwise.
7. Diagrams are NOT necessarily drawn to scale.
8. Number the answers correctly according to the numbering system used in this question paper.
9. Write legibly and present your work neatly.

QUESTION 1

1.1 Two schools, A and B, in the Lephalale circuit, are in competition. They want to see which one of them performed best in Mathematics during June Examination. Both schools have 25 learners.

The five number summary for school A is: (9; 23,5; 55; 75; 91).

The box and whisker diagram for the learners in school B is:



1.1.1 Draw the box and whisker diagram which represents the marks of school A on the grid in your answer book. Clearly indicate all relevant values. (2)

1.1.2 Determine which school performed better in the June Examination and give reasons for your conclusion. (3)

1.2 Learners at Bahananwa High School travel from 3 villages A, B and C. The table below shows the number of learners from each village and their mean travelling times from home to school.

VILLAGE	**A**	**B**	**C**
Number of learners	135	225	200
Mean travelling time (in minutes)	24	32	x

The mean travelling time for learners living in village C is the same as the mean travelling time for all 560 learners.

Calculate the mean travelling time for the learners from village C. (4)

[9]

QUESTION 2

Learners who scored a mark below 50% in a Mathematics Test were selected to use a remedial program to improve their marks. On completing the programme, these learners wrote a second test to determine the effectiveness of the intervention strategy. The mark scored by 10 of these learners in both tests is given in the table below.

Learner	1	2	3	4	5	6	7	8	9	10
Test 1	10	23	27	34	34	37	39	40	48	49
Test 2	33	32	58	49	48	41	55	50	68	60

2.1 Determine the equation of the least squares regression line. (3)

2.2 The total of Test 1 was 50. A learner's mark for Test 1 is 15, which is 30%. Predict his mark in % for Test 2. Give your answer to the nearest integer. (2)

2.3 For the 10 learners above, the mean mark for the second test is 49,4% and the standard deviation is 10,99%. The teacher discovered that he forgot to add the marks of the last question to the total mark of each of the learners. All the learners scored full marks in the last question. When the marks of the last question are added, the new mean mark is 57,4%.

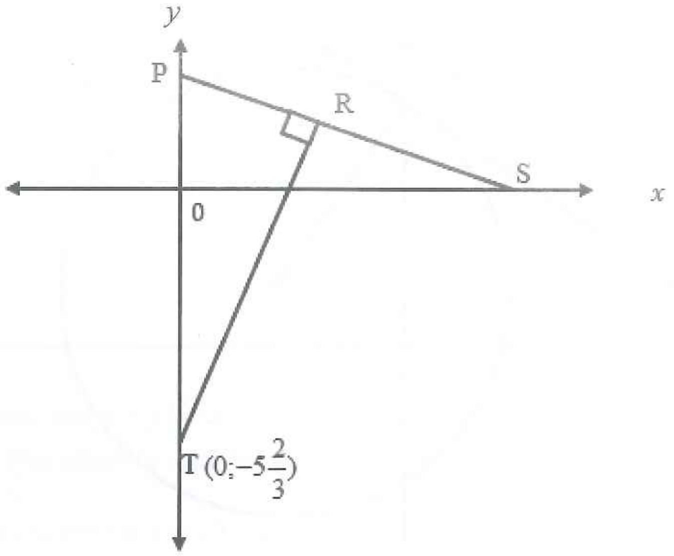
2.3.1 What is the standard deviation after the marks for the last question are added to each learner's total? (2)

2.3.2 What is the total mark (out of 50) of the last question? (2)

[9]

QUESTION 3

In the diagram the straight line SP is drawn with S and P as x- and y-intercepts respectively. The equation of SP is $x + ay - a = 0, a > 0$. It is also given that $OS = 3OP$. The straight line $TR \perp PS$ and TR cuts the y-axis at $T(0;-5\frac{2}{3})$

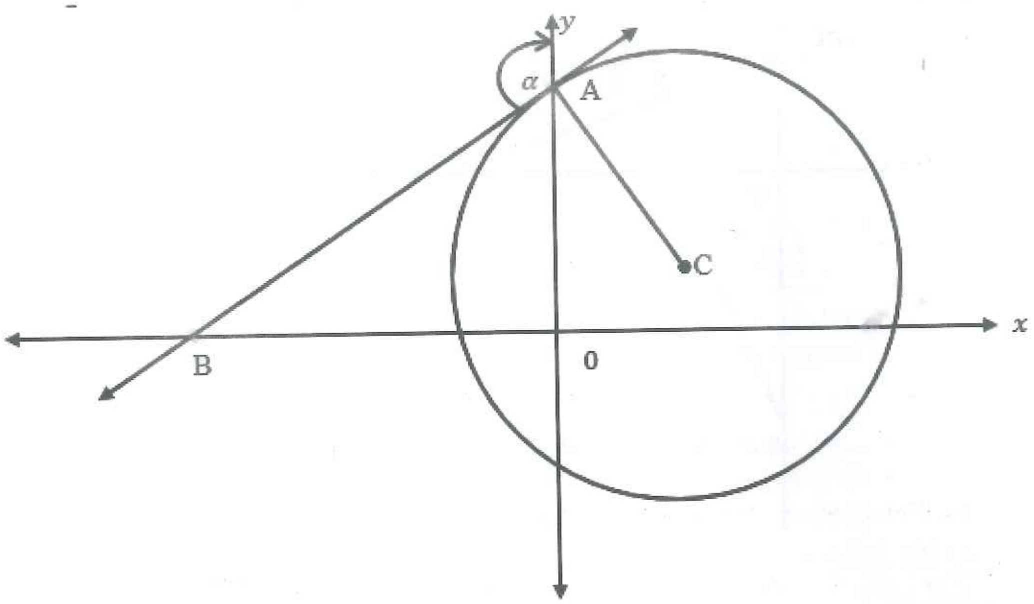


3.1	Calculate the coordinates of P.	(3)
3.2	Calculate the value of a.	(3)
3.3	Determine the equation of RT in the form $y = mx + c$.	(3)
3.4	Calculate the coordinates of R.	(4)
3.5	Calculate the area of ΔPRT.	(3)
3.6	Calculate, giving reasons, the radius of a circle passing through the points P,R and T.	(2)

[18]

QUESTION 4

In the diagram the circle $x^2 - 6x + y^2 - 4y = 12$ (with centre C) cuts he y-axis at A. BA is a tangent to the circle. B lies on the x-axis. $\hat{BAY} = \alpha$.



4.1 Determine:

4.1.1 The coordinates of C, and the length of the radius. (4)

4.1.2 The equation of the line BA. (8)

4.1.3 The size of α. (4)

4.2 If another circle $x^2 + 2x + y^2 - 4y = 44$ (with centre D) is drawn , determine, with calculations, whether circle C and D will touch, will not touch at all, or will intersect each other. (6)

[22]

QUESTION 5

NO CALCULATORS ALLOWED IN THIS QUESTION

5.1 A and B are complementary angles and $A > 0$ and $B > 0$.
If $12\tan A - 5 = 0$, calculate, by using a diagram, the value of
$13\sin A - 3\tan B$ (5)

5.2 Simplify the following expression:

$$\frac{\tan 300° + \cos(90° + x)}{\sin x + 2\cos(-30°)}$$ (6)

5.3.1 Given: $\sin(x + y) = \sin x \cos y + \cos x \sin y$.
Use this identity and derive an identity for $\cos(x + y)$ (3)

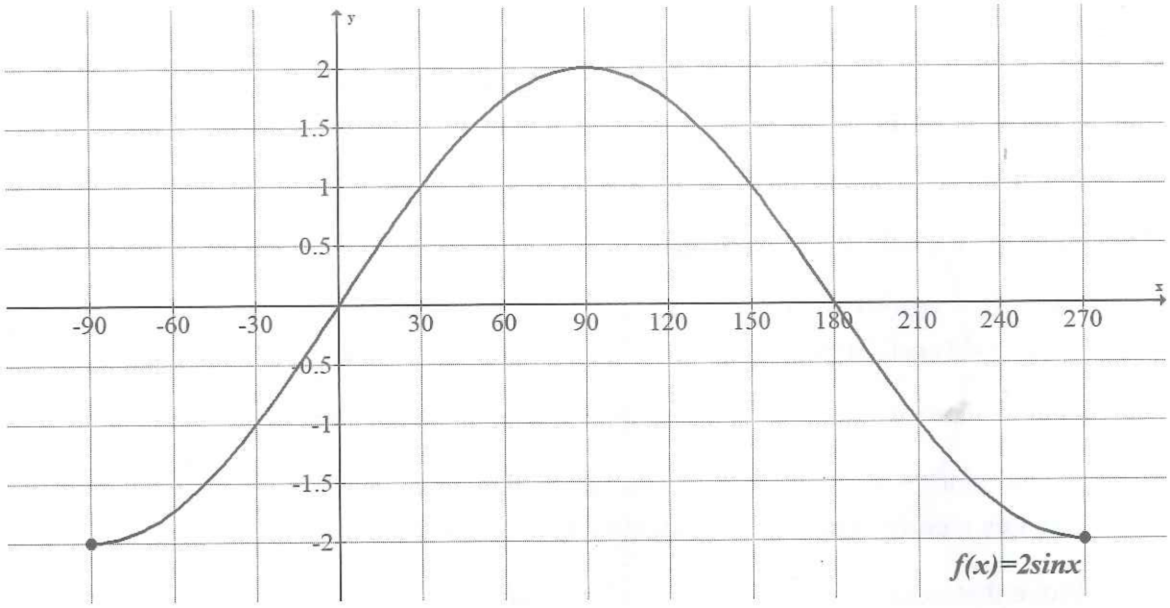
5.3.2 Prove that $\cos(x - y) - \cos(x + y) = 2\sin x \sin y$ (3)

5.3.3 Hence, or otherwise, calculate the numerical value of $2\sin 195° \sin 45°$ (5)

[22]

QUESTION 6

In the diagram the graph of $f(x) = 2\sin x$ is drawn for the interval $x \in [-90°;270°]$.



6.1 Draw the graph of g if $g(x) = \cos(x + 30°)$, for the interval $x \in [-90°;270°]$, in your answer book. Show all the intercepts with the axes, as well as the turning points. (3)

6.2 Show that $2\sin x = \cos(x + 30°)$ can also be expressed as $\tan x = \frac{\sqrt{3}}{5}$. (4)

6.3 Hence, determine the values of x, to ONE decimal, where $f(x) = g(x)$, for $x \in [-90°;270°]$. (2)

6.4 Use the solutions obtained in Question 6.3 and the graph drawn in Question 6.1 to determine for which values of x, $x \in [0°;270°]$ is:

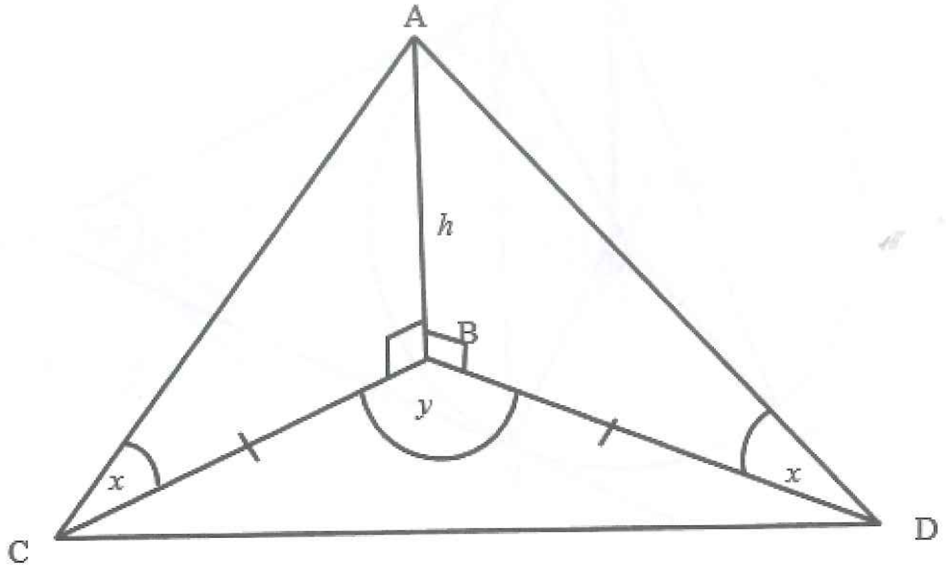
6.4.1 $f(x).g(x) \leq 0$ (3)

6.4.2 $f(x) \geq g(x)$ (2)

[14]

QUESTION 7

In the diagram C and D represent two boats in the same horizontal plane as B, the base of a lighthouse AB, which is h metres high. Boat C and D are equidistant from B. The angle of elevation from C and D to A, are both x. $C\hat{B}D = y$.

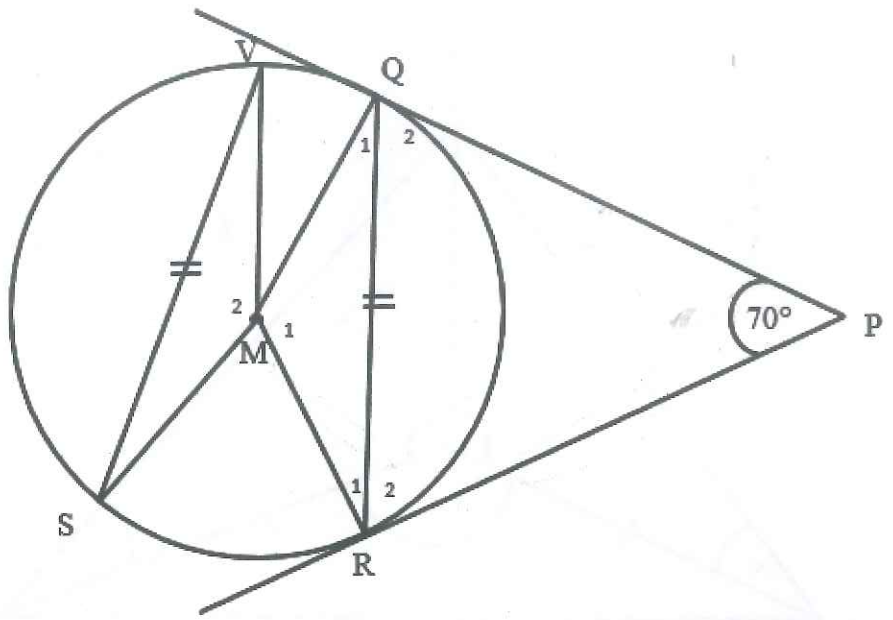


Prove that $CD = \dfrac{h}{\tan x}\sqrt{2 - 2\cos y}$ **[6]**

QUESTION 8

8.1 M is the centre of the circle SVQR and SV and QR are equal chords.

RP and QP are tangents to the circle at R and Q respectively. $R\hat{P}Q = 70°$.



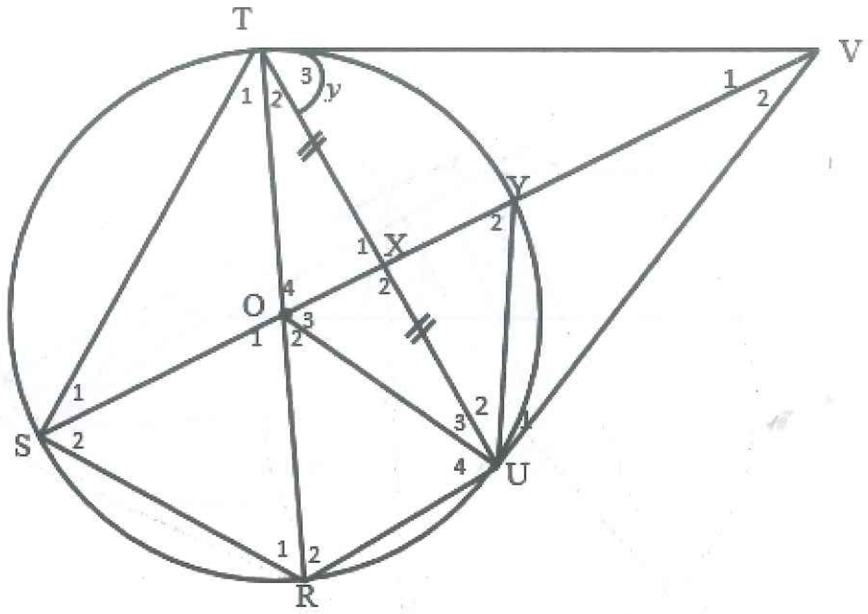
Calculate, giving reasons, the size of:

8.1.1 $\hat{R}_2$ (4)

8.1.2 $\hat{Q}_1$ (3)

8.1.3 $\hat{M}_2$ (3)

8.2 TV and VU are tangents to the circle with centre O at T and U respectively. TSRUY are points on the circle such that RT is the diameter. X is the midpoint of chord TU. $\hat{T}_3 = y$.



Prove that:

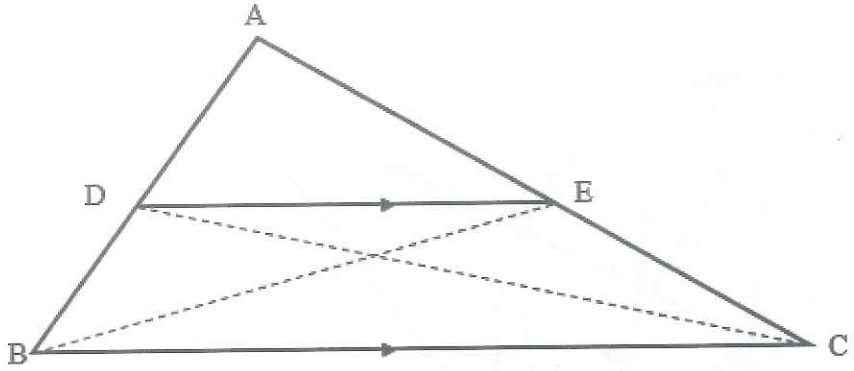
8.2.1 RU$\parallel$SY (5)

8.2.2 $\hat{T}_1 = \frac{1}{2}y$ (5)

[20]

QUESTION 9

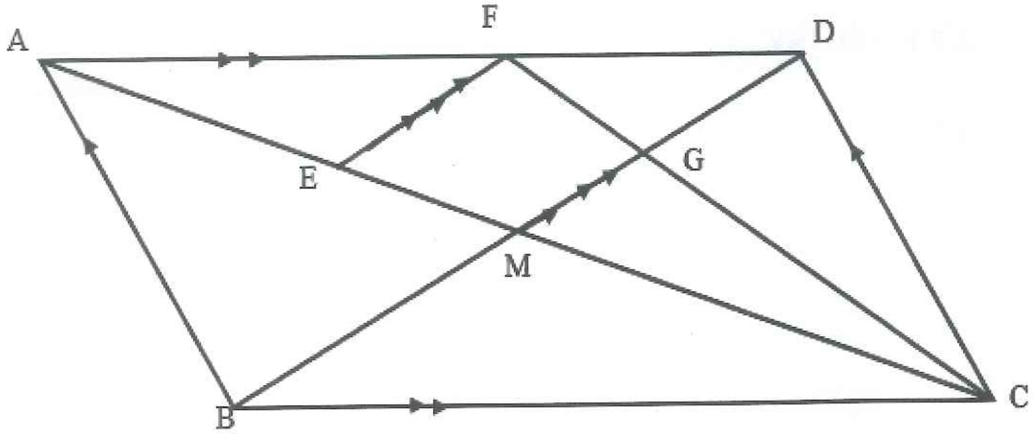
9.1 In the diagram, points D and E lie on AB and AC respectively, such that $DE \parallel BC$. DC and BE are joined.



Use the diagram to prove the theorem which states that:

if $DE \parallel BC$, then $\frac{AD}{DB} = \frac{AE}{EC}$. (6)

9.2 In the diagram ABCD is a parallelogram. The diagonals of ABCD intersect in M. F is a point on AD such that AF:FD = 4:3. E is a point on AM such that $EF \parallel BD$. FC and MD intersect in G.



Calculate, giving reasons:

9.2.1 $\frac{EM}{AM}$ (3)

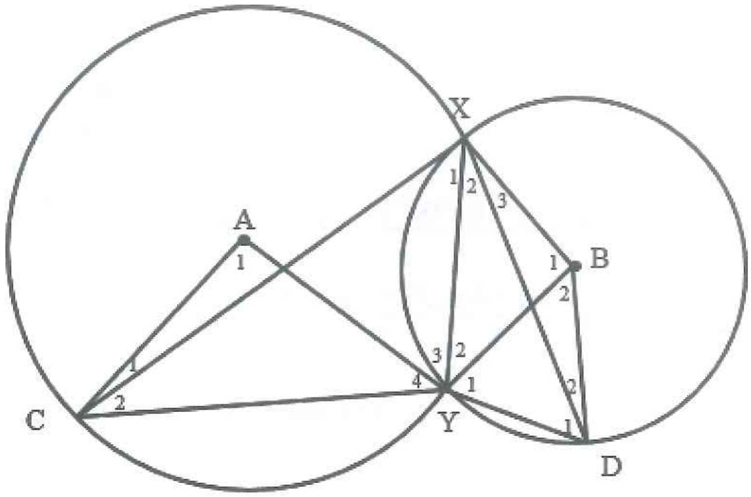
9.2.2 $\frac{CM}{ME}$ (3)

9.2.3 $\frac{\text{area}\,\Delta FDC}{\text{area}\,\Delta BDC}$ (4)

[16]

QUESTION 10

Two circles with centres A and B, intersect at X and Y. The radius of circle A is R and the radius of circle B is r. CX is a tangent to circle B at X and DX is a tangent to circle A at X.



Prove that:

10.1 $\Delta XYC\ |||\ \Delta DYX$ and hence deduce that $XY^2 = DY.YC$ (5)

10.2 $\hat{A}_1 = \hat{B}_1$ (4)

10.3 If $\Delta CAY\ |||\ \Delta YBX$, then $\dfrac{r^2}{R^2} = \dfrac{DY}{CY}$ (5)

[14]

GRAND TOTAL: 150

INFORMATION SHEET: MATHEMATICS

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $A = P(1+ni)$ $A = P(1-ni)$ $A = P(1-i)^n$ $A = P(1+i)^n$

$\sum_{i=1}^{n} 1 = n$ $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ $T_n = a + (n-1)d$ $S_n = \frac{n}{2}(2a + (n-1)d)$

$T_n = ar^{n-1}$ $S_n = \frac{a(r^n - 1)}{r-1}$; $r \neq 1$ $S_\infty = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x[(1+i)^n - 1]}{i}$ $P = \frac{x[1-(1+i)^{-n}]}{i}$ $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$ $y = mx + c$

$y - y_1 = m(x - x_1)$ $m = \frac{y_2 - y_1}{x_2 - x_1}$ $m = \tan\theta$ $(x-a)^2 + (y-b)^2 = r^2$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $a^2 = b^2 + c^2 - 2bc.\cos A$

$area\,\Delta ABC = \frac{1}{2} ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$ $\sin 2\alpha = 2\sin\alpha.\cos\alpha$

$(x\,;\,y) \to (x\cos\theta + y\sin\theta\,;\,y\cos\theta - x\sin\theta)$
$(x\,;\,y) \to (x\cos\theta - y\sin\theta\,;\,y\cos\theta + x\sin\theta)$

$\bar{x} = \frac{\sum fx}{n}$ $\sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$\hat{y} = a + bx$ $b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$