

education

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**NATIONAL
SENIOR CERTIFICATE**

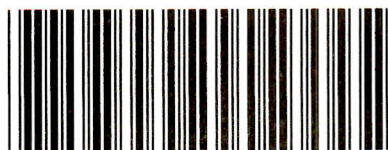
GRADE 12

MATHEMATICS P2

SEPTEMBER 2020

MARKS: 150

TIME: 3 hours



EMATHP2

**This question paper consists of 13 pages, 1 information sheet
and an answer book of 21 pages.**

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 10 questions.
2. Answer ALL the questions in the SPECIAL ANSWER BOOK provided.
3. Clearly show ALL calculations, diagrams, graphs, etc. that you have used in determining your answers.
4. Answers only will NOT necessarily be awarded full marks.
5. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
6. If necessary, round off answers to TWO decimal places, unless stated otherwise.
7. Diagrams are NOT necessarily drawn to scale.
8. An information sheet with formulae is included at the end of the question paper.
9. Write neatly and legibly.

QUESTION 1

During the 2019 Rugby World Cup the South African Springbok players played seven matches. The score obtained in each match by the Springbok players are given below:

k	19	$2k$	32	49	57	66
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The average score obtained by the Springbok players during the seven matches they played in the 2019 Rugby World Cup, was 37,43.

- 1.1 Show that $k = 13$. (2)
- 1.2 Calculate the standard deviation of the data. (2)
- 1.3 The two highest scores lie outside p standard deviation of the mean. Calculate the maximum value of p . (3)
- 1.4 Suppose EACH score is increased by adding a value of y to each score.
- 1.4.1 Write down the interquartile range of the new data set. (1)
- 1.4.2 Calculate the value of y if the lower quartile of the new data set is 30. (1)
- [9]**

QUESTION 2

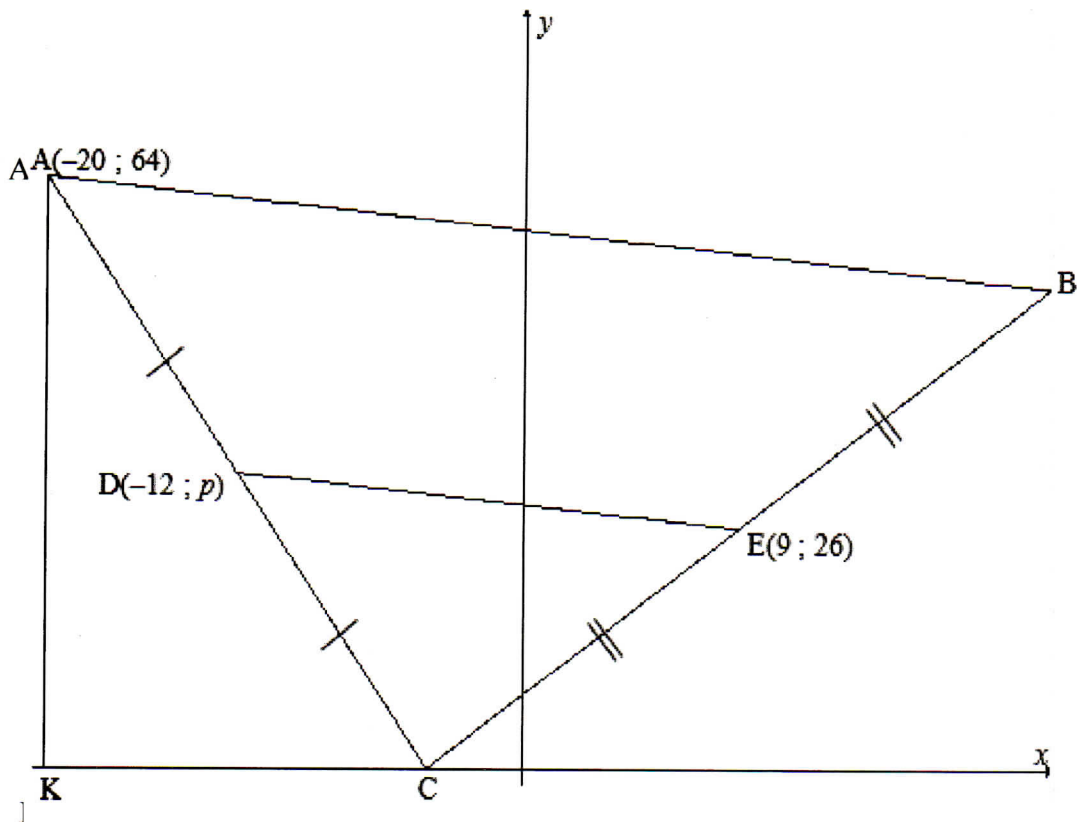
A town has two popular municipal swimming pools, namely the Madiba swimming pool and the Cronje swimming pool. The table below indicates the number of people that visited the **Madiba** swimming pool over a period of eight days. The corresponding maximum daily temperatures (in °C) for each day is also given in the table.

Maximum daily temperature (°C)	18	24	22	28	34	30	26	20
Number of visitors	180	210	200	350	410	300	280	195

- 2.1 Determine the equation of the least squares regression line of the data. (3)
- 2.2 Calculate the value of the correlation coefficient. (1)
- 2.3 The manager of the **Madiba** swimming pool asks a grade 12 Mathematics learner, Jan-Breet, to predict how many visitors can be expected on a certain day with a maximum daily temperature of 32 °C. Jan-Breet predicts that 365 visitors can be expected on that day. Should the manager take this prediction seriously and prepare for 365 visitors? Motivate your answer. (3)
- 2.4 Over the very same period of eight days, with the exact same daily maximum temperatures, the **Cronje** swimming pool also record the number of visitors for each day. The equation for the least squares regression line for the **Cronje** swimming pool is given as $\hat{y} = k + 12,85x$.
Which swimming pool had the highest number of visitors over the period of eight days? Motivate, **without any calculations**, your answer. (3)
- [10]

QUESTION 3

In the diagram below, $A(-20 ; 64)$, B and C are the vertices of ΔABC , with C a point on the x -axis. $D(-12 ; p)$ and $E(9 ; 26)$ are the midpoints of AC and BC . $KC = 16$ units.

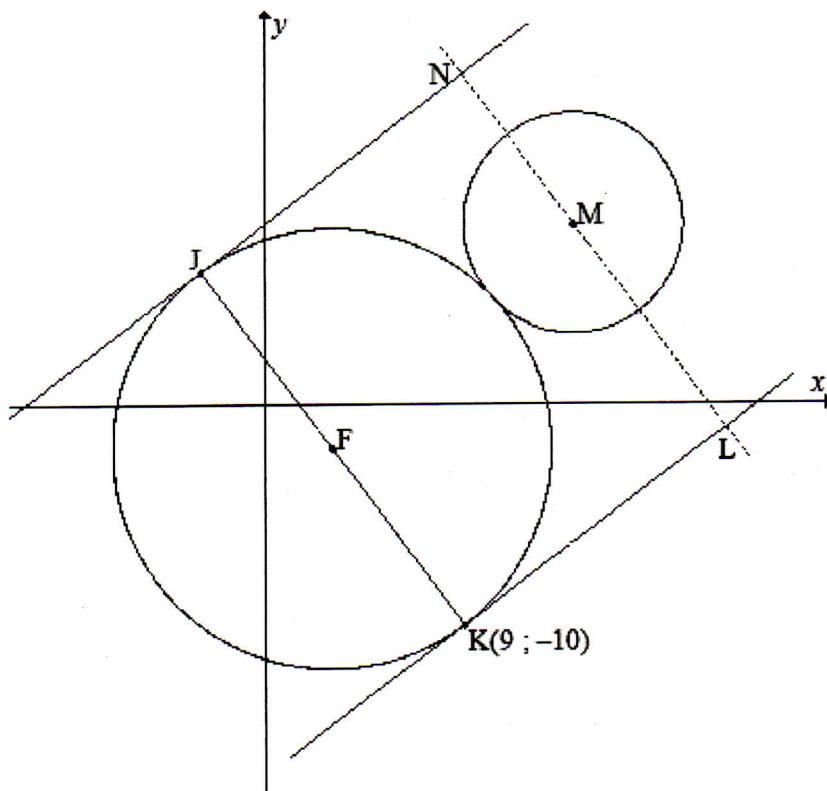


- 3.1 The gradient of AK is undefined. Write down the equation of the line AK . (1)
- 3.2 Calculate the x -intercept of the line AC . (1)
- 3.3 Calculate the length of AC . (2)
- 3.4 Show that $p = 32$. (1)
- 3.5 Determine the equation of the line AB . (5)
- 3.6 Calculate the size of \hat{KAB} . (5)
- 3.7 If $\hat{KAC} = 38,67^\circ$, determine the area of ΔABC . (6)

[21]

QUESTION 4

In the diagram below, the equation of the circle with centre F is $x^2 + y^2 - 6x + 4y = 87$.
 The equation of the circle with centre M is $(x - 14)^2 + (y - h)^2 = 25$.
 The point K(9 ; -10) lies on the circle with centre F. JN and KL are tangents to the circle F at the points J and K. The two circles touch externally.

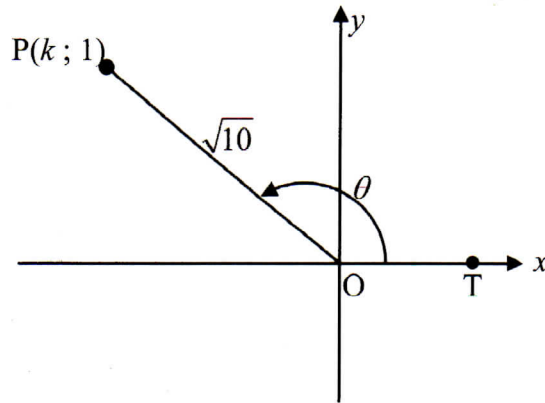


- 4.1 Write down the length of the radius of the circle with centre M. (1)
- 4.2 Determine the coordinates of F. (3)
- 4.3 Write down the length of the radius of the circle with centre F. (2)
- 4.4 Show, with calculations, that $h = \sqrt{104} - 2$. (4)
- 4.5 Show that the equation of the tangent JN is $y = \frac{3}{4}x + \frac{33}{4}$, if J(-3 ; 6) is given. (3)
- 4.6 A line NL, passes through the centre M, such that $NL \parallel JK$. The circle with centre M, is translated alongside the line NL and between the two tangents of circle F, for the interval $x \in [p; q]$. Give the minimum value of p, such that the circle M will not intersect the tangents to the circle with centre F, when translated alongside NL. (6)

[19]

QUESTION 5

- 5.1 In the diagram, $P(k; 1)$ is a point in the 2nd quadrant and is $\sqrt{10}$ units from the origin. T is a point on the positive x -axis and obtuse $\widehat{POT} = \theta$.



5.1.1 Calculate the value of k . (2)

5.1.2 **Without using a calculator**, calculate the value of:

(a) $-\cos \theta$ (1)

(b) $\sqrt{1 - \sin(180^\circ + \theta) \cdot \sin(360^\circ - \theta)}$ (4)

5.1.3 Show, **without using a calculator**, that

$$\sin\left(\frac{\theta}{2} - 15^\circ\right) \cdot \cos\left(\frac{\theta}{2} - 15^\circ\right) = \frac{\sqrt{3} + 3}{4\sqrt{10}} \quad (5)$$

5.2 Given: $\sin(\theta + 60^\circ) = d$

5.2.1 Write down the values of d such that $\sin(\theta + 60^\circ)$ is defined. (2)

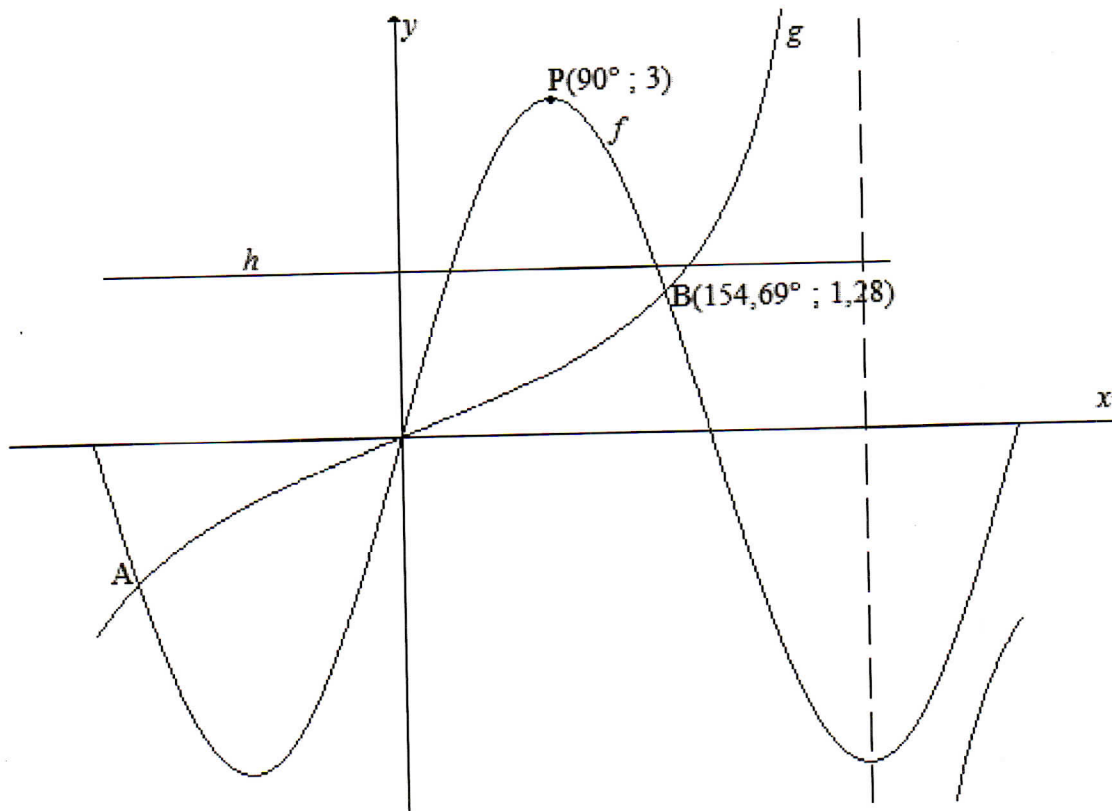
5.2.2 If $d = \sin \theta \cdot \cos 60^\circ + \sqrt{\frac{3}{8}}$, the general solution of $\sin(\theta + 60^\circ) = d$, is $\theta = \pm m^\circ + n \cdot 360^\circ$. Calculate, **without the use of a calculator**, the value of m . (5)

5.3 It is given that $A = 2^{\sin(a-m) \cdot \sin(b-m) \cdot \sin(c-m) \dots \sin(x-m)}$
Calculate the value of A . (3)

[22]

QUESTION 6

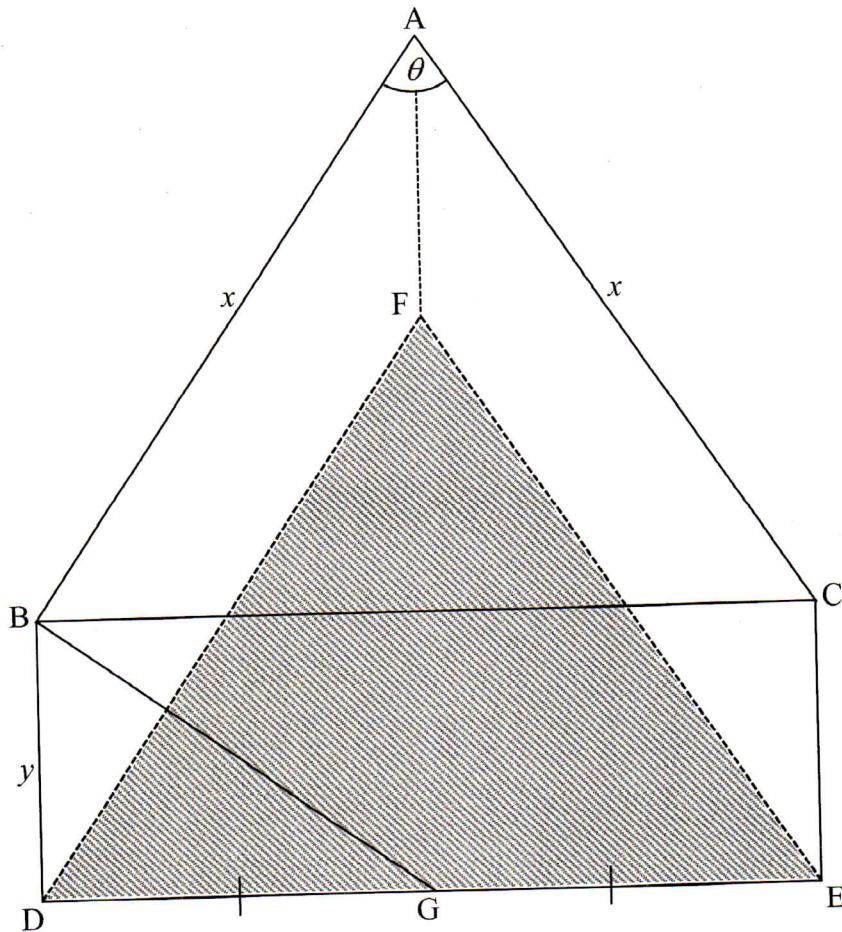
In the diagram below, the graphs of $f(x) = a\sin x$ and $g(x) = \tan bx$ are drawn for the interval $x \in [-180^\circ ; 360^\circ]$. The point $P(90^\circ ; 3)$ is on the graph of f . The function $h(x) = \frac{3}{2}$ intersects the graphs of f and g . The asymptote of g goes through one of the turning points of f . A and B($154,69^\circ ; 1,28$) are points of intersections of the graphs f and g .



- 6.1 Write down the values of a and b . (2)
 - 6.2 Write down the period of g . (1)
 - 6.3 Write down the coordinates of A. (2)
 - 6.4 It is given that $2 \tan 56,31^\circ - 6 \sin 168,93^\circ = t + 5$. Calculate in terms of t , the vertical distance between f and g , at the point where h intersects g . (4)
- [9]**

QUESTION 7

In the diagram below, steel poles AF, BD and CE are equal in length and vertical to the horizontal plane FDE. The steel poles hold $\triangle ABC$ that forms the roof of a storage area. G is the midpoint of DE. BCED is in the same vertical plane. $AB = AC = x$, $BD = y$ and $\hat{BAC} = \theta$.



7.1 Show that $BC = x\sqrt{2(1 - \cos \theta)}$ (3)

7.2 Write down the size of \hat{BDG} . (1)

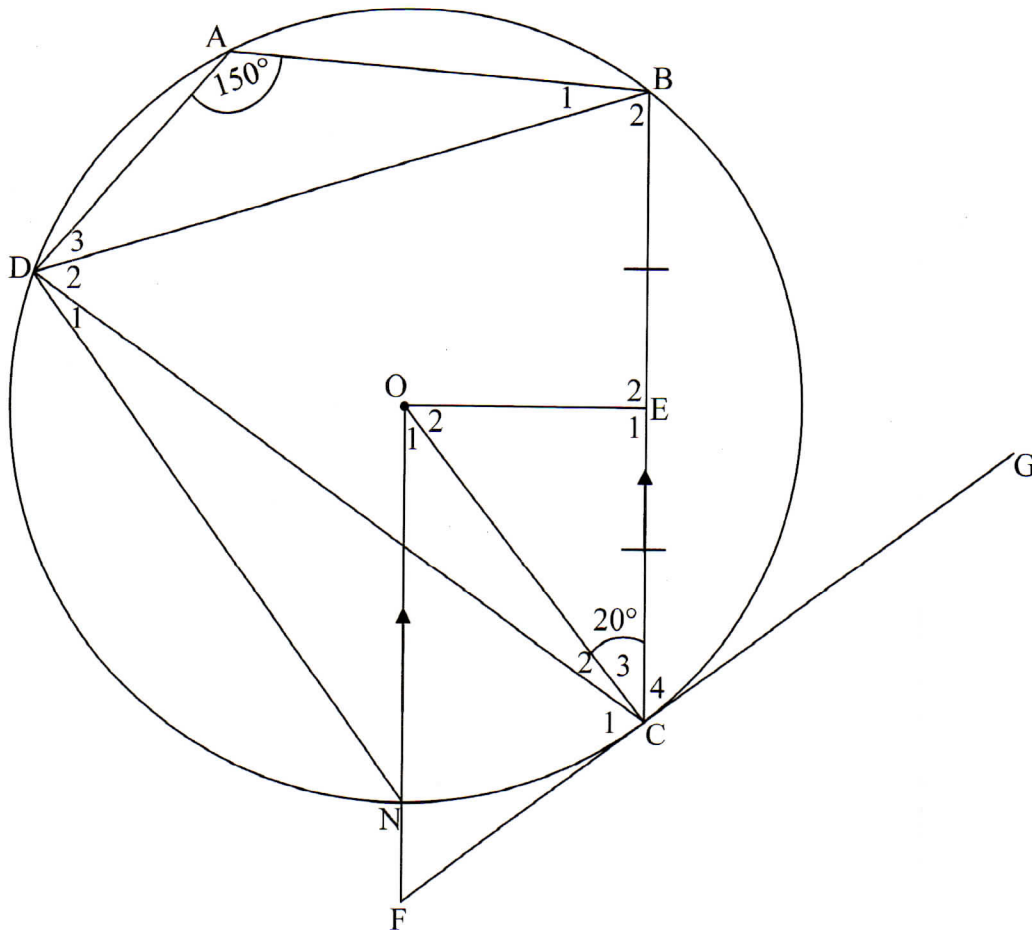
7.3 Show that $BG = \sqrt{\frac{2y^2 + x^2(1 - \cos \theta)}{2}}$ (3)

7.4 It is given that $y = \frac{8}{3}$ metres and $x = \frac{15}{2}$ metres. Calculate the maximum length of BG. (3) [10]

Give reasons for your statements in QUESTIONS 8, 9 and 10.

QUESTION 8

In the diagram, O is the centre of the circle with radius OC. ABCD is a cyclic quadrilateral. N is another point on the circle such that $ON \parallel BC$. Chord DN is drawn. OE bisects chord BC at E. GC is a tangent to the circle at C. ON and GC are produced to meet in F. $\hat{O}CE = 20^\circ$ and $\hat{D}AB = 150^\circ$.



8.1 Calculate, with reasons, the size of:

8.1.1 \hat{O}_1 (1)

8.1.2 \hat{D}_1 (2)

8.1.3 \hat{C}_2 (2)

8.1.4 \hat{D}_2 (4)

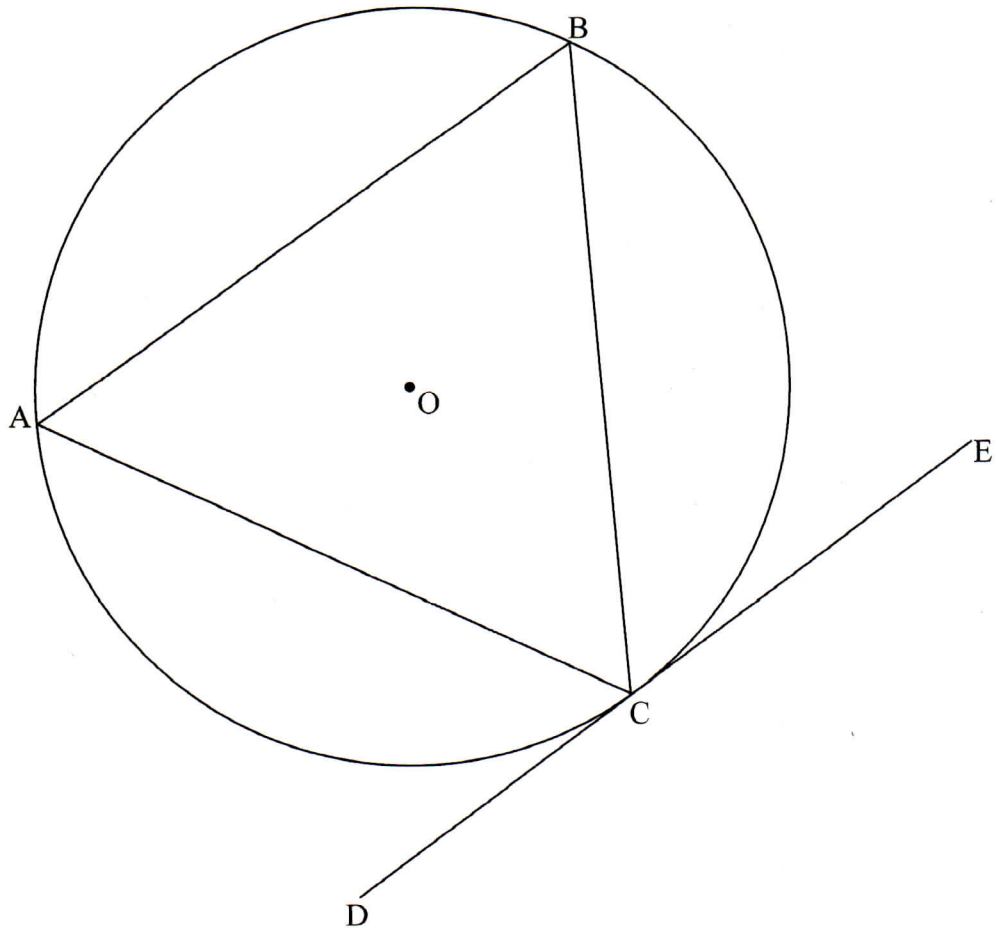
8.2 It is given that $OF = \sqrt{3}y$ and $EC = 2x$. The area of OECF is $\frac{3}{4}y^2 - x^2$.

Determine the length of OE in terms of x and y. (5)

[14]

QUESTION 9

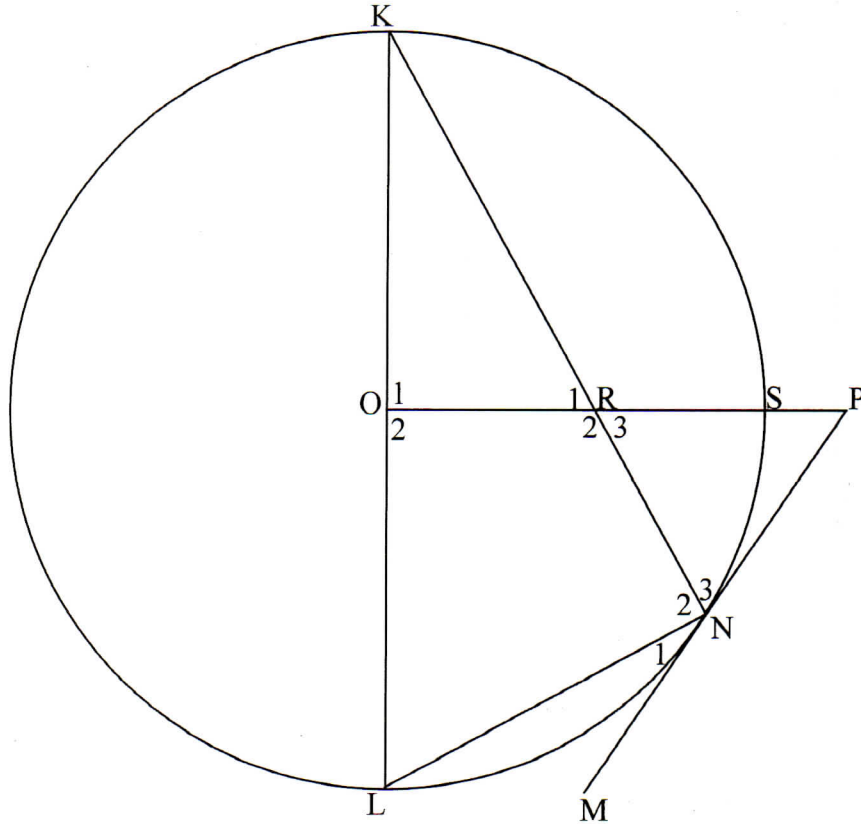
- 9.1 In the diagram, chords AB, BC and AC are drawn in the circle with centre O. DCE is the tangent to the circle at C.



Prove the theorem which states that $\hat{ACD} = \hat{ABC}$

(5)

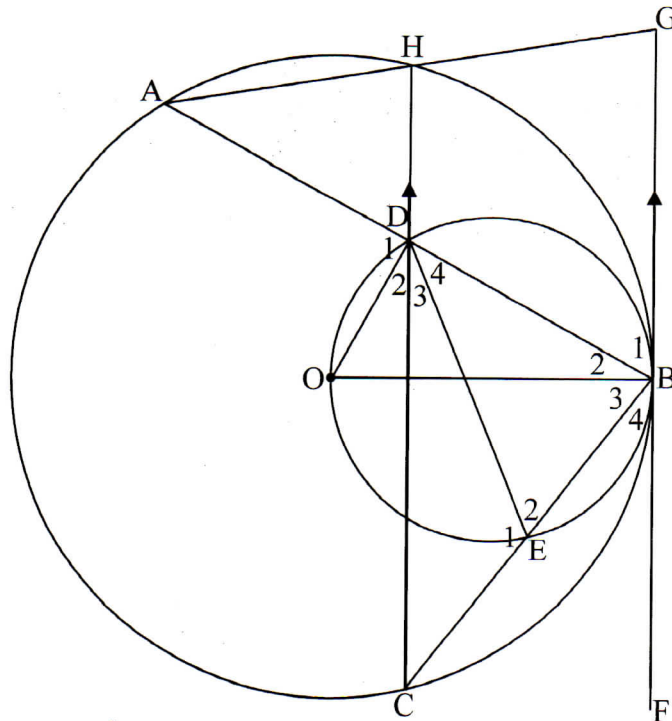
- 9.2 In the diagram below, the centre of the circle is O with the diameter KL. MNP is a tangent to the circle at N. Chords KN and LN are drawn. R is a point on KN and ORSP forms a straight line. $OP = 15$ units.



- 9.2.1 Give the length of OR if $OR : RP = 2 : 3$. (1)
- 9.2.2 It is given that $KR^2 - KO^2 = 36$. Prove that $OR \perp KL$. (2)
- 9.2.3 Prove that ORNL is a cyclic quadrilateral. (3)
- 9.2.4 MNP is translated to the left side of the circle, such that M' falls outside of the circle and N' inside of the circle. The point P' coincides with the point S. The points O, L and S remain fixed as indicated on the original diagram and $MNP \parallel M'N'P'$. Is it possible for $M'N'P'$ to be a tangent to the circle drawn through O, L and S? Motivate your answer. (4)
- [15]**

QUESTION 10

Two circles touch each other internally at B. O, the centre of the bigger circle, lies on the circumference of the smaller circle. FBG is a common tangent. AB and BC are chords of the bigger circle and intersect the smaller circle at D and E respectively. AHG forms a straight line. Chords OD, DE and OB of the smaller circle are drawn. $AH:AG = 1:2$.



10.1 Give a reason why:

10.1.1 $\hat{B}_4 = \hat{HCB}$ (1)

10.1.2 $\hat{B}_4 = \hat{D}_4$ (1)

10.2 Prove that:

10.2.1 $\triangle DBE \parallel \triangle DBC$ (3)

10.2.2 D is the midpoint of AB (3)

10.2.3 $\frac{AB^2}{4} = BC \cdot BE$ (4)

10.2.4 $CE:BC = 5:9$ if $DB = 3$ units and $BE = 2$ units. (3)

10.3 Calculate the ratio of $\frac{\text{area } \triangle AHD}{\text{area } \triangle HGB}$. (6)

[21]

TOTAL: 150

INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1 + ni) \quad A = P(1 - ni) \quad A = P(1 - i)^n \quad A = P(1 + i)^n$$

$$T_n = a + (n - 1)d \quad S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$T_n = ar^{n-1} \quad S_n = \frac{a(r^n - 1)}{r - 1}; r \neq 1 \quad S_\infty = \frac{a}{1 - r}; -1 < r < 1;$$

$$F = \frac{x[(1 + i)^n - 1]}{i} \quad P = \frac{x[1 - (1 + i)^{-n}]}{i} \quad f'(x) = \lim_{h \rightarrow 0} \frac{f(x + h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c \quad y - y_1 = m(x - x_1) \quad m = \frac{y_2 - y_1}{x_2 - x_1} \quad m = \tan \theta$$

$$(x - a)^2 + (y - b)^2 = r^2$$

$$\text{In } \triangle ABC: \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \quad a^2 = b^2 + c^2 - 2bc \cdot \cos A$$

$$\text{area } \triangle ABC = \frac{1}{2} ab \cdot \sin C$$

$$\sin(\alpha + \beta) = \sin \alpha \cdot \cos \beta + \cos \alpha \cdot \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cdot \cos \beta - \cos \alpha \cdot \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cdot \cos \beta - \sin \alpha \cdot \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cdot \cos \beta + \sin \alpha \cdot \sin \beta$$

$$\cos 2\alpha = \begin{cases} \cos^2 \alpha - \sin^2 \alpha \\ 1 - 2\sin^2 \alpha \\ 2\cos^2 \alpha - 1 \end{cases}$$

$$\sin 2\alpha = 2 \sin \alpha \cdot \cos \alpha$$

$$\bar{x} = \frac{\sum x}{n}$$

$$\sigma^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\hat{y} = a + bx$$

$$b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$$