Province of the
EASTERN CAPE
EDUCATION

NATIONAL SENIOR CERTIFICATE
NASIONALE SENIOR SERTIFIKAAT

GRADE/*GRAAD* 12

SEPTEMBER 2017

MATHEMATICS P2/*WISKUNDE V2*
MARKING GUIDELINE/*NASIENRIGLYN*

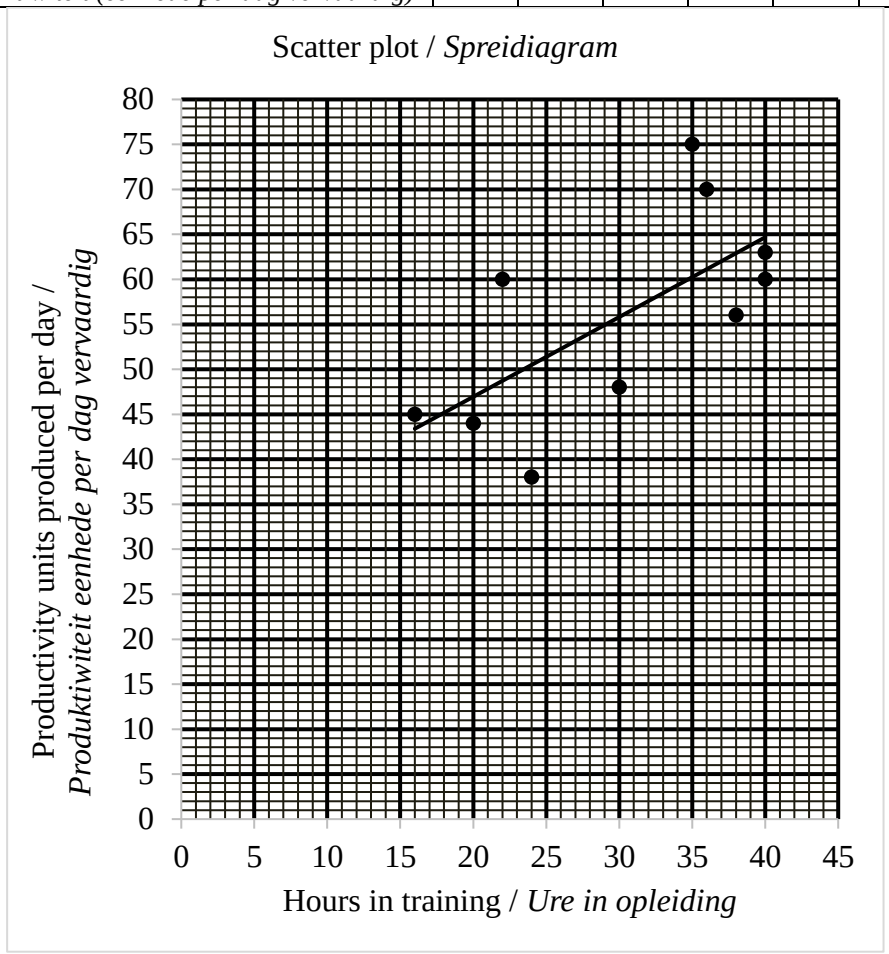
MARKS/*PUNTE*: 150

This marking guideline consists of 15 pages.
Hierdie nasienriglyn bestaan uit 15 bladsye.

QUESTION/*VRAAG* 1

Employee / *Werknemer*	1	2	3	4	5	6	7	8	9	10
Hours in training / *Ure in opleiding*	16	36	20	38	40	30	35	22	40	24
Productivity (units produced per day) *Produktiwiteit (eenhede per dag vervaardig)*	45	70	44	56	60	48	75	60	63	38

1.1



✓ 2-4 correct points / *korrekte punte*

✓ 5-7 correct points / *korrekte punte*

✓ plotting all points / *afsteek van alle punte*

(3)

1.2

$a = 29{,}22$

$b = 0{,}89$

$y = 29{,}22 + 0{,}89x$

✓ A
✓ B
✓ equation / *vergelyking*

(3)

1.3

$(30{,}9\,;55{,}50)$

y-int / *y-afsnit* 29,22

✓ mean point / *gemiddelde punt* $(30{,}90\,;55{,}50)$ and/*en* *y*-int/*y-afsnit* 29,22
✓ regression line / *regressielyn*

(2)

1.4

$y = 29{,}22 + 0{,}89(25)$

$= 51{,}47$

OR/*OF*

$y = 51{,}38$ [calculator use]/[*sakrekenaar gebruik*]

ANSWER ONLY FULL MARKS
SLEGS ANTWOORD VOLPUNTE

✓ subst./ *verv.*
✓ answer / *antwoord*

(2)

1.5

$r = 0{,}66$

✓ answer / *antwoord*

(1)

1.6

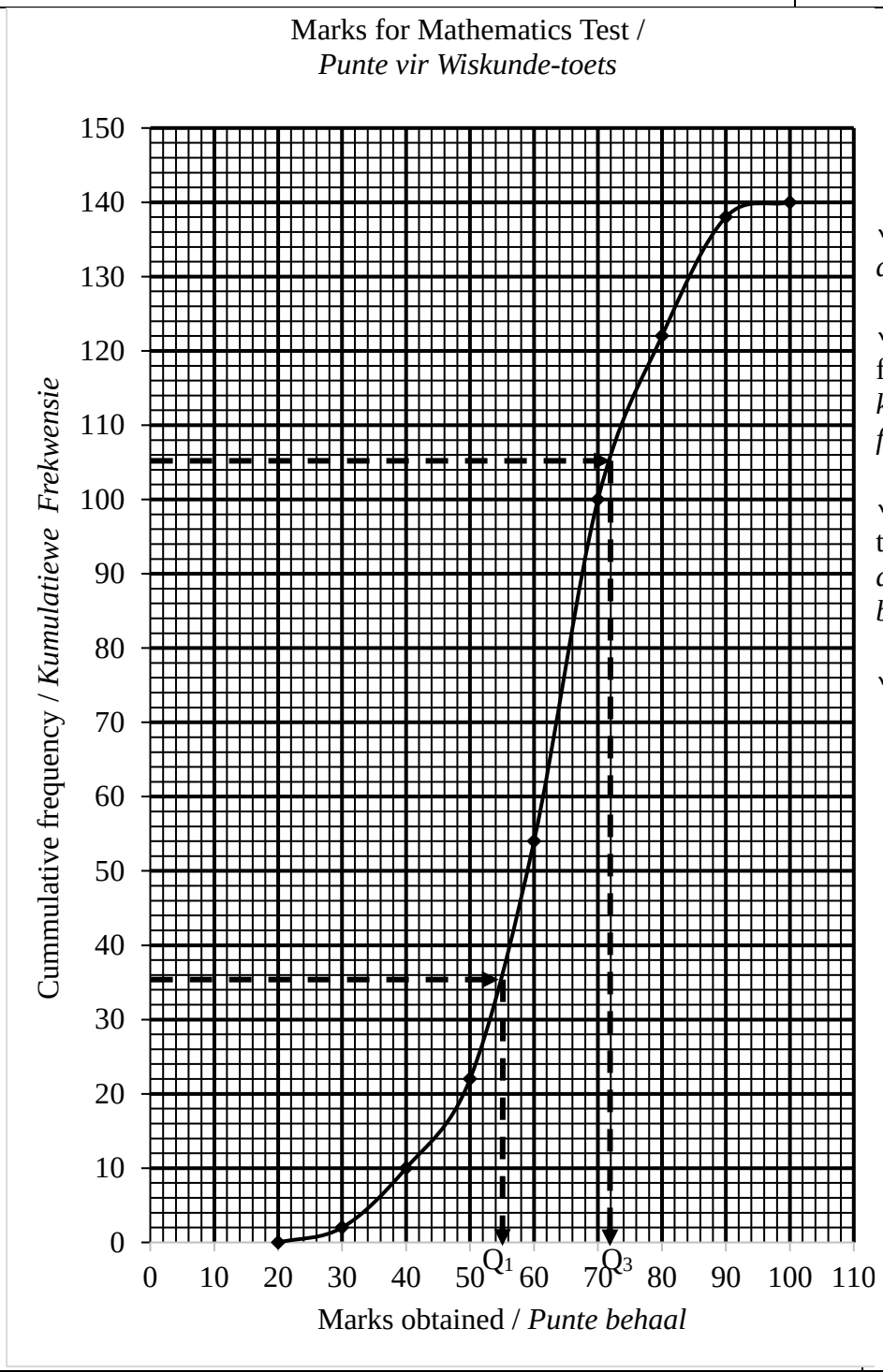
Moderately strong positive correlation /
Matige sterk positiewe korrelasie

✓ answer / *antwoord*

(1)

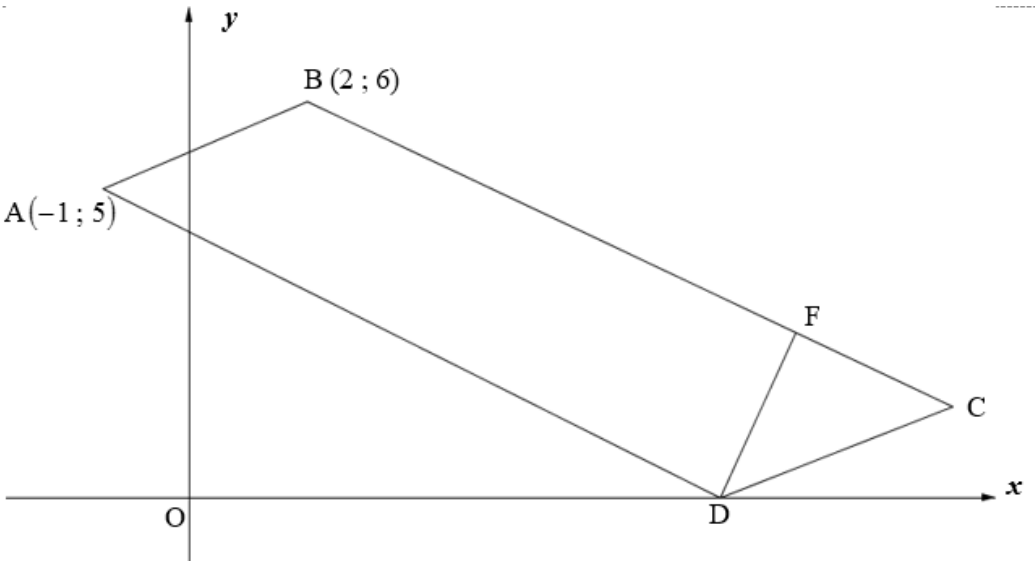
[12]

QUESTION/*VRAAG* 2

2.1	140 learners / *leerders*	✓answer / *antwoord*	(1)
2.2	$60 < x \leq 70$	✓answer / *antwoord*	(1)
2.3	 ✓ grounding / *anker* ✓cumulative frequency / *kumulatiewe frekwensie* ✓plotting against the upper limit / *afsteek teen die boonste limiet* ✓shape / *vorm*	(4)
2.4	$Q_3 - Q_1 = 72 - 55$ $= 17$	✓ Q_3 & Q_1 ✓ IQR	(2)

[8]

QUESTION/*VRAAG* 3

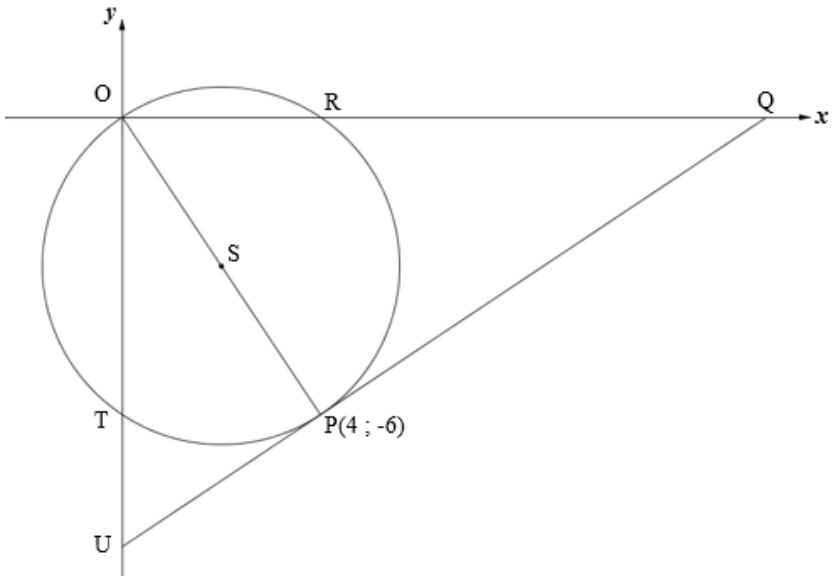


3.1	$BC: x+2y=14$ $y=-\frac{1}{2}x+7$ $m_{BC}=m_{AD}=-\frac{1}{2}$ $y-5=-\frac{1}{2}(x+1)$ $y=-\frac{1}{2}x+\frac{9}{2}$	✓ $m_{AD}=-\frac{1}{2}$ ✓ subst. *m* and A into correct formula / *verv. van m en A in korrekte formule* ✓ $y=-\frac{1}{2}x+\frac{9}{2}$	(3)
3.2	$-\frac{1}{2}x+\frac{9}{2}=0$ $x=9$ **OR/*OF*** $D(9;0)$ $m_{AD}=m_{BC}$ $\frac{0-5}{x+1}=-\frac{1}{2}$ $-10=-x-1$ $x=9$ $D(9;0)$	✓ $y=0$ ✓ $x=9$ ✓ $y=0$ ✓ $x=9$	(2)

3.3	$m_{FD} = \frac{2-0}{10-9}$ $= 2$ $m_{BC} \times m_{FD} = 2 \times -\frac{1}{2}$ $= -1$ FD $\perp$ BC $\quad [m_{BC} \times m_{FD} = -1]$	✓ correct subst. / *korrekte verv.* ✓ $m_{FD} = 2$ ✓ $m_{BC} \times m_{FD} = -1$	(3)
3.4	AD $= \sqrt{(-1-9)^2 + (5-0)^2}$ $= \sqrt{125}$ $= 5\sqrt{5}$	✓ subt. into correct formula / *verv. in korrekte formule* ✓ AD $= 5\sqrt{5}$	(2)
3.5	FD $= \sqrt{(9-10)^2 + (0-2)^2}$ $= \sqrt{5}$ A of ABCD $= b \times h$ $= 5\sqrt{5} \times \sqrt{5}$ $= 25$	✓ subst. into correct formula / *verv. in korrekte formule* ✓ FD $= \sqrt{5}$ ✓ subst into correct formula / *verv. in korrekte formule* ✓ answer / *antwoord*	(4)
3.6	$m_{AB} = m_{DC}$ $= \frac{6-5}{2-(-1)}$ $= \frac{1}{3}$ $\therefore$ *Inclination of / Helling van DC* $= 18,43^0$ *Inclination of / Helling van AD* $= 180^0 - \tan^{-1}\left(\frac{1}{2}\right)$ $= 153,43^0$ $A\hat{D}C = 153,43^0 - 18,43^0$ $= 135^0$ $\therefore A\hat{B}C = 135^0 \quad [opp \angle s\ of\ a\ parm. / teenoorst. \angle e\ van \|^m]$	✓ subst. into correct formula / *verv. in korrekte formule* ✓ $m_{AB} = \frac{1}{3}$ ✓ $\theta_{DC} = 18,43^0$ ✓ $\theta_{AD} = 153,43^0$ ✓ A$\hat{D}$C $= 135^0$ ✓ $A\hat{B}C = 135^0$	(6)

[20]

QUESTION/*VRAAG* 4

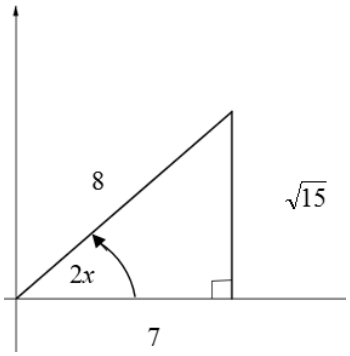


4.1	$x_S = \frac{0+4}{2}$ $y_S = \frac{0-6}{2}$ $= 2$ $= -3$ $S(2;-3)$ **Answer only full marks** ***Slegs antwoord volpunte***	✓ subst. into correct formula *verv. in korrekte formule* ✓ both coordinates / *beide koördinate*	(2)
4.2	$SP = \sqrt{(2-4)^2 + (-3+6)^2}$ $= \sqrt{13}$	✓ subst. into correct formula *verv. in korrekte formule* ✓ answer / *antwoord*	(2)
4.3	$(x-2)^2 + (y+3)^2 = 13$	✓ correct subt. centre / *korrekte verv. middelpunt* ✓ $r^2 = 13$.	(2)
4.4	Tangent $\perp$ radius / *Raaklyn* $\perp$ *radius*	✓ answer / *antwoord*	(1)
4.5	$m_{\text{rad}} = \frac{-3+6}{2-4}$ $= -\frac{3}{2}$ $m_{\text{tan}} = \frac{2}{3}$ $[\text{radius} \perp \tan]$ [*radius* $\perp$ *raaklyn*] $y+6 = \frac{2}{3}(x-4)$ $= \frac{2}{3}x - \frac{26}{3}$	✓ subst. into correct form. *verv. in korrekte formule* ✓ $m_{\text{rad}} = -\frac{3}{2}$ ✓ $m_{\text{tan}} = \frac{2}{3}$ ✓ subst. into correct form / *verv. in korrekte formule*	(4)

4.6	$(0-2)^2+(y+3)^2=13$ $(y+3)^2=9$ $y+3=\pm 3$ $y_T=-6$ $T(0;-6)$ **OR / *OF*** $(0-2)^2+(y+3)^2=13$ $y^2+6y=0$ $y(y+6)=0$ $y_T=-6$ $T(0;-6)$ **OR / *OF*** Draw horizontal line $y=-3$ with M on OT *Trek horisontale lyn* $y=-3$ *met* M *op* OT OM = MT $[\perp$ from centre bisect the chord$]$ $[\perp$ *vanaf middelpunt halveer koord*$]$ $OM=3$ $\therefore MT=3$ $\therefore OT=6$ $\therefore T(0;-6)$	✓ $x=0$ ✓ $(y+3)^2=9$ ✓ $y+3=\pm 3$ ✓ $y=-6$ ✓ subst. $x = 0$ in eqn of circle *verv. x = 0 in verg. van sirkel* ✓ standard form / *standaardvorm* ✓ factors / *faktore* ✓ $y_T=-6$ ✓ S/R ✓ length of / *lengte van* MT ✓ length of / *lengte van* OT ✓ answer / *antwoord*	(4)
4.7	*At U*, $y=-\frac{26}{3}$ $\therefore TU=\frac{26}{3}-6$ $=\frac{8}{3}$ $\frac{\text{Area }\Delta OTP}{\text{Area }\Delta PTU}=\frac{\frac{1}{2}\times 6\times 4}{\frac{1}{2}\times\frac{8}{3}\times 4}$ $=\frac{9}{4}$	✓ $U\left(0;-\frac{26}{3}\right)$ ✓ length of / *lengte van* TU ✓ length of / *lengte van* OT ✓ correct subst / *korrekte verv.* ✓ answer / *antwoord*	(5)

[20]

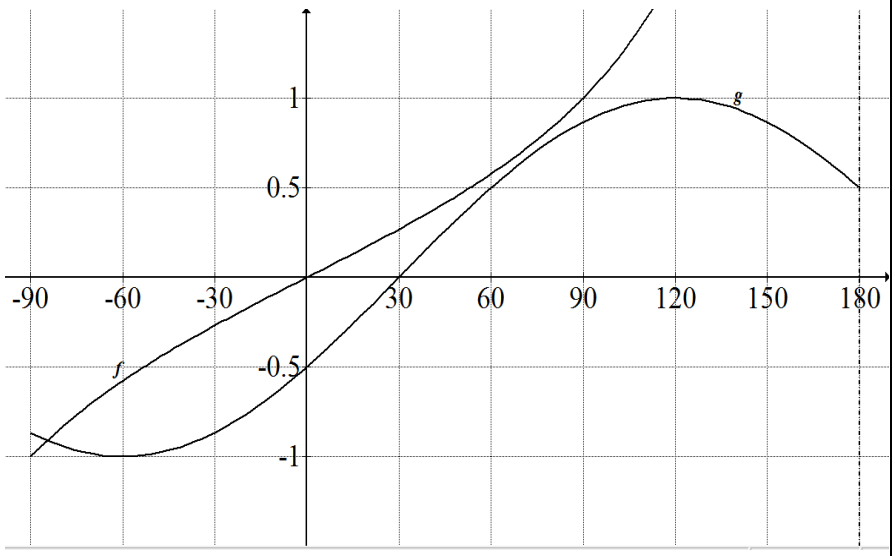
QUESTION/*VRAAG* 5

5.1.1	$\sin 2x = \frac{\sqrt{15}}{8}$  $\cos 2x = 2\cos^2 x - 1$ $2\cos^2 x = \cos 2x + 1$ $\cos x = \sqrt{\frac{\cos 2x + 1}{2}}$ $= \sqrt{\frac{\frac{7}{8} + 1}{2}}$ $= \frac{\sqrt{15}}{4}$ **OR / *OF*** $\cos 2x = 2\cos^2 x - 1$ $\frac{7}{8} = 2\cos^2 x - 1$ $\frac{15}{16} = \cos^2 x$ $\therefore \cos x = \frac{\sqrt{15}}{4}$	✓ diagram / *diagram* ✓ identity of cos2*x* *identiteit van cos2x* ✓ cos *x* subject of formula / cos *x onderwerp v/d formule* ✓ $\cos 2x = \frac{7}{8}$ ✓ answer / *antwoord* ✓ identity / *identiteit* ✓ $\cos 2x = \frac{7}{8}$ ✓ $\cos^2 x = \frac{15}{16}$ ✓ answer / *antwoord*	(5)
5.2	$\frac{\sin(180^0 - \theta).\sin(540^0 - \theta).\cos(\theta - 90^0)}{\tan(-\theta).\sin^2(360^0 - \theta)}$. $= \frac{(\sin\theta)(\sin\theta).(\sin\theta)}{(-\tan\theta)(-\sin\theta)^2}$. $= \frac{\sin\theta}{-\frac{\sin\theta}{\cos\theta}}$ $= -\cos\theta$	✓ sin θ ✓ sin θ ✓ sin θ ✓ $(-\tan\theta)$ ✓ $(-\sin\theta)^2$ ✓ $\frac{\sin\theta}{\cos\theta} = \tan\theta$ ✓ $(-\cos\theta)$	(7)

5.3.1	$\text{LHS} = \dfrac{\sin 5x.\cos 3x - \cos 5x.\sin 3x}{\tan 2x} - 1$ $= \dfrac{\sin(5x-3x)}{\tan 2x} - 1$ $= \dfrac{\sin 2x}{\dfrac{\sin 2x}{\cos 2x}} - 1$ $= \cos 2x - 1$ $= 1 - 2\sin^2 x - 1$ $= -2\sin^2 x$ $\therefore$ LHS = RHS		✓ $\sin(5x-3x)$ ✓ $\tan 2x = \dfrac{\sin 2x}{\cos 2x}$ ✓ simplification / *vereenvoudiging* ✓ identity/*identiteit* $1-2\sin^2 x$	(4)
5.3.2	$\tan 2x = 0$ $2x = 0^0$ *or / of* 180^0 $x = 0^0$ *or / of* 90^0	OR/*OF* $\tan 2x$ is undefined / *is ongedefinieerd* $2x = 90^0$ or/*of* 270^0 $x = 45^0$ or/*of* 135^0	✓ $\tan 2x = 0$ /undefined *ongedefinieerd* ✓ 0^0 or/of 180^0 ✓ 90^0 or/of 270^0 ✓ answers / *antwoorde*	(4)

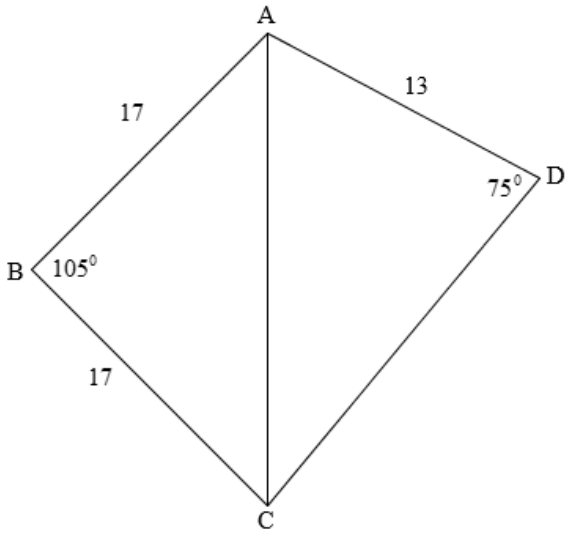
[20]

QUESTION/*VRAAG* 6

6.1		***f*** ✓ both intercepts / *beide afsnitte* ✓asymptote / *asimptote* ✓shape / *vorm* ***g*** ✓intercepts / *afsnitte* ✓min& max values / *waardes* ✓shape / *vorm*	(6)
6.2	Period(*e*) of/*van* $f(\frac{1}{2}x) = \frac{180^0}{\frac{1}{2}}$ $= 360^0$ Answer only full marks *Slegs antwoord volpunte*	✓ $\frac{180^0}{\frac{1}{2}}$ ✓ 360^0	(2)
6.3	$0^0 < x < 30^0$	✓critical values / *kritiese waardes* ✓notation / *notasie*	(2)
6.4	$x = 170^0$	✓ answer / *antwoord*	(1)

[11]

QUESTION/*VRAAG* 7

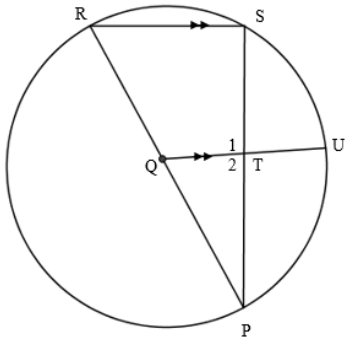


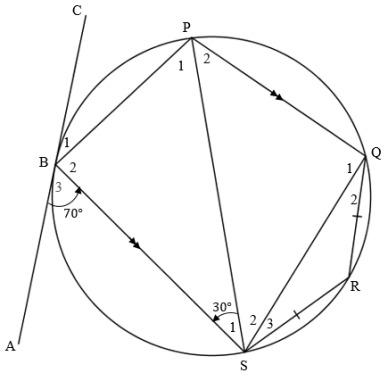
7.1	$\text{Area } \Delta ABC = \frac{1}{2}.AB.BC.\sin\hat{B}$ $= \frac{1}{2}\times 17\times 17\times \sin 105^0$ $= 139{,}58$	✓ Area rule formula / *Oppervlakte reël formule* ✓ correct subst. / *korrekte verv.* ✓answer / *antwoord*	(3)
7.2	$AC^2 = AB^2 + BC^2 - 2.AB.BC.\cos\hat{B}$ $= 17^2 + 17^2 - 2.17.17.\cos 105^0$ $= 727{,}5974081$ $\therefore AC = 26{,}97$	✓ cosine rule formula *cosinus-reël formule* ✓ correct subst into cosine rule *korrekte verv. in cosinus-reël* ✓ AC = 26,97	(3)
7.3	$\frac{\sin A\hat{C}D}{AD} = \frac{\sin\hat{D}}{AC}$ $\frac{\sin A\hat{C}D}{13} = \frac{\sin 75^0}{26{,}97}$ $\sin A\hat{C}D = 13\times\frac{\sin 75^0}{26{,}97}$ $= 0{,}46559272 31$ $A\hat{C}D = 27{,}75^0$	✓ sine rule formula *sinus-reël formule* ✓ correct subst. into sine rule *korrekte verv. in sinus-reël* ✓ answer / *antwoord*	(3)
7.4	Converse opp ∠s of a cyclic quad *Omgekeerde*: *∠e van koordevierhoek* OR / *OF* int. opp. ∠s of a quad supp *oorst. binne ∠e van koordevierhoek* suppl	✓ reason / *rede* OR/*OF* ✓ reason / *rede*	(1)

[10]

QUESTION/*VRAAG* 8

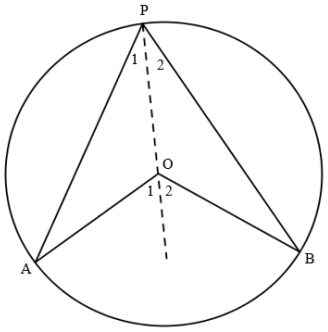
8.1

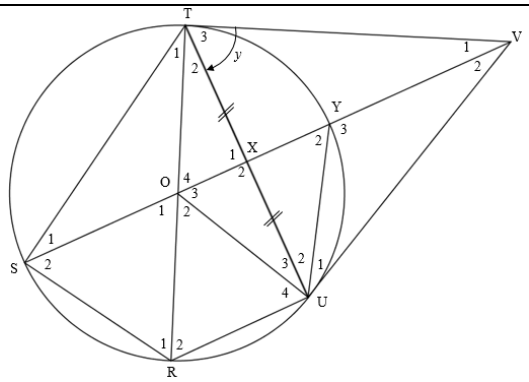


8.1.1	$\hat{S} = 90^0$ $[\angle \text{in semi} - \text{circle} / \angle \textit{in semi - sirkel}]$	✓ S ✓ R	(2)
8.1.2	$\hat{T}_2 = \hat{S} = 90^0$ $[\text{corresp.} \angle \text{s} / \textit{ooreenk.} \angle e, \text{RS} \parallel \text{QU}]$ $\therefore$ T is the midpt of SP [line from centre $\perp$ to chord] *T is die midpt van SP* [*lyn vanaf middelpunt* $\perp$ *op koord*] QP = 10 and/*en* TP = 8 $\text{QT}^2 = (10)^2 - (8)^2$ [Pyth. Theorem/Stelling] $\therefore$ QT = 6 QU = QP = 10 [radii] $\therefore$ TU = 4	✓ S/R ✓ S/R ✓ subst. into Pyth. *verv. in Pyth.* ✓ QT ✓ S/R ✓ TU	(6)
8.2			
8.2.1	$\hat{B}_1 = 30^0$ [tan chord theorem] / [*raaklyn koord stelling*]	✓ S ✓ R	(2)
8.2.2	$\hat{P}_2 = 30^0$ [*alt* $\angle s, BS \parallel PQ$] / [*verw.* $\angle e, BS \parallel PQ$]	✓ S ✓ R	(2)
8.2.3	$\hat{R} = 150^0$ [opp. $\angle$s of a cyclic quad] / [*oorst.* $\angle e$ *van 'n koordevierhoek*]	✓ S ✓ R	(2)
8.2.4	$\hat{Q}_2 = \hat{S}_3$ [$\angle$s opp = sides] / [$\angle e$ *teenoor* = *sye*] $\hat{Q}_2 = \frac{180^0 - 150^0}{2}$ [sum of $\angle$s in a Δ] / [*som van* $\angle e$ *in 'n* Δ] $= 15^0$	✓ S/R ✓ S ✓ answer/*antwoord*	(3)

[17]

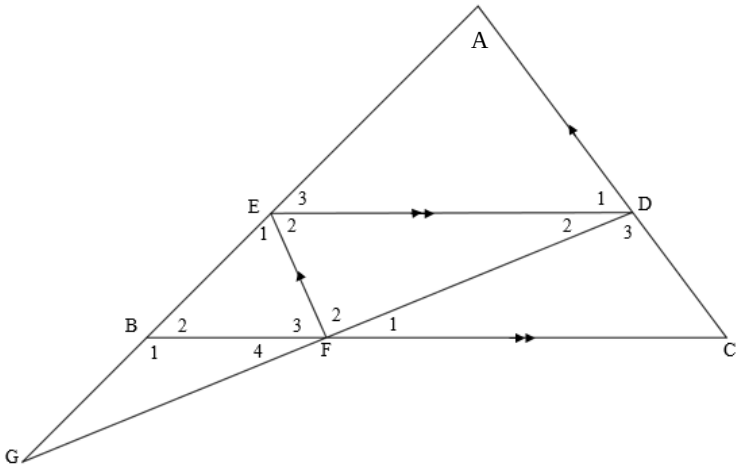
QUESTION/*VRAAG* 9



9.1	$\hat{O}_1 = \hat{P}_1 + A$ *and / en* $\hat{O}_2 = \hat{P}_2 + \hat{B}$ $[\text{ext} \angle \text{of a} \Delta] / [\textit{buite} \angle \textit{van 'n} \Delta]$ $AO = OP = OB$ $[\text{radii}] / [\textit{radiusse}]$ $\hat{P}_1 = \hat{A}$ *and / en* $\hat{P}_2 = \hat{B}$ $[\angle s \text{ opp} = \text{sides}] / [\angle e \textit{ teenoor} = \textit{sye}]$ $\therefore \hat{O}_1 = 2\hat{P}_1$ *and / en* $\hat{O}_2 = 2\hat{P}_2$ $\therefore \hat{O}_1 + \hat{O}_2 = 2\hat{P}_1 + 2\hat{P}_2$ $A\hat{O}B = 2\left(\hat{P}_1 + \hat{P}_2\right)$ $= 2A\hat{P}B$	✓S/R ✓S/R ✓S/R ✓S	(4)
9.2			
9.2.1	$R\hat{U}T = 90^0$ $[\angle \text{in a semi} - \text{circle}] / [\angle \textit{in 'n semi} - \textit{sirkel}]$ $\hat{X}_1 = 90^0$ $[\text{line from centre to midpoint}] /$ $[\textit{lyn vanaf middelpunt na middelpunt}]$ $\therefore RU \parallel SY$ $[\text{corresp.} \angle s =] / [\textit{ooreenk.} \angle e =]$ **OR/*OF*** $R\hat{U}T = 90^0$ $[\angle \text{in a semi} - \text{circle}] / [\angle \textit{in 'n halwe sirkel}]$ $\hat{X}_2 = 90^0$ $[\text{line from centre to the midpoint}] /$ $[\textit{lyn vanaf die middelpunt na die middelpunt}]$ $\therefore R\hat{U}T + \hat{X}_2 = 90^0 + 90^0 = 180^0$ $\therefore RU \parallel SY$ $[\text{co} - \text{int} \angle \text{s supp}] / [\textit{ko} - \textit{binne} \angle \textit{e suppl.}]$	✓S ✓R ✓S ✓R ✓R ✓S ✓R ✓S ✓R ✓R	(5)

9.2.2	$\hat{R}_2 = y$ [tan chord theorem]/[*raaklyn koord stelling*] $= \hat{O}_1$ [alt ∠s, RU ∥ SY]/[*verw.∠e*, RU ∥ SY] $\therefore \hat{T}_1 = \frac{1}{2}\hat{O}_1$ [∠ at centre = twice ∠ at circumf.] [*middelpunts∠ = tweemaal omtrekshoek*] $= \frac{1}{2}y$	✓S ✓R ✓S ✓R ✓R	(5)
9.2.3	$\hat{O}_4 = y = \hat{O}_1$ [vert opp. ∠s]/[*regoorst.∠e*] $T\hat{U}V = y$ [tan from same point = in length] [*raaklyne vanaf dieselfde punt is = in lengte*] ∴ TOUV is a cyclic quad [converse same segment] *TOUV is 'n koordevierhoek* [*omgekeerde dieselfde segment*] **OR/*OF*** $V\hat{T}O = 90^0$ [tan ⊥ radius]/[*raaklyn ⊥ radius*] $V\hat{U}O = 90^0$ [tan ⊥ radius]/[*raaklyn ⊥ radius*] ∴ $V\hat{T}O + V\hat{U}O = 180^0$ ∴ TOUV is a cyclic quad [converse opp ∠s of cyclic quad. supp] *TOUV is 'n koordevierhoek* [*omgekeerde teenoorst.∠e van koordevierhoek*]	✓S ✓R ✓S ✓R ✓R ✓S ✓R ✓S ✓R ✓R	(5)
			[19]

QUESTION/*VRAAG* 10



10.1	$\frac{BA}{EA}=\frac{BC}{FC}$ $\left[\text{prop theorem; EF}\parallel\text{AC}\right]$ / [*Ewerdigh.stelling; EF* $\parallel$ *AC*] $=\frac{CA}{DA}$ $\left[\text{prop theorem; ED}\parallel\text{BC}\right]$ / [*Eweredigh.stelling; ED* $\parallel$ *BC*] $\therefore\frac{BC}{FC}=\frac{CA}{DA}$	✓S ✓R ✓S ✓R	(4)
10.2	$\hat{B}_2=\hat{E}_3$ $\left[\text{corresp}\angle\text{s, }ooreenk.\angle e\text{: ED}\parallel\text{BC}\right]$ $\hat{E}_1=\hat{A}$ $\left[\text{corresp}\angle\text{s, }ooreenk.\angle e\text{: EF}\parallel\text{AC}\right]$ $\therefore\hat{F}_3=\hat{D}_1$ $\left[\text{sum of}\angle\text{s of}\Delta\text{ / }som\,van\,\angle e\,van\,\Delta\right]$ $\therefore\Delta\text{BFE}\|\|\|\Delta\text{EDA}$ $\left[\angle\angle\angle\right]$	✓S/R ✓S/R ✓R	(3)
10.3.1	$\frac{AD}{FE}=\frac{ED}{BF}$ $\left[\Delta\text{BFE}\|\|\|\Delta\text{EDA}\right]$ $\frac{AD}{2}=\frac{10}{3,5}$ $AD=\frac{40}{7}=5,71$	✓S ✓R ✓subst *verv.* ✓AD	(4)
10.3.2	DC = EF = 2 $\left[\text{opp.sides of a parm}\right]$ / [*oorst.sye van 'n*$\|\|^m$]	✓S ✓R	(2)
			[13]
		TOTAL/*TOTAAL*:	**150**