



Province of the
EASTERN CAPE
EDUCATION

**NATIONAL/NASIONALE
SENIOR
CERTIFICATE/SERTIFIKAAT**

GRADE/GRAAD 12

JUNE/JUNIE 2017

**MATHEMATICS P2/WISKUNDE V2
MEMORANDUM**

MARKS/PUNTE: 150

This memorandum consists of 12 pages.
Hierdie memorandum bestaan uit 12 bladsye.

QUESTION 1 / VRAAG 1

1.1	Percentages / <i>Persentasies</i>	Frequency / <i>Frekwensie</i>	Cumulative Frequency / <i>Kumulatiewe Frekwensie</i>	✓ 3, 12 ✓ 24, 35, 44 ✓ 50 (3)
	$30 \leq x < 40$	1	1	
	$40 \leq x < 50$	2	3	
	$50 \leq x < 60$	9	12	
	$60 \leq x < 70$	12	24	
	$70 \leq x < 80$	11	35	
	$80 \leq x < 90$	9	44	
	$90 \leq x < 100$	6	50	

1.2	Ogive	✓ upper limits / <i>bo-limiete</i> ✓ cum <i>f</i> / <i>kum. f</i> ✓ shape / <i>vorm</i> ✓ grounded / <i>ge-anker</i> (4)

1.3	Approx. 30 [accept between 28 – 32] Ongeveer 30 [aanvaar tussen 28 – 32]	✓✓ answer/ indicated on graph. antwoord / op grafiek aangedui (2)
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[9]

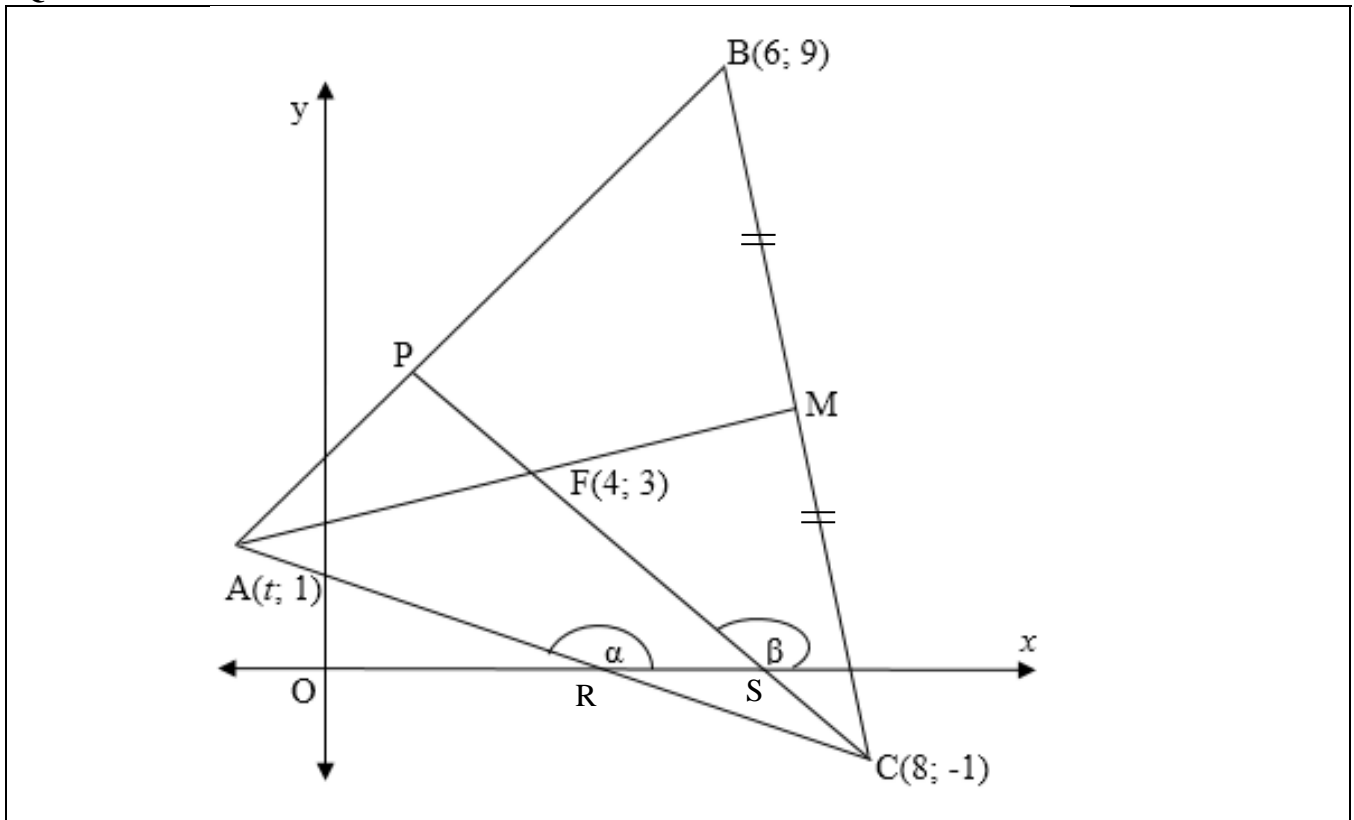
QUESTION 2 / VRAAG 2

	12,4	15,1	18,9	19,7	19,7	20,0	
	20,9	23,7	23,8	31,1	33,6	34,5	
	34,9	36,5	40,1				
2.1	Minimum / <i>Minimum</i> = 12.4 Lower quartile / <i>Onderste kwartiel</i> (Q_1) = 19.7 Median / <i>Mediaan</i> (Q_2) = 23.7 Upper quartile / <i>Boonste kwartiel</i> (Q_3) = 34.5 Maximum / <i>Maksimum</i> = 40.1						✓ min & max ✓ Q_1 ✓ Q_2 ✓ Q_3 (4)

2.2		✓ min / max ✓ Q_1 / Q_3 ✓ Q_2 (3)
2.3	Skewed positively to the right. Skeef positief na regs	✓ positively skewed / <i>positief skeef</i> (1)
2.4	$SD/SA = 8,36$	✓✓ answer / <i>antwoord</i> (2)
2.5	A small standard deviation indicates that the data is clustered around the mean. OR/OF A large standard deviation indicates that the data is more spread out. <i>’n Klein standaardafwyking dui aan dat die data rondom die gemiddelde gegroepeer is. ’n Groot standaardafwyking dui aan dat die data meer versprei is.</i>	✓ answer / <i>antwoord</i> (1)

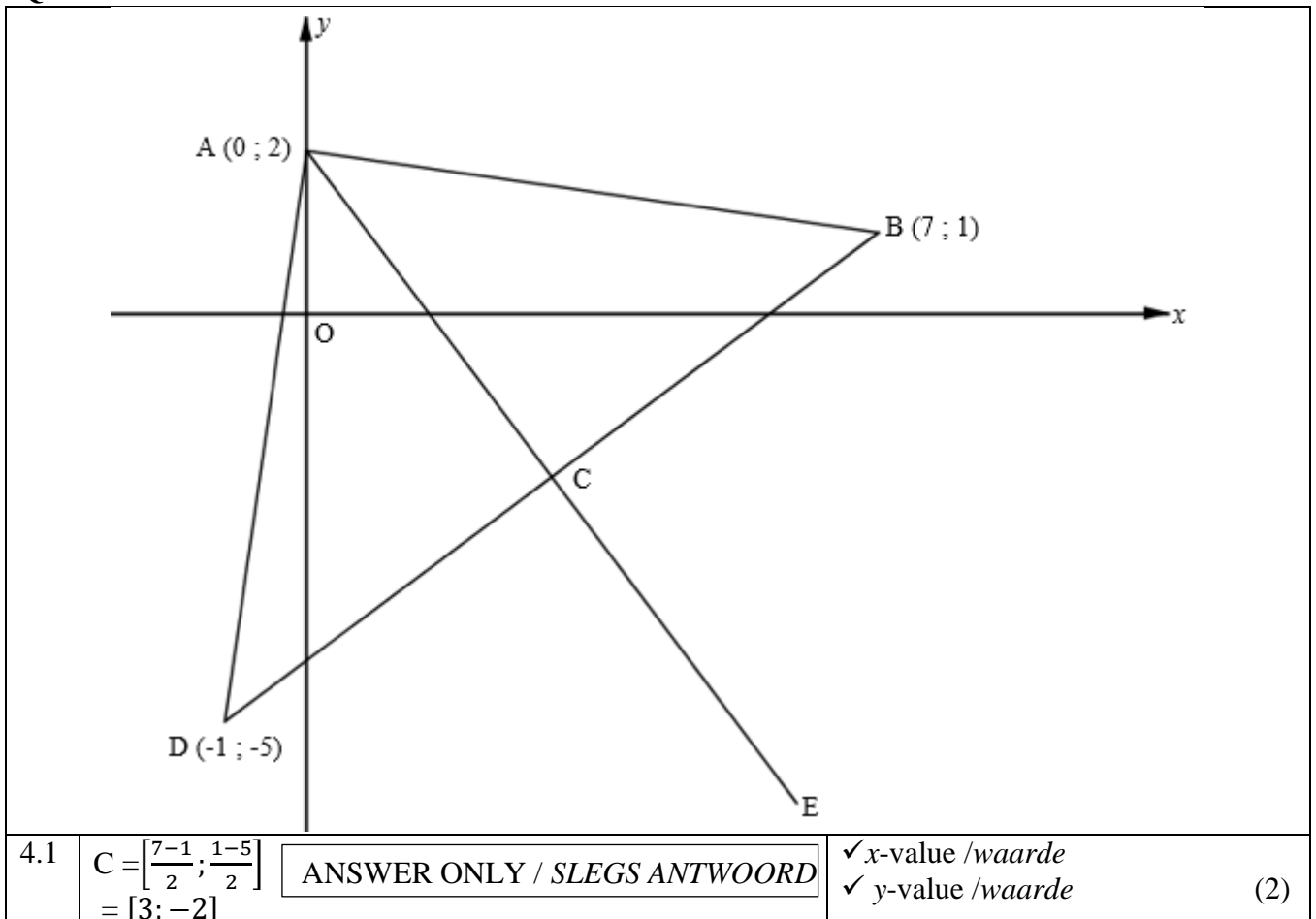
[11]

QUESTION 3 / VRAAG 3



3.1	$M = \left[\frac{6+8}{2} ; \frac{9-1}{2} \right]$ $M = (7 ; 4)$	✓ x- value of M / x-waarde van M ✓ y- value of M / y-waarde van M (2)
3.2	$m_{FM} = \frac{4-3}{7-4} = \frac{1}{3}$ $y - y_1 = \frac{1}{3}(x - x_1) \quad m = \frac{1}{3}$ $y - 4 = \frac{1}{3}(x - 7) \quad M = (7; 4)$ $\therefore y = \frac{1}{3}x + \frac{5}{3}$	✓ substituting / <i>vervang</i> ✓ value of m_{FM} / <i>waarde van m_{FM}</i> ✓ substituting $M(7; 4)$ / <i>vervang $M(7; 4)$</i> ✓ answer / <i>antwoord</i> (4)

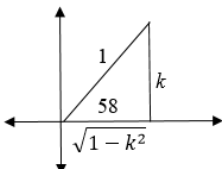
3.3	$1 = \frac{1}{3}t + \frac{5}{3}$ $t = -2$ <p style="text-align: center;">OR / OF</p> $m_{AF} = m_{FM}$ $\frac{3-1}{4-t} = \frac{1}{3}$ $4-t = 6$ $t = -2$	<ul style="list-style-type: none"> ✓ substitution into line equation / <i>vervang in lyn vergelyking</i> ✓ answer (as negative) / <i>antwoord (as negatief)</i> <p style="text-align: center;">OR / OF</p> <ul style="list-style-type: none"> ✓ substitution into grad eqn / <i>vervang in gradiënt vergelyking</i> ✓ answer as negative / <i>antwoord as negatief</i> <p style="text-align: right;">(2)</p>
3.4	$m_{PC} = \frac{3-(-1)}{4-8} = -1$ <div style="border: 1px solid black; padding: 5px; margin-top: 10px;"> <p style="text-align: center;">ANSWER ONLY FULL MARKS/ SLEGS ANTWOORD</p> </div>	<ul style="list-style-type: none"> ✓ substitution / <i>vervang</i> ✓ answer / <i>antwoord</i> <p style="text-align: right;">(2)</p>
3.5	$\tan \beta = -1$ $\beta = 135^\circ$	<ul style="list-style-type: none"> ✓ $\tan \beta = -1$ ✓ $\beta = 135^\circ$ <p style="text-align: right;">(2)</p>
3.6	$\tan \alpha = \frac{-2}{10} = -\frac{1}{5}$ $\therefore \alpha = 180^\circ - 11.31^\circ$ $= 168.69^\circ$ $A\hat{C}P = \alpha - \beta$ $= 33.69^\circ$	<ul style="list-style-type: none"> ✓ $\tan \alpha = -\frac{1}{5}$ ✓ $\alpha = 168.69^\circ$ ✓ $A\hat{C}P = \alpha - \beta$ ✓ answer / <i>antwoord</i> <p style="text-align: right;">(4)</p>

[16]**QUESTION 4 / VRAAG 4**

4.2	$CA^2 = (3 - 0)^2 + (2 + 2)^2$ $CA^2 = 25$ $CA = 5$ $CB^2 = (7 - 3)^2 + (1 + 2)^2$ $CB^2 = 25$ $CB = 5$ $\therefore CA = CB$	✓ substitution / <i>substitusie</i> ✓ answer for CA <i>antwoord vir CA</i> ✓ answer for CB <i>antwoord vir CB</i> (3)
4.3	$m_{AD} = \frac{2+5}{0+1}$ $= 7$ $m_{AB} = \frac{2-1}{0-7}$ $= -\frac{1}{7}$ $m_{AD} \times m_{AB} = 7 \times \left(-\frac{1}{7}\right) = -1$ $\therefore AD \perp AB \quad [m_{AD} \times m_{AB} = -1]$ $\therefore \hat{DAB} = 90^\circ$	✓ substitution ✓ $m_{AD} = 7$ ✓ substitution ✓ m_{AB} ✓ $m_{AD} \times m_{AB} = -1$ (5)
4.4	$(x - 3)^2 + (y + 2)^2 = 25$	✓ correct centre / <i>korrek middelpunt</i> ✓ correct / <i>korrekte r²</i> (2)
4.5	$m_{BC} = \frac{1 - (-2)}{7 - 3}$ $= \frac{3}{4}$	✓ substitution ✓ m_{BC} (2)
4.6	$m_{\tan} = -\frac{4}{3}$ $y - 1 = -\frac{4}{3}(x - 7)$ $y = -\frac{4}{3}x + \frac{31}{3}$	✓ m_{\tan} ✓ subst $m = -\frac{4}{3}$ and B(7;1) verv. $m = -\frac{4}{3}$ en B(7;1) ✓ answer / <i>antwoord</i> (3)
4.7	AE = DB \therefore ABED is a rectangle	[diameters of same circle] [diagonals =] ✓ AE = DB ✓ reason ✓ reason (3)

[20]

QUESTION 5 / VRAAG 5

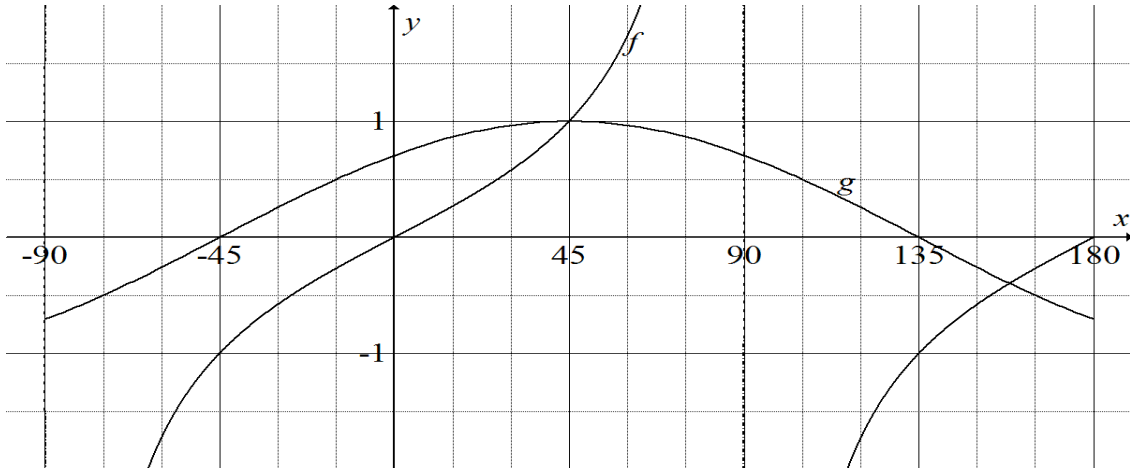
5.1.1	$\sin 238^\circ = -\sin 58^\circ$ $= -k$	✓ reduction / <i>reduksie</i> ✓ answer / <i>antwoord</i> (2)
5.1.2	$\cos 58^\circ = \sin 32^\circ$ $= \sqrt{1 - k^2}$ 	✓ $\sin 32^\circ$ ✓ answer / <i>antwoord</i> (2)

5.2	$\frac{\tan(180^\circ - 30^\circ) \cdot \sin(360^\circ - 60^\circ) \cdot \sin 10^\circ}{\cos(180^\circ + 45^\circ) \cdot \sin(180^\circ - 45^\circ) \cdot \cos(90^\circ - 10^\circ)}$ $= \frac{(-\tan 30^\circ)(-\sin 60^\circ) \sin 10^\circ}{(-\cos 45^\circ)(\sin 45^\circ) \sin 10^\circ}$ $= \frac{\frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{2}}{\frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}}}$ $= -1$	<ul style="list-style-type: none"> ✓ - tan 30° ✓ - sin 60° ✓ - cos 45° ✓ sin 45° ✓ sin 10° ✓ simplification / vereenvoudiging ✓ answer / antwoord (7)
5.3	$\sin(\alpha + \beta) = \cos[90^\circ - (\alpha + \beta)]$ $= \cos[(90 - \alpha) - \beta]$ $= \cos(90^\circ - \alpha) \cos \beta - \sin(90^\circ - \alpha) \sin \beta$ $= \sin \alpha \cos \beta - \cos \alpha \sin \beta$	<ul style="list-style-type: none"> ✓ cos [90° - (α + β)] ✓ cos[(90 - α) - β] ✓ cos(90° - α) cos β - sin(90° - α) sin β ✓ sin α cos β - cos α sin β (4)
5.4	$\frac{\cos 2x + 1}{\sin 2x \cdot \tan x} = \frac{2 \cos^2 x - 1 + 1}{2 \sin x \cos x \cdot \frac{\sin x}{\cos x}}$ $= \frac{2 \cos^2 x}{2 \sin^2 x}$ $= \frac{1}{\tan^2 x}$	<ul style="list-style-type: none"> ✓ identity numerator <i>identiteit teller</i> ✓ identity denominator <i>identiteit noemer</i> ✓ $\frac{\sin x}{\cos x}$ ✓ $\frac{\cos^2 x}{\sin^2 x}$ simplification / vereenvoudiging (4)
5.5.1	$\frac{\sin x}{\cos x} = 2 \sin x$ $\sin x = 2 \sin x \cos x$ $\sin x - 2 \sin x \cos x = 0$ $\sin x(1 - 2 \cos x) = 0$ $\sin x = 0 \quad \text{or/of} \quad \cos x = \frac{1}{2}$	<ul style="list-style-type: none"> ✓ identity / <i>identiteit</i> ($\frac{\sin x}{\cos x}$) ✓ simplification / vereenvoudiging ✓ factors / <i>faktore</i> (3)
5.5.2	$\sin x = 0 \quad \text{or} \quad \cos x = \frac{1}{2}$ $x = 0^\circ + 360^\circ k, k \in Z$ <p>OR</p> $x = 180^\circ + 360^\circ k$ $x = \pm 60^\circ + 360^\circ k$ $k \in Z$	<ul style="list-style-type: none"> ✓ $x = 0^\circ$ ✓ $x = 180^\circ$ ✓ $x = \pm 60^\circ$ ✓ $360^\circ k, k \in Z$ (4)

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QUESTION 6 / VRAAG 6

6.1



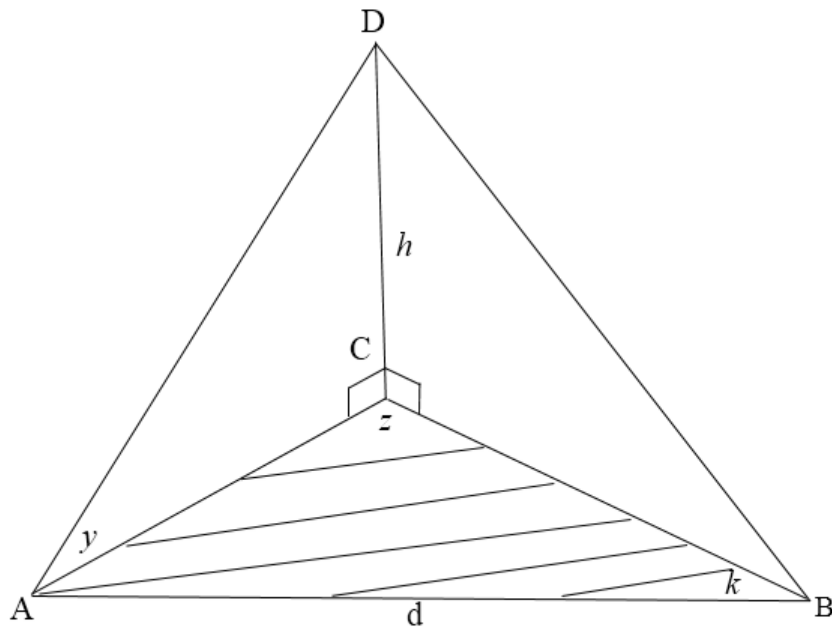
- ✓ Asymptotes / Asimptote (*f*) ✓ Shape / Vorm (*g*)
- ✓ (45° ; 1) (*f*) ✓ (-45° ; 0) / (135° ; 0) / *x*-intercepts (*g*)
- ✓ Endpoints / Eindpunte (*f*) ✓ Endpoints / Eindpunte (*g*)

(6)

6.2.1	$x = -45^\circ$	✓✓ -45°	(2)
6.2.2	$(-90^\circ; 45^\circ]$ OR/OF $-90^\circ < x \leq 45^\circ$	✓ -90° and 45° ✓ correct inequalities	(2)
6.3	90°	✓ answer / <i>antwoord</i>	(1)

[11]

QUESTION 7 / VRAAG 7

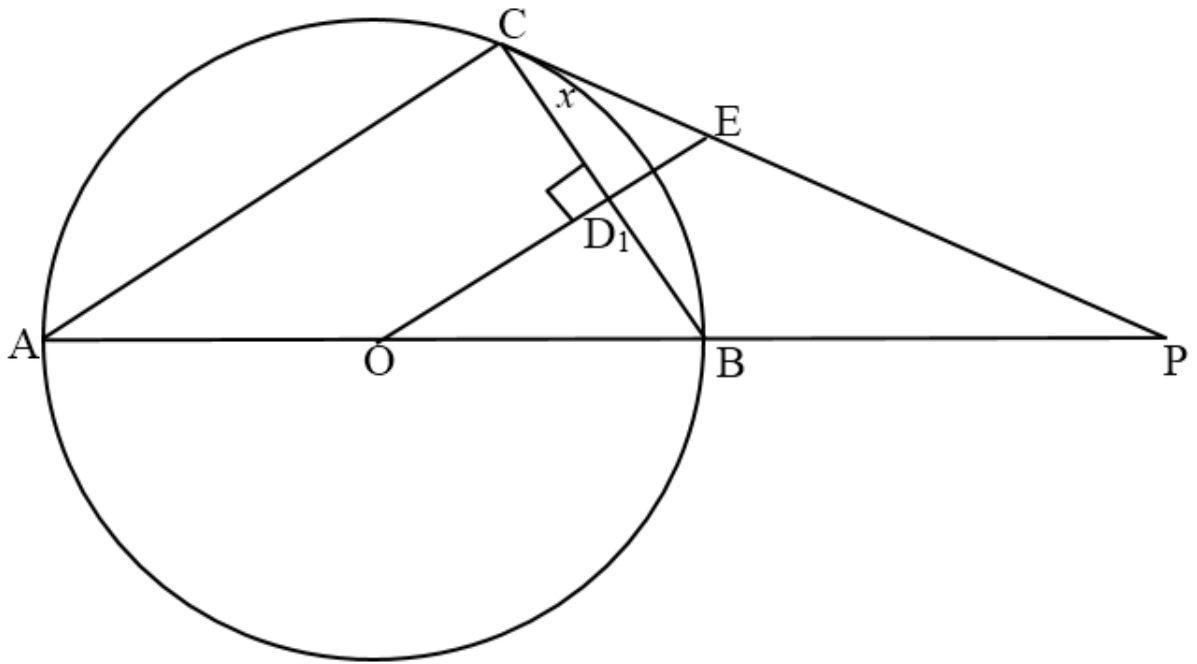


7.1	In $\triangle ABC$ $\frac{AC}{\sin k} = \frac{d}{\sin z}$ $\therefore AC = \frac{d \cdot \sin k}{\sin z}$	ANSWER ONLY/ SLEGS ANTWOORD	✓ proportion / <i>verhouding</i> ✓ answer / <i>antwoord</i>	(2)
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7.2	<p>In $\triangle ADC$</p> $\frac{AC}{\sin(90^\circ - y)} = \frac{h}{\sin y}$ $AC = \frac{h \cdot \cos y}{\sin y}$ $AC = \frac{h}{\tan y}$ <p style="text-align: center;">OR/OF</p> $\frac{AC}{h} = \frac{1}{\tan y}$ $AC = \frac{h}{\tan y}$	<p>✓ proportion / <i>verhouding</i></p> <p>✓ answer / <i>antwoord</i></p> <p>✓ $\frac{AC}{h} = \frac{1}{\tan y}$</p> <p>✓ $AC = \frac{h}{\tan y}$ (2)</p>
7.3	$h = \frac{AC \cdot \sin y}{\cos y}$ $h = \frac{d \sin k \cdot \sin y}{\cos y \cdot \sin z}$ $h = \frac{d \sin k \cdot \sin y}{\sin z}$ <p style="text-align: center;">OR/OF</p> $AC = \frac{h}{\tan y}$ $AC = \frac{d \cdot \sin k}{\sin z}$ $\therefore \frac{h}{\tan y} = \frac{d \cdot \sin k}{\sin z}$ $\therefore h = \frac{d \sin k \cdot \tan y}{\sin z}$	<p>✓ subst/verv. $AC = \frac{d \cdot \sin k}{\sin z}$</p> <p style="text-align: center;">OR/OF</p> <p>✓ equating AC / <i>gelykstel aan AC</i> (1)</p>
7.4	$\therefore h = \frac{d \sin k \cdot \tan y}{\sin z}$ $h = \frac{80 \cdot \sin 38^\circ \cdot \tan 40^\circ}{\sin 125}$ $= 50,45\text{m}$	<p>✓ substitution / <i>vervanging</i></p> <p>✓ answer / <i>antwoord</i> (2)</p>

[7]

QUESTION 8 / VRAAG 8



8.1	Line from centre perpendicular to chord, bisects the chord. / <i>Lyn vanaf die middelpunt loodreg op die koord, halveer die koord.</i>	✓ answer / <i>antwoord</i> (1)
8.2.	$\widehat{ACB} = 90^\circ$ [angle in semi-circle] / [<i>hoek in semi-sirkel</i>] $\widehat{ACB} = \widehat{D_1}$ [both = 90°] / [<i>beide = 90°</i>] $\therefore OE \parallel AC$ [<i>corresp \angle's equal</i>] / [<i>ooreenkomstige \angle's is gelyk</i>]	✓ S ✓ R ✓ R (3)
8.3	$\widehat{A} = x$ [<i>tan chord</i>] / [<i>raaklyn koord</i>] $\widehat{EOB} = x$ [<i>corresp \angle's ; $AC \parallel OE$</i>] / [<i>ooreenkomstige \angle's ; $AC \parallel OE$</i>]	✓ S ✓ R ✓ S ✓ R (4)
8.4	$\widehat{EOB} = \widehat{ECB}$ [both = x] / [<i>beide = x</i>] $\therefore OBEC$ is cyclic quad [<i>converse angles in same segment</i>] <i>OBEC is 'n koordevierhoek</i> [<i>hoeke in dieselfde segment</i>]	✓ S ✓ R (2)

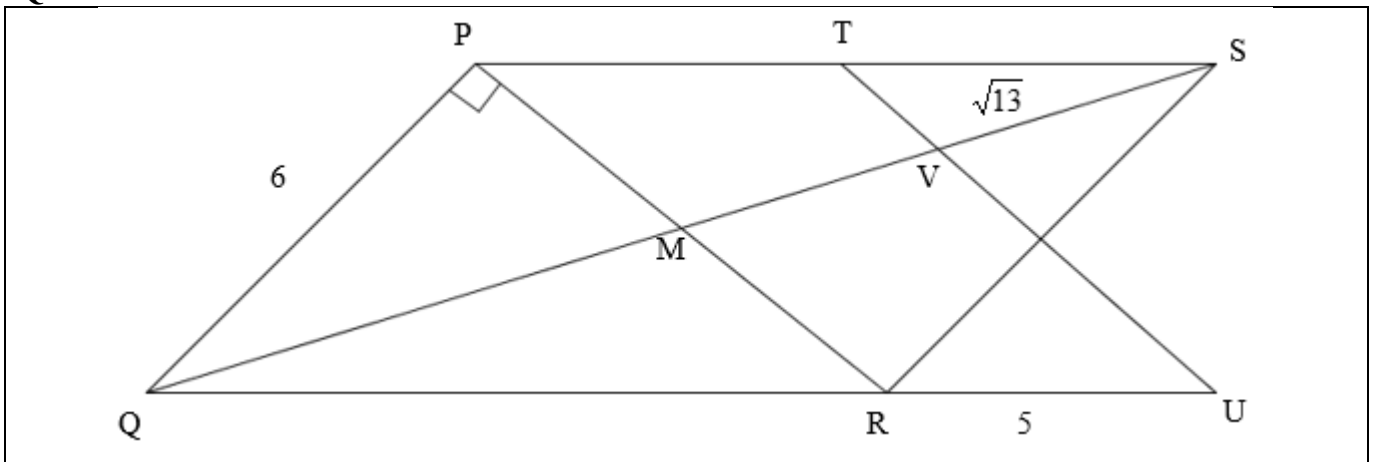
[10]

QUESTION 9 / VRAAG 9

9.1	$\hat{A} = 49^\circ$ [\angle at centre = 2 \angle at circumf.] / [<i>Middelpunts \angle</i>]	\checkmark S \checkmark R (2)
9.2	$\hat{C}_1 = \hat{B}_1$ [angles opp equal sides] / [<i>hoeke teenoor gelyke sye</i>] $\hat{B}_1 = \frac{180^\circ - 98^\circ}{2}$ [angles of Δ] / [<i>hoeke van Δ</i>] $\hat{B}_1 = 41^\circ$	\checkmark R \checkmark S (3)
9.3	$\hat{BCD} = 90^\circ$ [\angle 's in semi-circle] / [<i>\angle in 'n semi-sirkel</i>] $\hat{B}_2 = \hat{C}_3 = 26^\circ$ [\angle 's in same segment] / [<i>\angle in dieselfde segment</i>] $\hat{C}_2 = 23^\circ$	\checkmark S/R \checkmark S/R \checkmark S (3)

[8]

QUESTION 10 / VRAAG 10



10.1.1	$QR^2 = PQ^2 + PR^2$ $= 6^2 + 8^2$ $\therefore QR = 10$ $\therefore \frac{UR}{RQ} = \frac{5}{10}$ $= \frac{1}{2}$	Pyth.Theo	\checkmark subst. in Pyth \checkmark QR = 10 $\checkmark \frac{UR}{RQ} = \frac{1}{2}$ (3)
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<p>10.1.2</p>	<p>PM = 4 [diagonals bisect each other] $QM^2 = 6^2 + 4^2$ [Pyth. Theo] $QM = 2\sqrt{13}$ $MS = QM = 2\sqrt{13}$ [Diagonals bisect each other] $\therefore MV = \sqrt{13}$ $\therefore \frac{VM}{MQ} = \frac{\sqrt{13}}{2\sqrt{13}}$ $= \frac{1}{2}$</p>	<p>✓ R ✓ $QM = 2\sqrt{13}$ ✓ $MV = \sqrt{13}$ ✓ $\frac{VM}{MQ} = \frac{1}{2}$ (4)</p>
<p>10.2</p>	<p>$\frac{UR}{RQ} = \frac{VM}{MQ}$ [both = $\frac{1}{2}$] $\therefore MR \parallel VU$ [line divides two sides of Δ in prop]</p>	<p>✓ S ✓ R (2)</p>

[9]

QUESTION 11 / VRAAG 11

<p>11.1</p>	<p>Constr/<i>Konstr</i>: On AB mark off AG = DE / <i>Merk</i> AG = DE af op AB On AC mark off AH = DF / <i>Merk</i> AH = DF af op AC Join GH. / <i>Verbind</i> GH Proof / <i>Bewys</i>: In ΔAGH & ΔDEF: i) $AG = DE$ (constr) / (<i>konstr.</i>) ii) $\hat{A} = \hat{D}$ (given) / (<i>gegee</i>) iii) $AH = DF$ (constr) / (<i>konstr.</i>) $\therefore \Delta AGH \parallel \Delta DEF$ (SAS) / (<i>SHS</i>) $\therefore \hat{G}_1 = \hat{E}$ But / <i>Maar</i> $\hat{B} = \hat{E}$ given/gegee $\therefore \hat{G}_1 = \hat{B}$ $\therefore GH \parallel BC$ (corresp angles equal) / (<i>ooreenk. hoeke gelyk</i>) $\therefore \frac{AB}{AG} = \frac{AC}{AH}$ $\therefore \frac{AB}{DE} = \frac{AC}{DF}$ (AG= DE, AH = DF) $\therefore \frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF}$</p>	<p>✓ constr / <i>konstr.</i> ✓ S ✓ S/R ✓ S ✓ R ✓ S ✓ R (7)</p>

<p>11.2</p>		
<p>11.2.1</p>	$R_2 = x$ [tan chord] : [raaklyn koord] $T_1 = x$ [\angle 's opp equal sides] : [\angle 'e teenoor gelyke sye] $Q_3 = x$ [tan chord] : [raaklyn koord] $R_1 = x$ [tan from same point] : [raaklyne vanaf dieselfde punt]	<p>✓ S/R ✓ S/R ✓ S/R ✓ S/R (any three) / (enige drie) (3)</p>
<p>11.2.2</p>	$Q_2 = 180^\circ - 2x$ [angles of Δ] : [hoeke van Δ]	<p>✓ S ✓ R (2)</p>
<p>11.2.3</p>	$\hat{P} = 180^\circ - 2x$ [sum of angles of Δ PQR] $R_3 = Q_2 = 180^\circ - 2x$ [tan chord] : [raaklyn koord] $\therefore TR \parallel QP$ [corresp \angle 's =] : [ooreenkomstige \angle 'e =]	<p>✓ S ✓ S/R ✓ R (3)</p>
<p>11.2.4</p>	<p>In ΔSTR & ΔSRQ $\hat{S} = \hat{S}$ common / gemeen $\hat{R}_3 = \hat{Q}_2$ tan chord / raaklyn koord $\therefore \Delta STR \parallel \Delta SRQ$ [AAA] / [HHH]</p>	<p>✓ S ✓ S ✓ R (3)</p>
<p>11.2.5</p>	$\frac{ST}{SR} = \frac{SR}{SQ}$ $\Delta STR \parallel \Delta SRQ$ $RS^2 = ST \cdot SQ$	<p>✓ S ✓ R (2)</p>
<p>11.2.6</p>	$\frac{SP}{PR} = \frac{SQ}{TQ}$ [line \parallel to one side of a Δ] $= \frac{5}{3}$ $PQ = PR$ [tan from same point] $\frac{SP}{PQ} = \frac{5}{3}$	<p>✓ S/R ✓ R ✓ value of $\frac{SP}{PQ} = \frac{5}{3}$ (3)</p>

[23]

TOTAL/TOTAAL: 150