Possible "solutions" to 2019 DBE Paper 1

Please note that this is **NOT** an official document. It has been created mainly as a service to IEB pupils as another source of practice ahead of their upcoming examination.

It is **NOT** a marking memo in the sense that it does not contain mark allocations. Nor does it contain alternate methods. It is likely not even fully correct.

For all the above reasons I would caution against sharing it with pupils in the DBE who, having seen it may be concerned about how they have fared if their answers don't match these. This will only serve to stress them out when they should be focused on their upcoming examinations. As we know, one can get plenty of part marks despite getting the final answer wrong. However, pupils won't necessarily appreciate that.

QUESTION 1

1.1

1.1.1

$$x^2+5x-6=0$$
$$\therefore (x+6)(x-1)=0$$
$$\therefore x=-6 \text{ or } 1$$

1.1.2

$$4x^2+3x-5=0$$
$$\therefore x=0.80 \text{ or } -1.55$$

1.1.3

$$4x^2-1<0$$
$$\therefore (2x-1)(2x+1)<0$$
$$\therefore -\frac{1}{2}<x<\frac{1}{2} \left(\begin{array}{l}\text{concave up parabolas are}\\ \text{negative between their roots}\end{array}\right)$$

1.1.4

$$\left(\sqrt{\sqrt{32}+x}\right)\left(\sqrt{\sqrt{32}-x}\right)=x$$

$$\therefore \sqrt{32-x^2}=x$$

$$\therefore 32-x^2=x^2$$

$$\therefore 2x^2=32$$

$$\therefore x^2=16$$

$$\therefore x=\pm 4$$

a check reveals $x=4$ *only*

1.2

$y+x=12$ *and* $xy=14-3x\ (2)$

$y=12-x$

substituting into (2) *gives*:

$$x(12-x)=14-3x$$

$$\therefore -x^2+12x+3x-14=0$$

$$\therefore x^2-15x+14=0$$

$$\therefore (x-14)(x-1)=0$$

$$\therefore x=14 \text{ or } 1$$

$$\therefore y=-2 \text{ or } 11$$

$$\therefore (x;y)=(14;-2) \text{ or } (1;11)$$

1.3

The powers of 3 are 3, 9 and 27 – they contain 6 threes.
Other multiples of 3: 6 ; 12; 15 ; 21 ; 24 ; 30 each of which contain 1 three
And 18 which contains 2.
So, a total of 14 threes.
So, k=14

QUESTION 2

2.1

2.1.1

209 and 186

2.1.2

$$second\ difference = 2$$
$$so\ a = 1$$
$$now\ c = T_0 = 354$$
$$and\ a + b + c = 321$$
$$\therefore 1 + b + 354 = 321$$
$$\therefore b = -34$$
$$\therefore T_n = n^2 - 34n + 354$$

2.1.3

$$n^2 - 34n + 354 = 74$$
$$\therefore n^2 - 34n - 280 = 0$$
$$\therefore (n-20)(n-14) = 0$$
$$\therefore T_{14} = T_{20} = 74$$

2.1.4.

$$T_n = n^2 - 34n + 354$$
$$= (n-17)^2 + 354 - 289$$
$$= (n-17)^2 + 65$$

$so,\ the\ smallest\ term\ is\ 65,\ and\ it\ is\ the\ 17^{th}\ term$

2.2

2.2.1

$$a = \frac{5}{8}\ \ and\ \ r = \frac{1}{2}$$
$$now\ S_n = \frac{a\left(1 - r^n\right)}{1 - r}$$
$$\therefore K = S_{21} = \frac{\frac{5}{8}\left(1 - \left(\frac{1}{2}\right)^{21}\right)}{1 - \frac{1}{2}}$$
$$\therefore K = 1.25\ (2\ d.p.)$$

2.2.2

$$T_n = ar^{n-1}$$

$$\therefore \frac{5}{8192} = \frac{5}{8}\times\left(\frac{1}{2}\right)^{n-1}$$

$$\therefore \frac{1}{1024} = \left(\frac{1}{2}\right)^{n-1}$$

$$\therefore \left(\frac{1}{2}\right)^{10} = \left(\frac{1}{2}\right)^{n-1}$$

$$\therefore n = 11$$

$$so, T_{10} \text{ is the last term to be bigger than } \frac{5}{8192}$$

$$so\ n = 10$$

QUESTION 3

3.1

$$\sum_{y=3}^{10}\frac{1}{y-2} - \sum_{y=3}^{10}\frac{1}{y-1}$$

$$= \left(\frac{1}{1}+\frac{1}{2}+....+\frac{1}{8}\right) - \left(\frac{1}{2}+\frac{1}{3}+.....\frac{1}{9}\right)$$

$$= \frac{1}{1}-\frac{1}{9}$$

$$= \frac{8}{9}$$

3.2

$$\text{Area of open sides} = 2\times\left(\frac{1}{3}\times\frac{2}{3}+\frac{2}{3}\times\frac{2}{3}+\frac{3}{3}\times\frac{2}{3}+....\text{ to } 12 \text{ terms}\right)$$

$$= 2\times\frac{2}{3}\left(\frac{1}{3}+\frac{2}{3}+\frac{3}{3}+....\text{ to } 12 \text{ terms}\right)\quad \left(\begin{array}{l}\text{in brackets we have an}\\ \text{arithmetic series with}\\ a=\frac{1}{3} \text{ and } d=\frac{1}{3}\end{array}\right)$$

$$= 2\times\frac{2}{3}\left(\frac{12}{2}\left(\frac{2}{3}+11\left(\frac{1}{3}\right)\right)\right)$$

$$= \frac{4}{3}\left(6\left(\frac{13}{3}\right)\right)$$

$$= \frac{104}{3}\ m^2$$

QUESTION 4

4.1

$p = -1$

4.2

$g(x) = \frac{a}{x-1}$

but $(0;-3)$ *lies on g*

so $-3 = \frac{a}{0-1}$

$\therefore a = 3$

now $f(x) = x^2 + bx - 3$

but $(1;0)$ *lies on f*

$\therefore 0 = 1 + b - 3$

$\therefore b = 2$

4.3

$f(x) = x^2 + 2x - 3$

$= (x+1)^2 - 3 - 1$

$= (x+1)^2 - 4$

$C = (-1;-4)$

4.4

$y \geq -4$

4.5

$y - -4 = 1(x - -1)$

$\therefore y = x - 3$

4.6

No, since it **cuts** *f* at that point

Or, $f'(-1) = 0$ which is not equal to the gradient of the line.

4.7

$f(x) = x^2 + 2x - 3$

now $f(m-x) + q = (m-x)^2 + 2(m-x) - 3 + q$

$= x^2 - 2mx - 2x + m^2 + 2m - 3 + q$

$= x^2 - (2m+2)x + (m^2 + 2m - 3 + q)$

now for the parabola to turn on the origin we need

$(2m+2) = 0$ *so* $m = -1$

and $(m^2 + 2m - 3 + q) = 0$

$\therefore q = 4$

QUESTION 5

5.1

$f(x) = k^x$

$but\ f(4) = 16$

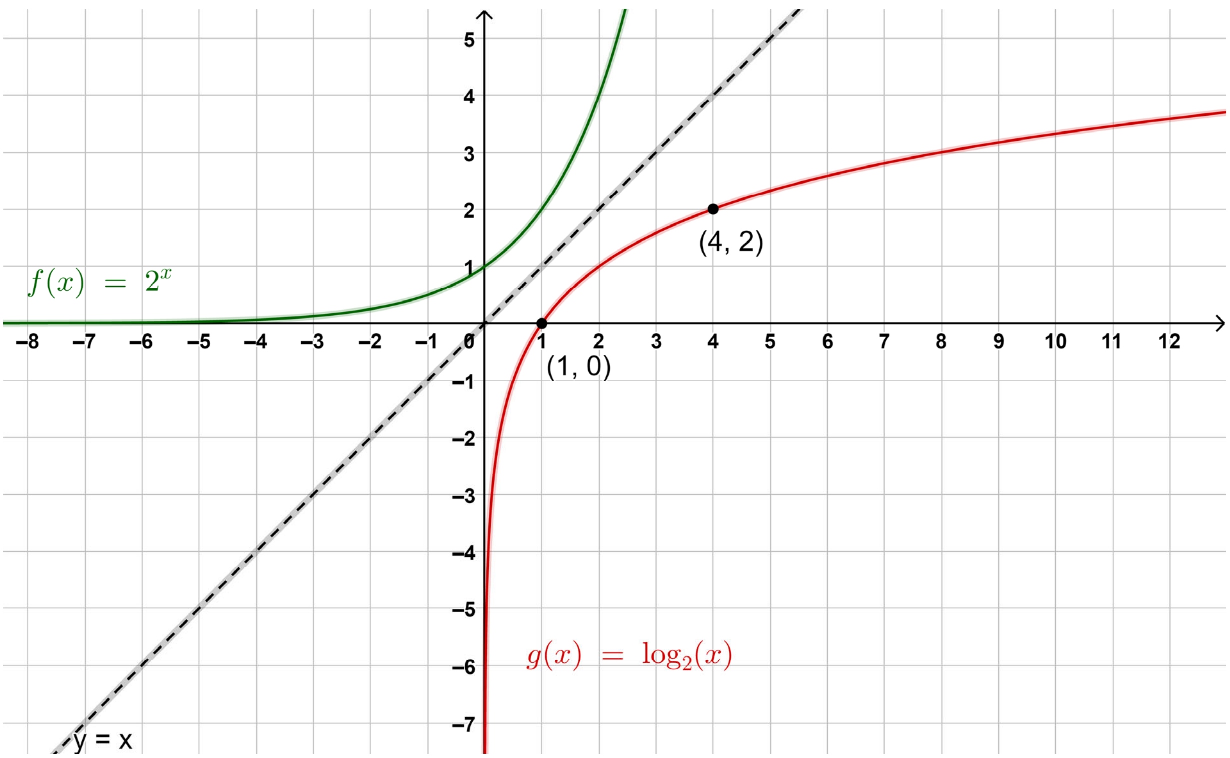
$\therefore k^4 = 16$

$\therefore k = 2$

5.2

$y = \log_2 x$

5.3



5.4

5.4.1

$x > 1$

5.4.2

$0 < x \leq \frac{1}{2}$

5.5

$$f(x) - h(x) = \frac{15}{4}$$

$$\therefore 2^x - 2^{-x} = \frac{15}{4} \ \left(a \ quadratic \ in \ 2^x\right)$$

$$\therefore 2^{2x} - 1 = \frac{15}{4}\left(2^x\right)$$

$$\therefore 4\left(2^x\right)^2 - 15\left(2^x\right) - 4 = 0$$

$$\therefore \left(4 \times 2^x + 1\right)\left(2^x - 4\right) = 0$$

$$\therefore \therefore 4^x = -\frac{1}{4} \ \left(impossible\right) \quad or \ x = 2$$

QUESTION 6

6.1

$$Kuda : \left(5000\left(1 + 4 \times 0.083\right)\right) \times 1.04$$

$$= 6660 \times 1.04$$

$$= R6\,926.40$$

$$Thabo : 5000\left(1 + \frac{0.081}{12}\right)^{48}$$

$$= R6\,905.71$$

So Kuda is better off

6.2

6.2.1

$$P_v = x\left[\frac{1-(1+i)^{-n}}{i}\right]$$

$$\therefore 525000 = 6000\left[\frac{1-\left(1+\frac{0.1}{12}\right)^{-n}}{\frac{0.1}{12}}\right]$$

$$\therefore \frac{525}{6} = \frac{1-\left(1+\frac{0.1}{12}\right)^{-n}}{\frac{0.1}{12}}$$

$$\therefore \left(1+\frac{0.1}{12}\right)^{-n} = 1-\left(\frac{525}{6}\right)\left(\frac{0.1}{12}\right)$$

$$\therefore n = -\log_{1+\frac{0.1}{12}}\left(1-\left(\frac{525}{6}\right)\left(\frac{0.1}{12}\right)\right)$$

$$\therefore n = 157{,}4$$

so 157 *payments of* R6 000 *and one last*, *lesser payment*

6.2.2

She paid an extra R933.64 extra each month for 108 months.
The future value of this is

$$933.64\left(\frac{\left(1+\frac{0.1}{12}\right)^{108}-1}{\frac{0.1}{12}}\right)$$

$$= R162\,503.51 \quad \left(\begin{array}{l} \textit{actually } R16502.88 \textit{ if one} \\ \textit{uses full accuracy of actual} \\ \textit{minimum payment} \end{array}\right)$$

ALTERNATIVELY, we could work out the difference in the outstanding balances with the two payments:

$$\textit{With exact minimum payment of } R5066.36$$

$$OB = 5066.36\left[\frac{1-\left(1+\frac{0.1}{12}\right)^{-132}}{\frac{0.1}{12}}\right]$$

$$= R404\,665.59$$

$$\textit{With payments of } R6000$$

$$OB = 525000\left(1+\frac{0.1}{12}\right)^{108} - 6000\left[\frac{\left(1+\frac{0.1}{12}\right)^{108}-1}{\frac{0.1}{12}}\right]$$

$$= R242\,162.72$$

$$\textit{Difference is } R162\,502.87$$

QUESTION 7

7.1

$$f'(x)=\lim_{h\to 0}\frac{4-7(x+h)-(4-7x)}{h}$$
$$=\lim_{h\to 0}\frac{4-7x-7h-4+7x}{h}$$
$$=\lim_{h\to 0}\frac{-7h}{h}$$
$$=-7$$

7.2

$$y=4x^8+\sqrt{x^3}$$
$$\therefore y=4x^8+x^{\frac{3}{2}}$$
$$\therefore \frac{dy}{dx}=32x^7+\frac{3}{2}x^{\frac{1}{2}}$$

7.3

7.3.1

$$\frac{dy}{dx}=2ax$$

7.3.2

$$\frac{dy}{da}=x^2+1$$

7.4

$$y=x+12x^{-1}$$
$$pt.\ A\ is\ (2;8)$$
$$\frac{dy}{dx}=1-\frac{12}{x^2}$$
$$when\ x=2,\ \frac{dy}{dx}=-2$$
$$so\ perpendicular\ line\ has\ a\ gradient\ of\ \frac{1}{2}$$
$$\therefore y-8=\frac{1}{2}(x-2)$$
$$\therefore y=\frac{1}{2}x+7$$

QUESTION 8

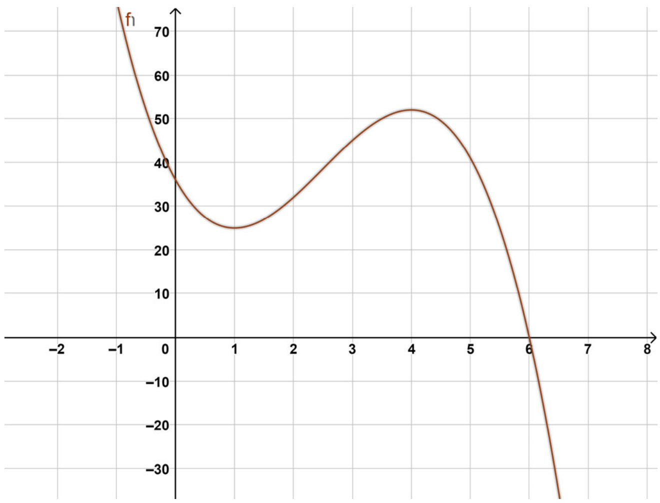
8.1

$$h(t)=(t-6)\left(-2t^2+3t-6\right)$$
$$=-2t^3+15t^2-24t+36$$
$$36\ cm$$

8.2

Only once

Graph of $h(x)$ looks like this. Surely this model only applies for $0\le t\le 6$?



8.3

$$h(t)=-2t^3+15t^2-24t+36$$
$$h'(t)=-6t^2+30t-24$$
$$h''(t)=-12t+30$$
$$-6t^2+30t-24=0$$
$$t^2-5t+4=0$$
$$(t-4)(t-1)=0$$
$$t=1\ or\ 4$$
$$h''(1)>0\ so\ local\ min$$
$$h''(4)<0\ so\ local\ max$$
$$h(4)=52$$
$$so,\ 52\ cm\ is\ \text{max. ht reached}$$

QUESTION 9

9.1

$3x^3 = 9x^2$

$\therefore 3x^3 - 9x^2 = 0$

$\therefore 3x^2(x-3) = 0$

$\therefore x = 0 \ \ or\ 3$

9.2

9.2.1

$f(x)$ *and* $f'(x)$ since both their derivatives will be zero at $x = 0$

9.2.2

The stationary point on f is a horizontal point of inflection, not a turning point

The stationary point on f' is a minimum turning point

9.3

$f'(x) = 9x^2 \ and\ f''(x) = 18x$

$f'(1) = 9 \ and\ f''(x) = 18$

so the vertical distance between them is 9

9.4

$3x^3 - 9x^2 < 0$

$\therefore 3x^2(x-3) < 0$

$\therefore x - 3 < 0 \ \ (3x^2 \ is\ positive)$

$\therefore x < 3 \ but\ x \neq 0$

QUESTION 10

Sample space is

$$(A,B) = \begin{Bmatrix} (M,M)\,(M,T)\,(M,W)\,(M,Th) \\ (T,M)\,(T,T)\,(T,W)\,(T,Th) \\ (W,M)..... \\ ....(Th,Th) \end{Bmatrix}$$

10.1

$P(study\ same\ day) = \frac{1}{4} \times \frac{1}{4} \times 4 = \frac{4}{16} = \frac{1}{4}$

10.2

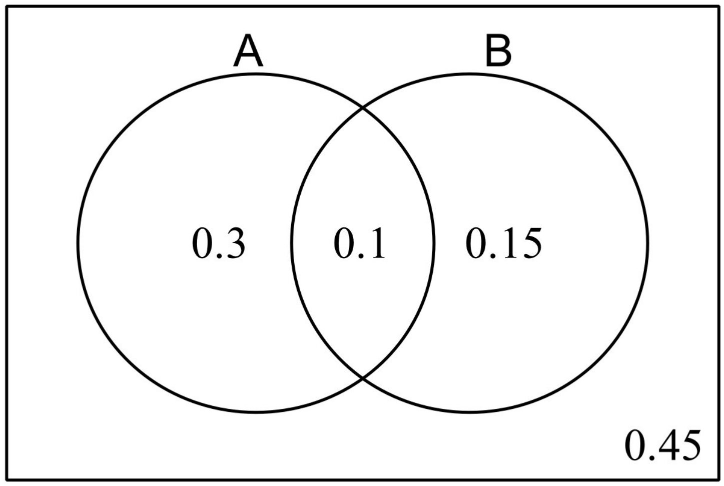
$P(consecutive\ days) = \frac{6}{16} \quad (MT, TM, TW, WTh, WT, ThW)$

QUESTION 11

11.1

Since A and B are independent $P(A \cap B) = 0.4 \times 0.25 = 0.1$

11.1.1



11.1.2

$P(A \cup \bar{B}) = 0.85$

11.2

The total number of combinations is
$5 \times 4 \times 6 = 120$
but of these:
there are 5 which have blue interior and blue exterior
and there are 5 which have red interior and red exterior
so, they need space for 110 cars
but this will require 550 m^2 and they only have 500m^2
So, no they won't have enough space.