Basic Education

KwaZulu-Natal Department of Education
REPUBLIC OF SOUTH AFRICA

MATHEMATICS P1

PREPARATORY EXAMINATION

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MEMORANDUM

NATIONAL SENIOR CERTIFICATE

GRADE 12

MARKS: 150

This memorandum consists of 11 pages.

QUESTION 1

1.1.1	$x(x-5)=0$ $x=0 \quad \text{or} \quad x=5$	✓✓ Answers	2
1.1.2	$4x^2-5x=3$ $4x^2-5x-3=0$ $x=\dfrac{-(-5)\pm\sqrt{(-5)^2-4(4)(-3)}}{2(4)}$ $x=1{,}69 \quad or \quad x=-0{,}44$	✓ equation in std. form ✓ substitution ✓✓ answers	4
1.1.3	$2^x(3x+1)<0$ $2^x>0$ for all $x\in R$ $\therefore 3x+1<0$ $x<-\dfrac{1}{3}$	✓ $2^x>0$ and $x\in R$ ✓ $3x+1<0$ ✓answer	3
1.1.4	$x-3x^{\frac{1}{2}}=4$ $x-4=3\sqrt{x}$ $9x=x^2-8x+16$ $x^2-17x+16=0$ $(x-1)(x-16)=0$ $x=1 \quad \text{or} \quad x=16$ n/a **OR** $x-3x^{\frac{1}{2}}-4=0$ $\left(x^{\frac{1}{2}}-4\right)\left(x^{\frac{1}{2}}+1\right)=0$ $x^{\frac{1}{2}}=4 \quad \text{or} \quad x^{\frac{1}{2}}=-1$ $x=16 \qquad$ no solution	✓ isolating $3\sqrt{x}$ or $3x^{\frac{1}{2}}$ ✓ squaring both sides ✓ trinomial ✓ factors ✓ both answers ✓ selection ✓ standard form ✓factors ✓ $x^{\frac{1}{2}}=4$ ✓ $x^{\frac{1}{2}}=-1$ ✓answer ✓rejecting	6 6
1.2	$\dfrac{\sqrt{9^{2028}}}{\sqrt{9^{2030}}-\sqrt{9^{2026}}}$ $\dfrac{\sqrt{9^{2028}}}{\sqrt{9^{2026}}\left(\sqrt{9^4}-1\right)}$ $=\dfrac{\sqrt{9^2}}{80}$ $=\dfrac{9}{80}$	✓ factorising denominator ✓ simplifying ✓answer	3

1.3	$2^{3x+1} = 4^y$ and $x^2 + 2y = 29$ $3x+1 = 2y \quad \rightarrow \quad (1)$ $x^2 + 2y = 29 \quad \rightarrow \quad (2)$ Subst. (1) into (2) $x^2 + 3x + 1 = 29$ $x^2 + 3x - 28 = 0$ $(x+7)(x-4) = 0$ $x = -7$ or $x = 4$ $y = -10$ or $y = \frac{13}{2}$	✓ for $2y$ as subject ✓ substitution of $2y$ ✓ std. form ✓factors ✓x values ✓y values	6
			[24]

QUESTION 2

2.1.1	3 ; 2 ; 6 ; 2 ; 9 ; 2 ; ... 12 ; 2	✓✓ answers	2
2.1.2	$a = 3; d = 3; n = 50$ $S_{50} = \frac{50}{2}[3+150]$ or $\frac{50}{2}[2(3)+3(50-1)]$ $= 3825$ $a = 2$ and is a constant sequence $\therefore S_{50} = 50 \times 2$ $= 100$ $S_{100} = 3825 + 100$ $= 3925$	✓ for a, d and n values ✓ for substitution into formula ✓ 3825 ✓100 ✓answer	5
2.2	$S_n = a + ar + ar^2 + ... + ar^{n-1} \quad \rightarrow \quad (1)$ $rS_n = ar + ar^2 + ... \quad + ar^n \quad \rightarrow \quad (2)$ $(2) - (1)$: $rS_n - S_n = ar^n - a$ $S_n(r-1) = a(r^n - 1)$ $\therefore S_n = \frac{a(r^n - 1)}{r-1}$	✓ for equation (1) ✓ for equation (2) ✓ subtraction on LHS and RHS ✓ factorising	4
			[11]

QUESTION 3

3.1	$\frac{24}{x}+12+6x+3x^2+....$ $a=6, r=2$ $S_{15}=\frac{6\left(2^{15}-1\right)}{2-1}$ $=196602$	✓a value ✓r value ✓substitution into formula ✓answer	4
3.2	$-1<r<1$ $-1<\frac{1}{2}x<1$ $-2<x<2$	✓ for condition ✓ for $r=\frac{1}{2}x$ ✓ answer	3
3.3	$r>1$ $\frac{1}{2}x>1$ $x>2$	✓ $r>1$ ✓answer	2
			[9]

QUESTION 4

4.1	$2a=4$ $a=2$ $3a+b=2$ $b=-4$ $a+b+c=5$ $c=7$ $T_n=2n^2-4n+7$ **OR** $2a=4$ $a=2$ $3a+b=2$ $b=-4$ $T_0=c=7$ $T_n=2n^2-4n+7$ **OR**	✓ a value ✓b value ✓c value ✓general term ✓ a value ✓b value ✓c value ✓general term	4 4

	$T_n = T_1 + (n-1)d_1 + \frac{(n-1)(n-2)}{2}d_2$ $= 5 + (n-1)(2) + \frac{(n-1)(n-2)}{2}(4)$ $= 5 + 2n - 2 + 2n^2 - 6n + 4$ $= 2n^2 - 4n + 7$	✓formula ✓substituting first and second difference values ✓simplifying ✓general term	4
4.2	First Difference nth term = (4n-2) $4n - 2 = 2018$ $4n = 2020$ $n = 505$ $T_{506} = 2(506)^2 - 4(506) + 7 = 510055$ $T_{505} = 2(505)^2 - 4(505) + 7 = 508037$ Between the 505th and the 506th terms.	✓$4n - 2 = 2018$ ✓$n = 505$ ✓answer	3
	OR $T_n - T_{n-1} = 2n^2 - 4n + 7 - \left[2(n-1)^2 - 4(n-1) + 7\right]$ $= 2n^2 - 4n + 7 - \left[2n^2 - 4n + 2 - 4n + 4 + 7\right]$ $= 4n - 6$ $4n - 6 = 2018$ $4n = 2024$ $n = 506$ Between the 505th and the 506th terms.	✓$T_n - T_{n-1}$ substitution ✓$4n - 6 = 2018$ ✓answer	3
			[7]

QUESTION 5

5.1	$A = P(1+i)^n$ $2x = x(1+0{,}085)^n$ $n = \dfrac{\log 2}{\log(1+0{,}085)}$ $= 8{,}5$ or 9 years or 8 years and 6 months	✓✓correct substitution into correct formula ✓making n the subject ✓answer	4
5.2.1	$A = P(1+i)^n$ $= 350\,000\left(1+\dfrac{0{,}135}{12}\right)^2$ $=$ R357919,30	✓correct substitution into correct formula ✓answer	2
5.2.2	$P = \dfrac{x[1-(1+i)^{-n}]}{i}$ $357919{,}30 = \dfrac{x\left[1-\left(1+\dfrac{0{,}135}{12}\right)^{-238}\right]}{\dfrac{0{,}135}{12}}$ $x = \dfrac{357919{,}30\times\dfrac{0{,}135}{12}}{\left[1-\left(1+\dfrac{0{,}135}{12}\right)^{-238}\right]}$ $x =$ R4328,60	✓✓correct substitution into correct formula ✓ x as subject ✓ answer	4
5.2.3	Balance on Loan after the 180th payment $= 357919{,}30\left(1+\dfrac{0{,}135}{12}\right)^{178} - \dfrac{4328{,}60\left[\left(1+\dfrac{0{,}135}{12}\right)^{178}-1\right]}{\dfrac{0{,}135}{12}}$ $=$ R188118,54 Total Payments $= R4328{,}60\times 60 = R259716$ Savings $= R259716$ - R188118,54 $=$ R71597,46	✓✓substitution into formulae ✓R188118,54 ✓R1030206,80 ✓ answer	5
			[15]

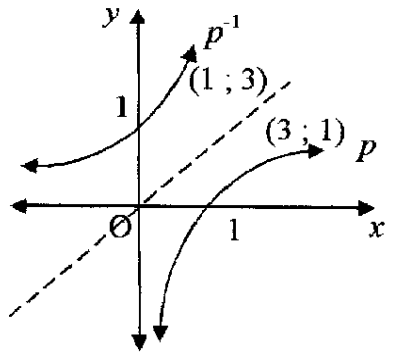
QUESTION 6

6.1	$f(x)=\frac{a}{x+p}+q$. P(–2; 4) is a point on f $p=1$ $q=1$ $y=\frac{a}{x+1}+1$ *Now subst.* $P(-2;4)$: $4=\frac{a}{-2+1}+1$ $\therefore a=-3$	✓ p value ✓ q value ✓ subst. p, q and point P ✓a value	4
6.2	$\frac{-3}{x+1}+1=0$ $x+1=3$ $x=2$ $T(2\,;0)$	✓$y=0$ ✓ $x+1=3$ ✓answer	3
6.3	$y=(x+p)+q$ $y=(x+1)+1$ $y=x+2$ $\therefore c=2$ **OR** $y=x+c$ Point of intersection of asymptotes $(-1\,;1)$ $-1=1+c$ $c=2$	✓substitution of p and q values into equation of line of symmetry ✓answer ✓substitution of p and q values into equation of line of symmetry ✓answer	2 2
			[9]

QUESTION 7

7.1	$g(x)=-x^2+2x+3$ and $h(x)=ax+q$. CD = 6 units. $-x^2+2x+3=0$ $(x+1)(x-3)=0$ $x=-1$ *or* $x=3$ B$(3;0)$ C$(0;3)$	✓$y=0$ ✓factors ✓coordinates of B ✓coordinates of C	4
7.2	D$(0;-3)$	✓✓ answer	2
7.3	$a=1$ *and* $q=-3$	✓✓ answers	2
7.4	$-x^2+2x+3=x-3$ $x^2-x-6=0$ $(x+2)(x-3)=0$ $x=-2$ or $x=3$ $y=-5$ or $y=0$ $E(-2;-5)$	✓equating both equations ✓ factors ✓ x values ✓y values ✓answer	5
7.5	$g: y=-(x-1)^2+4$ $-1<x<1$ or $x>3$	✓Axis of symmetry value ✓ ✓ end points and inequality ✓ $x>3$	4
			[17]

QUESTION 8

8.1	$p(x)=\log_3 x$ $x=\log_3 y$ $p^{-1}: y=3^x$	✓✓ answer	2
8.2		✓✓ graph of p ✓✓graph of p^{-1}	4
8.3	$\log_3 x=2$ $x=9$ $0<x\leq 9$	✓✓for end points and inequality	2
8.4	x intercept of p is (1 ; 0) x intercept of h is (–1 ; 0)	✓✓answer	2
			[10]

QUESTION 9

9.1	$f(x)=x^2+3x$ $f'(x)=\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$ $=\lim\limits_{h\to 0}\dfrac{(x+h)^2+3(x+h)-(x^2+3x)}{h}$ $=\lim\limits_{h\to 0}\dfrac{x^2+2xh+h^2+3x+3h-x^2-3x}{h}$ $=\lim\limits_{h\to 0}\dfrac{2xh+h^2+3h}{h}$ $=\lim\limits_{h\to 0}\dfrac{h(2x+h+3)}{h}$ $=2x+3$	✓ $f(x+h)=(x+h)^2+3(x+h)$ ✓ substitution into correct formula ✓ simplifying ✓factors ✓ answer	5
9.2.1	$\dfrac{dy}{dx}$ if $y=3x^2.\sqrt[3]{8x^4}$ $y=6x^{\frac{10}{3}}$ $\dfrac{dy}{dx}=20x^{\frac{7}{3}}$ or $20\sqrt[3]{x^7}$	✓ 6 ✓ $x^{\frac{10}{3}}$ ✓ answer	3
9.2.2	$f'(x)$ if $f(x)=\dfrac{x^3-5x^2+4x}{x-4}$ $f(x)=\dfrac{x(x^2-5x+4)}{(x-4)}$ $=\dfrac{x(x-1)(x-4)}{(x-4)}$ $=x^2-x$ $f'(x)=2x-1$	✓factorising ✓factors ✓simplification of f ✓answer	4
			[12]

QUESTION 10

10.1	$y=\frac{4}{x}$ & the gradient of the tangent to the curve is -1. $y=4x^{-1}$ $\frac{dy}{dx}=-4x^{-2}=\frac{-4}{x^2}$ $\frac{-4}{x^2}=-1$ $x^2=4$ $\therefore x=-2$ or $x=2$ $y=-2$ or $y=2$ $(-2;-2)$ $(2;2)$	✓exponential form ✓ derivative ✓ derivative $=-1$ ✓ x - values ✓y – values	5
10.2.1	$y=(x-1)(x-4)^2$ $=(x-1)(x^2-4x+4)$ $=x^3-9x^2+24x-16$	✓$(x-1)(x-4)^2$ ✓squaring binomial	2
10.2.2	$y=x^3-9x^2+24x-16$ $\frac{dy}{dx}=3x^2-18x+24=0$ $x^2-6x+8=0$ $(x-2)(x-4)=0$ $x=2$ *or* $x=4$ $y=4$ *or* $y=0$ $B(2;4)$	✓derivative and equal to 0 ✓factors ✓ y – values ✓coordinates of B	4
10.2.3	$k<-16$	✓inequality ✓-16	2
10.2.4	$f''(x)=6x-18>0$ $x>3$	✓ $6x-18>0$ ✓answer	2
			[15]

QUESTION 11

11.1	$V = l \times b \times h$ $= 5x(9-2x)(x)$ $= 45x^2 - 10x^3$	✓formula ✓ substitution	2
11.2	$V' = 90x - 30x^2$ $90x - 30x^2 = 0$ $30x(3-x) = 0$ $x = 0$ or $x = 3$ Therefore the box will have a maximum at $x = 3$	✓derivative ✓derivative equal to 0 ✓factors ✓x values ✓choosing $x = 3$	5
			[7]

QUESTION 12

12.1	$(x+0,2)\times 0,5 = 0,2$ $x + 0,2 = 0,4$ $x = 0,2$ $0,2 + 0,2 + 0,3 + y = 1$ $y = 0,3$	✓P(A) x P(B) = P(A and B) ✓x value… P(A only) ✓ $y = 1 - 0,7$ ✓y value	4
12.2	P(X and Y together) $= \frac{2!5!}{6!}$ $= \frac{1}{3}$ P(X and Y not together) $= 1 - \frac{1}{3}$ $= \frac{2}{3}$	✓$\frac{2!5!}{6!}$ ✓$\frac{1}{3}$ ✓answer	3
12.3.1	4 digit numbers(with repetitions) =9x10x10x10 =9000	✓product ✓answer	2
12.3.2	4 digit numbers(without repetitions) =9x9x8x7 =4536	✓ 9x9 ✓ 8x7 ✓ answer	3
12.3.3	4 digit numbers(with repetitions and last digit = 0) =9x10x10x1 =900	✓ product ✓ answer	2
			[14]

TOTAL MARKS: 150