Basic Education

KwaZulu-Natal Department of Basic Education
REPUBLIC OF SOUTH AFRICA

MATHEMATICS P1

PREPARATORY EXAMINATION

SEPTEMBER 2015

NATIONAL SENIOR CERTIFICATE

GRADE 12

Marks: 150

Time: 3 hours

N.B: This question paper consists of 9 pages and 1 information sheet

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 10 questions.
2. Answer ALL the questions.
3. Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining your answers.
4. Answers only will not necessarily be awarded full marks.
5. An approved scientific calculator (non-programmable and non-graphical) may be used, unless stated otherwise.
6. If necessary, answers should be rounded off to TWO decimal places, unless stated otherwise.
7. Diagrams are NOT necessarily drawn to scale.
8. Number the answers correctly according to the numbering system used in this question paper. Write neatly and legibly.

QUESTION 1

1.1 Solve for x in each of the following :

1.1.1 $x(x-5)=0$ (2)

1.1.2 $4x^2-5x=3$ (Give answer correct to TWO decimal places) (4)

1.1.3 $2^x(3x+1)<0$ (3)

1.1.4 $x-3x^{\frac{1}{2}}=4$ (6)

1.2 Calculate, **without using a calculator**:

$$\frac{\sqrt{9^{2028}}}{\sqrt{9^{2030}}-\sqrt{9^{2026}}}$$ (3)

1.3 Solve for x and y simultaneously:

$2^{3x+1}=4^y$ and

$x^2+2y=29$ (6)

[24]

QUESTION 2

2.1 Given the combined arithmetic and constant sequences :

3 ; 2 ; 6 ; 2 ; 9 ; 2 ; ...

2.1.1 Write down the next **two** terms in the sequence. (2)

2.1.2 Calculate the sum of the first 100 terms of the sequence. (5)

2.2 Prove that: $a+ar+ar^2+...(to\ n\ terms)=\frac{a(r^n-1)}{r-1};\quad r\neq 1$ (4)

[11]

QUESTION 3

Given the geometric series: $\frac{24}{x}+12+6x+3x^2+....$

3.1 If $x=4$, then determine the sum to 15 terms of the sequence. (4)

3.2 Determine the values of x for which the original series converges. (3)

3.3 Determine the values of x for which the original series will be increasing. (2)

[9]

QUESTION 4

Given the quadratic sequence: 5 ; 7 ; 13 ; 23 ; . . .

4.1 Calculate the n^{th} term of the quadratic sequence. (4)

4.2 Determine between which two consecutive terms of the quadratic sequence the first difference will be equal to 2018. (3)
[7]

QUESTION 5

5.1 How long will it take for a motor car to double in value if the annual inflation rate is 8,5 % ? (4)

5.2 A loan of R350 000, taken on 1 January 2005, is to be repaid in regular fixed instalments at the end of each month. Interest was charged at 13,5 % p.a. compounded monthly for 20 years. The client made the first payment on 31 March 2005.

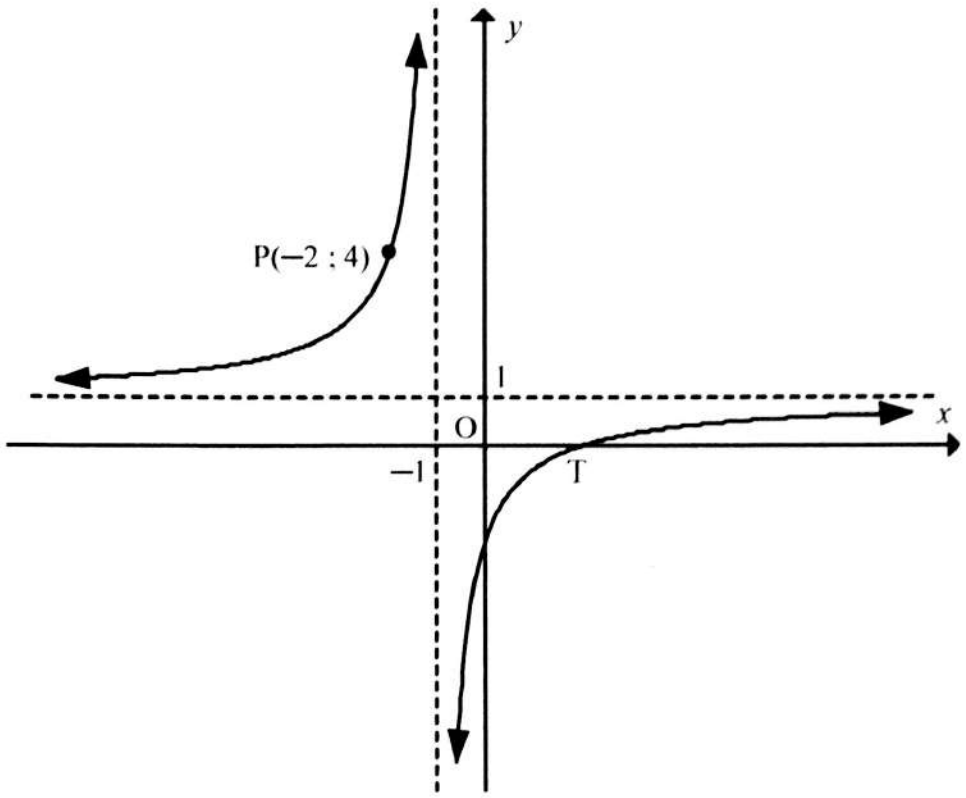
5.2.1 Calculate the value of the loan payable on 28 February 2005. (2)

5.2.2 Determine the monthly repayment that will settle the loan within the 20 year period. (4)

5.2.3 The client wishes to settle the loan at the end of the 180^{th} month. Calculate the savings made as a result of settling this loan earlier. (5)
[15]

QUESTION 6

The diagram above shows the graph of $f(x) = \frac{a}{x+p} + q$. The lines $x = -1$ and $y = 1$ are the asymptotes of f. P$(-2;4)$ is a point on f and T is the x–intercept of f.



6.1 Determine the values of a, p, and q. (4)

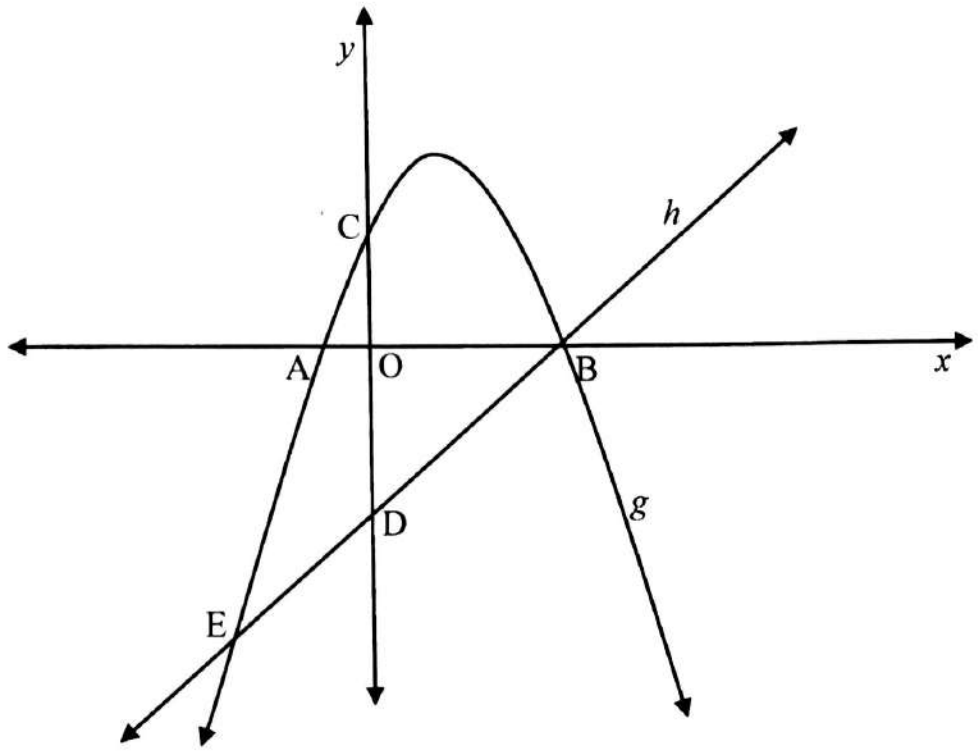
6.2 Calculate the coordinates of T, the x – intercept of f. (3)

6.3 If the graph of f is symmetrical with respect to the line $y = x + c$, determine the value of c. (2)

[9]

QUESTION 7

The sketch below shows the graphs of $g(x) = -x^2 + 2x + 3$ and $h(x) = ax + q$. The graphs intersect at B and E. The graph of g intersects the x – axis at A and B and has a turning point at C. The graph of h intersects the y – axis at D. The length of CD is 6 units.



7.1 Determine the coordinates of B and C. (4)

7.2 Write down the coordinates of D. (2)

7.3 Write down the values of a and q. (2)

7.4 Determine the coordinates of E. (5)

7.5 Determine the value(s) of x for which $g'(x).g(x) > 0$. (4)
[17]

QUESTION 8

Given $p(x) = \log_3 x$.

8.1 Write down the equation of p^{-1}, the inverse of p, in the form $y = \ldots$ (2)

8.2 Sketch in your ANSWER BOOK the graphs of p and p^{-1} on the same system of axes. Show clearly all the intercepts with the axes and at least one other point on each graph. (4)

8.3 Determine the values of x for which $p(x) \leq 2$ (2)

8.4 Write down the x intercept of h if $h(x) = p(-x)$. (2)
[10]

QUESTION 9

9.1 Determine the derivative of $f(x) = x^2 + 3x$ from first principles. (5)

Evaluate:

9.2.1 $\frac{dy}{dx}$ if $y = 3x^2 \cdot \sqrt[3]{8x^4}$ (3)

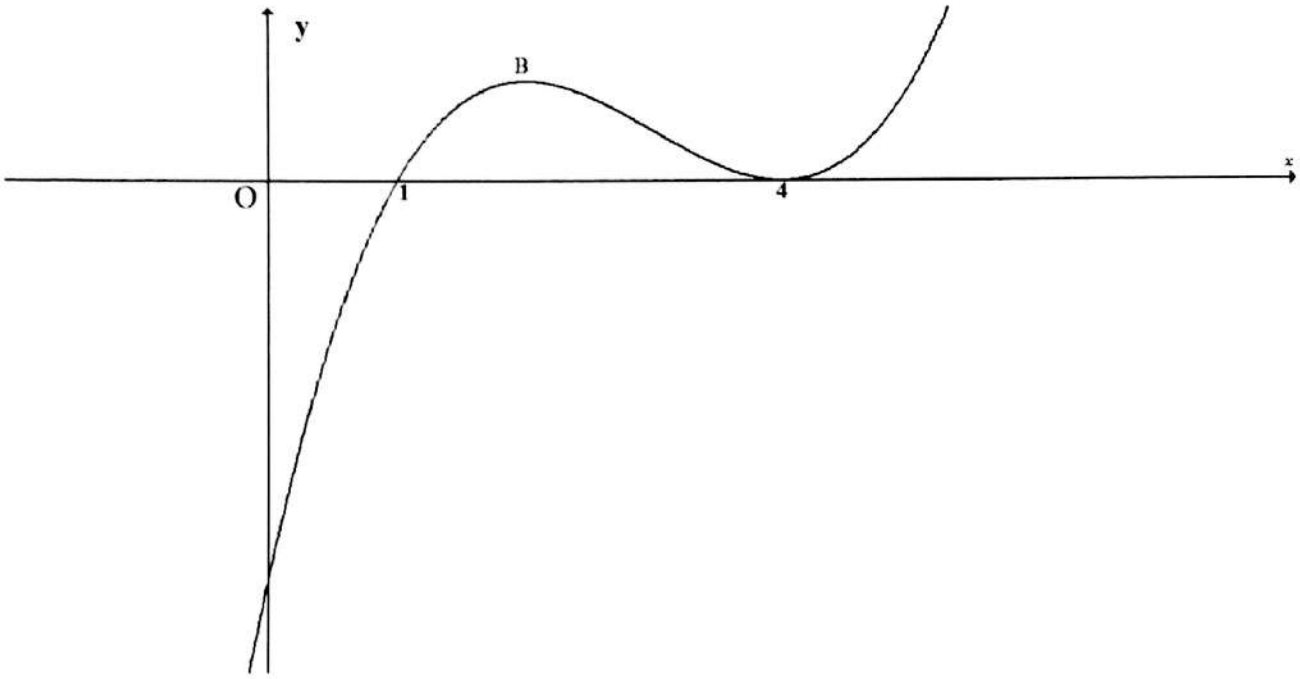
9.2.2 $f'(x)$ if $f(x) = \frac{x^3 - 5x^2 + 4x}{x - 4}$ (4)
[12]

QUESTION 10

10.1 Determine the points on the curve $y = \frac{4}{x}$ where the gradient of the tangent to the curve is -1. (5)

10.2 The graph of a cubic function with equation $f(x) = x^3 + ax^2 + bx + c$ is drawn.

- $f(1) = f(4) = 0$
- f has a a local maximum at B and a local minimum at x = 4.



10.2.1 Show that $a = -9$, $b = 24$ and $c = -16$. (2)

10.2.2 Calculate the coordinates of B. (4)

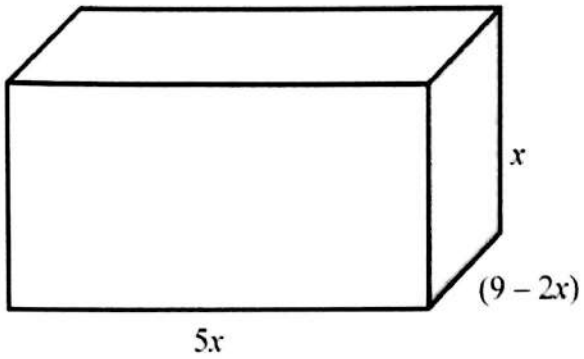
10.2.3 Determine the value(s) of k for which $f(x) = k$ has negative roots only. (2)

10.2.4 Determine the value(s) of x for which f is concave up. (2)

[15]

QUESTION 11

A rectangular box has a length of $5x$ units, breadth of $(9-2x)$ units and its height of x units.



11.1 Show that the volume (V) of the box is given by $V = 45x^2 - 10x^3$. (2)

11.2 Determine the value of x for which the box will have maximum volume. (5)

[7]

QUESTION 12

12.1 Given that A and B are independent events. Determine the values of x and y if:

- P(B only) = 0,3
- P(A and B) = 0,2
- P(A only) = x
- P(not A or B) = y

(4)

12.2 Six players of a volleyball team stand at random positions in a row before the game begins. X and Y are two players in this team.

Determine the probability that:

X and Y will not stand next to each other. (3)

12.3 Determine how many 4 – digit numbers can be formed from 10 digits 0 to 9 if:

12.3.1 repetition of digits is allowed. (2)

12.3.2 repetition of digits is not allowed. (3)

12.3.3 the last digit must be 0 and repetition of digits is allowed. (2)

[14]

TOTAL: 150

INFORMATION SHEET: MATHEMATICS

INLIGTINGSBLAD: WISKUNDE

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1+ni) \qquad A = P(1-ni) \qquad A = P(1-i)^n \qquad A = P(1+i)^n$$

$$T_n = a + (n-1)d \qquad S_n = \frac{n}{2}\left(2a + (n-1)d\right)$$

$$T_n = ar^{n-1} \qquad S_n = \frac{a\left(r^n - 1\right)}{r-1} \; ; \quad r \neq 1 \qquad S_\infty = \frac{a}{1-r} \; ; \; -1 < r < 1$$

$$F = \frac{x\left[(1+i)^n - 1\right]}{i} \qquad P = \frac{x[1-(1+i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad \mathrm{M}\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c \qquad y - y_1 = m(x - x_1) \qquad m = \frac{y_2 - y_1}{x_2 - x_1} \qquad m = \tan\theta$$

$$(x-a)^2 + (y-b)^2 = r^2$$

In ΔABC: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$ $\qquad a^2 = b^2 + c^2 - 2bc.\cos A$ $\qquad$ area $\Delta ABC = \dfrac{1}{2}ab.\sin C$

$$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta \qquad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$$

$$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta \qquad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases} \qquad \sin 2\alpha = 2\sin\alpha.\cos\alpha$$

$$\bar{x} = \frac{\sum fx}{n} \qquad \sigma^2 = \frac{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}{n}$$

$$\mathrm{P}(A) = \frac{n(\mathrm{A})}{n(\mathrm{S})} \qquad \mathrm{P(A \text{ or } B)} = \mathrm{P(A)} + \mathrm{P(B)} - \mathrm{P(A \text{ and } B)}$$

$$\hat{y} = a + bx \qquad b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$$