GAUTENG PROVINCE
EDUCATION
REPUBLIC OF SOUTH AFRICA

PREPARATORY EXAMINATION
2016
MEMORANDUM

MATHEMATICS (FIRST PAPER) (10611)

15 pages

GAUTENG DEPARTMENT OF EDUCATION

PREPARATORY EXAMINATION

MATHEMATICS
(First Paper)

MEMORANDUM

	QUESTION 1		
1.1	1.1.1	$3x^2+5x=2$ $3x^2+5x-2=0$ $(3x-1)(x+2)=0$ $x=\frac{1}{3}...OR...x=-2$	✓ standard form ✓ factors ✓ both x values
			(3)
	1.1.2	$\sqrt{x+7}-1=x$ $\sqrt{x+7}=x+1$ $x+7=(x+1)^2$ $x+7=x^2+2x+1$ $x^2+x-6=0$ $(x+3)(x-2)=0$ $x=-3 \quad OR \quad x=2$ N/A. $\therefore x=2$	✓ $x+7=(x+1)^2$ ✓standard form ✓factors ✓N/A ✓ $x=2$
			(5)
	1.1.3	$x^2-8x=10$ $x^2-8x-10=0$ $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ $x=\frac{-(-8)\pm\sqrt{(-8)^2-4(1)(-10)}}{2(1)}$ $x=9,1 \qquad OR \qquad x=-1,1$	✓standard form ✓substitution ✓ $x=9,1$ ✓ $x=-1,1$
			(4)

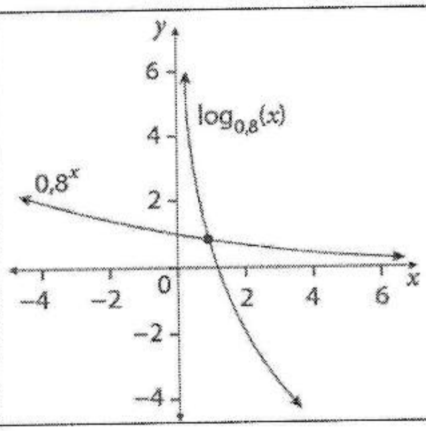
	1.1.4	$3^x + 3^{-x+1}.5 = 8$ $3^x + \frac{3.5}{3^x} = 8$ $3^{2x} - 8.3^x + 15 = 0$ $(3^x - 5)(3^x - 3) = 0$ $3^x = 5 \quad OR \quad 3^x = 3$ $x = \log_3 5 \qquad x = 1$ $x = 1{,}46$ **OR** $Let\ 3^x = k$ $k + \frac{3.5}{k} = 8$ $k^2 - 8k + 15 = 0$ $(k-3)(k-5) = 0$ $k = 3 \quad OR \quad k = 5$ $3^x = 3 \qquad 3^x = 5$ $x = 1 \qquad x = \log_3 5$ $x = 1{,}46$	✓standard form ✓✓factors ✓ $x = 1$ ✓ $x = \log_3 5$ OR $x = 1{,}46$ ✓substitute with k ✓standard form ✓factors ✓ $x = 1$ ✓ $x = \log_3 5$ OR $x = 1{,}46$
			(5)
1.2	$\frac{2^{2015} + 2^{2013}}{4^{1006}}$ $= \frac{2^{2013}(2^2 + 1)}{2^{2012}}$ $= 2(5)$ $= 10$		✓factors ✓ 2^{2012} ✓answer
			(3)
			[20]

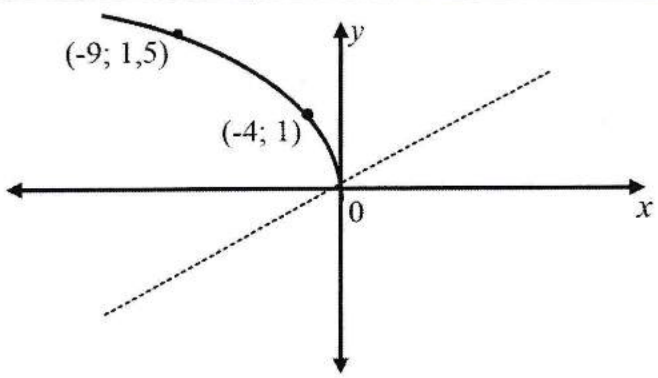
	QUESTION 2		
2.1	$kx^2 + kx + 1 = 0$ $\Delta = b^2 - 4ac$ $= (k)^2 - 4(k)(1)$ $\cdots = k^2 - 4k$ $= k(k-4)$ *for non-real roots* $\Delta < 0$ $k(k-4) < 0$ $\therefore 0 < k < 4$		✓correct substitution in Δ ✓ $\Delta < 0$ ✓0 and 4 ✓ inequality signs
			(4)
2.2	$2^{x+1} = 4^y$.......1 $x^2 + 2y = 3$......2 $2^{x+1} = 2^{2y}$ $x + 1 = 2y$ $x = 2y - 1$.......3 *Substitute in* 2 $(2y-1)^2 + 2y = 3$ $4y^2 - 4y + 1 + 2y - 3 = 0$ $4y^2 - 2y - 2 = 0$ $2(2y+1)(y-1) = 0$ $\ldots y = -\frac{1}{2}$ ***OR*** $y = 1$ *Substitute in* 3 $x = 2\left(-\frac{1}{2}\right) - 1 \cdots\cdots x = 2(1) - 1$ $x = -2 \cdots\cdots\cdots\cdots x = 1$ **OR**	$2^{x+1} = 4^y$.......1 $x^2 + 2y = 3$......2 $2^{x+1} = 2^{2y}$ $x + 1 = 2y$.........3 *Substitute in* 2 $x^2 + x + 1 = 3$ $x^2 + x - 2 = 0$ $(x+2)(x-1) = 0$ $x = -2 \cdots$ ***OR*** $\cdots x = 1$ *Substitute in* 3 $-2 + 1 = 2y \cdots 1 + 1 = 2y$ $y = -\frac{1}{2} \cdots\cdots\cdots y = 1$	✓ simplify equation ✓ substitute ✓ factors ✓ both y values ✓both x values
			(5)
			[9]

	QUESTION 3				
3.1	-8 -2 8 22 6 10 14 4 4				
	3.1.1	40			✓answer
					(1)
	3.1.2	$2a=4$ $a=2$ $\therefore T_n=2n^2-10$	$3a+b=6$ $3(2)+b=6$ $b=0$	$a+b+c=-8$ $2+c=-8$ $c=-10$	✓second difference = 4 ✓ $a=2$ ✓$b=0$ ✓$c=-10$
					(4)
3.2					
	3.2.1	$a=5$ $T_6=10T_3$ $a+5d=10(a+2d)$ $5+5d=10(5+2d)$ $5d-20d=50-5$ $d=-3$			✓$T_6=10T_3$ ✓$a+5d=10(a+2d)$ ✓simplification ✓$d=-3$
					(4)
	3.2.2	$S_n=\frac{n}{2}[2a+(n-1)d]$ $S_{20}=\frac{20}{2}[2(5)+(20-1)(-3)]$ $=-470$			✓ correct formula ✓correct substitution ✓answer
					(3)

3.3	3.3.1	$\sum_{k=1}^{\infty} 2\left(\frac{1}{2}\right)^k$ $1;\ \frac{1}{2};\ \frac{1}{4};.....$ $r=\frac{1}{2}$ $S_\infty=\frac{a}{1-r}$ $=\frac{1}{1-\frac{1}{2}}$ $=2$	✓$r=\frac{1}{2}$ ✓correct substitution in S_∞ ✓$S_\infty=2$
			(3)
	3.3.2	$\sum_{k=0}^{7} 2\left(\frac{1}{2}\right)^k$ $2;\ 1;\ \frac{1}{2};.....$ $r=\frac{1}{2}$ $S_n=\frac{a(r^n-1)}{r-1}$ $=\frac{2\left(\left(\frac{1}{2}\right)^8-1\right)}{\frac{1}{2}-1}$ $=\frac{255}{64}\cdots$ ***OR*** $\cdots 3{,}98$	✓$n=8$ ✓correct substitution in S_n ✓$S_n=\frac{255}{64}$ OR 3,98
			(3)
	3.3.3	$T=2-\frac{255}{64}$ $=-1\frac{63}{64}$ ***OR*** $=\frac{-127}{64}$ ***OR*** $=-1{,}98$	✓answer
			(1)

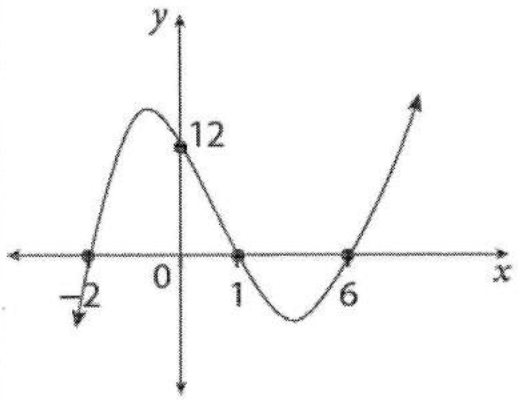
3.4	$19;\ 18\frac{1}{5};\ 17\frac{2}{5};\ \ldots$ $d=-\frac{4}{5}$ $T_n=a+(n-1)d$ $19+(n-1)\left(\frac{-4}{5}\right)<0$ $19-\frac{4}{5}n+\frac{4}{5}<0$ $\frac{-4}{5}n<-19\frac{4}{5}$ $n>24\frac{3}{4}\cdots OR\cdots\frac{99}{4}\cdots 24{,}75$ $\therefore$ *The 25th Term*	✓$d=-\frac{4}{5}$ ✓correct substitution in correct formula ✓$T_n<0$ ✓inequality ✓answer ✓conclusion
		(6)
		[25]
	QUESTION 4	
4.1	$p=4$ $q=2$ $T(5;3): y=\frac{a}{x-4}+2$ $3=a+2$ $a=1$	✓$p=4$ ✓$q=2$ ✓$a=1$
		(3)
4.2	$h(x)=\frac{1}{-x-4}+2$ **OR** $h(x)=\frac{-1}{x+4}+2$	✓answer
		(1)
4.3	$y=-x+c\cdots\cdots\cdots(4;2)$ $2=-(4)+c$ $c=6$	✓correct substitution of (4; 2) ✓answer **(2)**
		[6]

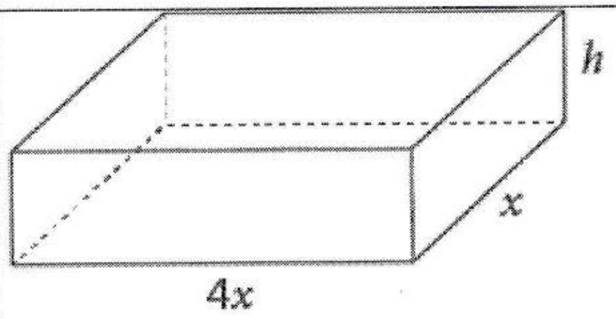
	QUESTION 5	
5.1	$f(x) = a^x$ and passes through the point $\left(-2; 1\frac{9}{16}\right)$. $\therefore 1\frac{9}{16} = a^{-2}$ $\therefore a^{-2} = \frac{25}{16}$ $\therefore a^2 = \frac{16}{25}$ $\therefore a = \sqrt{\frac{16}{25}}$ $\therefore a = \frac{4}{5}$	✓substitution ✓value of a
		(2)
5.2	$y = \left(\frac{4}{5}\right)^x$ $x = \left(\frac{4}{5}\right)^y$ $y = \log_{\frac{4}{5}} x$	✓interchange x and y ✓equation
		(2)
5.3	 	✓shape f ✓shape f^{-1} ✓asymptotes of both graphs
		(3)
5.4	$x > 0$	✓answer
		(1)
5.5	$y = \left(\frac{4}{5}\right)^x$ $h(x) = -\left(\frac{4}{5}\right)^x$	✓answer
		(1)
5.6	About the line $y = x$	✓answer
		(1)
5.7	$y > -1$	✓answer
		(1)
		[11]

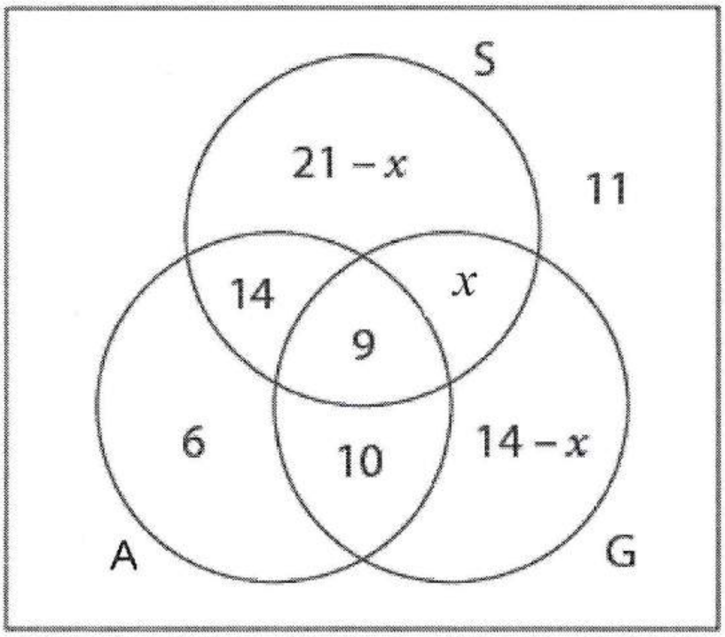
	QUESTION 6	
6.1	$y = ax^2$ $-4 = a(1)^2$ $-4 = a \qquad ;x \geq 0$ $h(x) = -4x^2$	✓substitution (1; -4) ✓value of a ✓equation
		(3)
6.2	$y = -4x^2$ $-9 = -4x^2$ $x^2 = \frac{9}{4}$ $x = \pm\sqrt{\frac{9}{4}}$ $x \geq 0 \quad \therefore T\left(\frac{3}{2}; -9\right)$	✓simplification $x^2 = \frac{9}{4}$ ✓answer
		(2)
6.3	$Gradient\ of\ ST = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{-9-(-4)}{1{,}5-1}$ $= -10$	✓substitution ✓answer
		(2)
6.4	$x = -4y^2\ ; y \geq 0$ $y^2 = -\frac{x}{4}$ $y = \pm\sqrt{-\frac{x}{4}}$ $h^{-1} = \sqrt{-\frac{x}{4}}$	✓interchange x and y ✓restriction ✓the value of y^2 ✓equation
		(4)
6.5		✓shape inverse ✓point on inverse ✓axis of symmetry
		(3)
		[14]

	QUESTION 7		
7.1		$1+0,164=\left[1+\frac{i_{nom}}{12}\right]^{12}$ $\sqrt[12]{1,164}=1+\frac{i_{nom}}{12}$ $i_{nom}=15,28\%$	✓correct substitution into correct formulae ✓ $\sqrt[12]{1,164}$ ✓answer
			(3)
7.2		$F=\frac{x\left[(1+i)^n-1\right]}{i}$ $2\ 300\ 000=\frac{x\left[\left(1+\frac{0,12}{4}\right)^{32}-1\right]}{\frac{0,12}{4}}$ $x=R43\ 807,22$	✓ 2 300 000 ✓ $\left[\left(1+\frac{0,12}{4}\right)^{32}-1\right]$ ✓ $n=32$ ✓answer
			(4)
7.3			
	7.3.1	$A=P(1+i)^n$ $A=R135\ 000\left(1+\frac{0,1475}{12}\right)^2$ $A=\ R138\ 339,15$	✓substitution ✓ $A=R138\ 339,15$
			(2)
	7.3.2	$P=\frac{x\left[1-(1+i)^{-n}\right]}{i}$ $R138\ 339,15=\frac{x\left[1-\left(1+\frac{0,1475}{12}\right)^{-18}\right]}{\frac{0,1475}{12}}$ $x=R8\ 613,99$	✓correct formula (P) ✓correct substitution ✓ $n=18$ ✓answer
			(4)
			[13]

	QUESTION 8		
8.1	$f(x)=4-3x^2$ $f(x+h)=4-3(x+h)^2$ $=4-3x^2-6xh-3h^2$ $f'(x)=\lim_{h\to0}\frac{f(x+h)-f(x)}{h}$ $=\lim_{h\to0}\frac{(4-3x^2-6xh-3h^2)-(4-3x^2)}{h}$ $=\lim_{h\to0}\frac{-6xh-3h^2}{h}$ $=\lim_{h\to0}\frac{h(-6x-3h)}{h}$ $=\lim_{h\to0}(-6x-3h)$ $=\quad -6x$		✓simplification of $f(x+h)$ ✓formula ✓simplification $-6xh-3h^2$ ✓common factor ✓ answer $-6x$
			(5)
	8.2.1	$D_x\left[x^4-2x+\frac{1}{x^2}\right]$ $=4x^3-2-2x^{-3}$	✓ $4x^3$ ✓ -2 ✓ $-2x^{-3}$
			(3)
	8.2.2	$y=\frac{2x-3}{\sqrt[4]{x}}$ $=2x^{\frac{3}{4}}-3x^{-\frac{1}{4}}$ $\frac{dy}{dx}=\frac{3}{2}x^{-\frac{1}{4}}+\frac{3}{4}x^{-\frac{5}{4}}$	✓simplification ✓both values of derivatives
			(2)
8.3	$f(x)=x^3-7x^2$ $f'(x)=3x^2-14x$ $f'(x)=5$ $3x^2-14x=5$ $3x^2-14x-5=0$ $(3x+1)(x-5)=0$ $x=-\frac{1}{3}$ ***OR*** $x=5$		✓ $f'(x)$ ✓ $f'(x)=5$ ✓factors ✓both answers **NOTE** *NO PENALTY IF CANDIDATE COMMITS AN ERROR IN WRITING AN ANSWER WITH POSITIVE EXPONENTS.*
			(4)
			[14]

	QUESTION 9	
9.1		✓y-intercept ✓x-intercept ✓shape
		(3)
9.2	$f(x) = a(x+2)(x-1)(x-6)$ $12 = a(0+2)(0-1)(0-6)$ $a = 1$ $\therefore f(x) = (x+2)(x-1)(x-6)$ $= x^3 - 5x^2 - 8x + 12$	✓correct formulae ✓substitution ✓$a = 1$ ✓expansion
		(4)
9.3	$f'(x) = 0$ $\therefore 3x^2 - 10x - 8 = 0$ $(3x+2)(x-4) = 0$ $x = -\frac{2}{3}$ ***OR*** $x = 4$ $y = \frac{400}{27}$ ***OR*** $y = -36$ Turning points are $\left(-\frac{2}{3}\ ;\ \frac{400}{27}\right)$ **AND** (4 ; -36)	✓equating to zero ✓factors ✓answers x ✓answers y ✓TPs
		(5)
9.4	$-\frac{2}{3} < x < 4$	✓answer
		(1)
9.5	$f''(x) = 6x - 10$ $6x - 10 > 0$ $x > \frac{5}{30}$	✓second derivative ✓$6x - 10 > 0$ ✓answer
		(3)
		[16]

	QUESTION 10	
	 	
10.1	$V = 128$ $4x.x.h = 128$ $h = \frac{128}{4x^2}$	✓substitution ✓answer
		(2)
10.2	$= 4x.x + 2xh + 8xh$ $= 4x^2 + 10xh$ Total surface area $= 4x^2 + 10x.\frac{32}{x^2}$ $= 4x^2 + \frac{320}{x}$	✓formula ✓simplification ✓answer
		(3)
10.3	$Dx\left(4x^2 + \frac{320}{x}\right) = 0$ $8x - \frac{320}{x^2} = 0$ $8x^3 - 320 = 0$ $8x^3 = 320$ $x^3 = \frac{320}{8}$ $x = \sqrt[3]{\frac{320}{8}}$ $x = \sqrt[3]{40}$ $x = 2\sqrt[3]{5}$ **Height =** $\frac{32}{\left(2\frac{3}{5}\right)^2} = 2{,}74$	✓derivative ✓equate to zero ✓simplification ✓ answer
		(4)
		[9]

	QUESTION 11	
11.1	Let S represent the participants for swimming; G represent the gymnastics participants and A for the athletics participants. 	✓6 ✓$21-x$ ✓$14-x$ ✓ all other values in the Venn diagram correct (CA)
		(4)
11.2	$21-x+x+14-x+9+14+10+6+11=80$ $85-x=80$ $x=5$	✓equation ✓answer
		(2)
11.3	P (Participants in at least 2 events) $=\dfrac{5+14+10+9}{80}$ $=0{,}475$	✓equation ✓division by 80 ✓answer
		(3)
		[9]

	QUESTION 12	
12.1	9! = 362 880 (Any other valid representation of the answer)	✓answer
		(1)
12.2	$\frac{9!}{5!}$ = 3 024 **OR** 9 x 8 x 7 x 6 = 3 024	✓ 9! ✓ 5! ✓answer ✓✓ 9 x 8 x 7 x 6 ✓answer
		(3)
		[4]
	TOTAL: 150	