GAUTENG PROVINCE
EDUCATION
REPUBLIC OF SOUTH AFRICA

GAUTENG DEPARTMENT OF EDUCATION
PREPARATORY EXAMINATION
2016

10611
MATHEMATICS
FIRST PAPER

TIME: 3 hours

MARKS: 150

11 pages + 1 information sheet and 1 answer sheet

X10

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(First Paper)

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INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 12 questions.
2. Answer ALL the questions.
3. Clearly show ALL calculations, diagrams, graphs, etc. which were used in determining the answers.
4. Answers only will not necessarily be awarded full marks.
5. Use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
6. Where necessary, answers should be rounded-off to TWO decimal places, unless stated otherwise.
7. Diagrams are NOT necessarily drawn to scale.
8. An ANSWER SHEET for answering Question 5.3 and Question 6.5 is located at the end of the question paper. This page must be submitted together with your ANSWER BOOK.
9. An INFORMATION SHEET appears on Page 12 of the question paper.
10. Number the answers correctly according to the numbering system used in the question paper.
11. Write neatly and legibly.

QUESTION 1

1.1 Solve for x:

1.1.1 $3x^2 + 5x = 2$ (3)

1.1.2 $\sqrt{x+7} - 1 = x$ (5)

1.1.3 $x^2 - 8x = 10$ (Round the answer off to ONE decimal place.) (4)

1.1.4 $3^x + 3^{-x+1} . 5 = 8$ (5)

1.2 Simplify without the use of a calculator:

$$\frac{2^{2015} + 2^{2013}}{4^{1006}}$$

(3)
[20]

QUESTION 2

2.1 Calculate the possible values of k if the roots of $kx^2 + kx + 1 = 0$ are non-real. (4)

2.2 Solve for x and y in the following simultaneous equations:

$$2^{x+1} = 4^y$$

$$x^2 + 2y = 3$$

(5)
[9]

QUESTION 3

3.1 Given the sequence:

$-8;\ -2;\ 8;\ 22;\ \ldots$

3.1.1 Write down the next term. (1)

3.1.2 Determine the formula for the general term of the sequence. (4)

3.2 The first term of an arithmetic sequence is 5 and the sixth term is 10 times the third term.

3.2.1 Calculate the constant first difference. (4)

3.2.2 If $d = -3$, calculate the sum of the first 20 terms. (3)

3.3 Calculate:

3.3.1 $\sum_{k=1}^{\infty} 2\left(\frac{1}{2}\right)^k$ (3)

3.3.2 $\sum_{k=0}^{7} 2\left(\frac{1}{2}\right)^k$ (3)

3.3.3 the value of T if:

$$T = \sum_{k=1}^{\infty} 2\left(\frac{1}{2}\right)^k - \sum_{k=0}^{7} 2\left(\frac{1}{2}\right)^k$$ (1)

3.4 Given the arithmetic sequence:

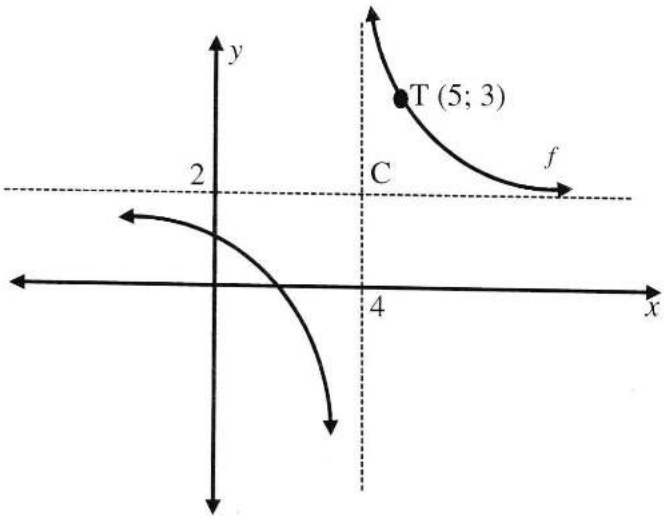
$19;\ 18\frac{1}{5};\ 17\frac{2}{5};\ \ldots$

Determine which term in this sequence will be the FIRST to be negative. (6)
[25]

QUESTION 4

The graph of $f(x)=\frac{a}{x+p}+q$ is sketched below with asymptotes $x=4$ and $y=2$.

$T(5;3)$ is a point on f and C is the point of intersection of the asymptotes.



4.1 Determine the values of a, p and q. (3)

4.2 Give the equation of h, the reflection of f in the y-axis. (1)

4.3 If the graph of f is symmetrical about the line $y=-x+c$, determine the value of c. (2)

[6]

QUESTION 5

Given that $f(x) = a^x$ with $a > 0$, and point $Q\left(-2\,;\,1\frac{9}{16}\right)$ lies on the graph of f.

5.1 Show that the value of $a = \frac{4}{5}$. (2)

5.2 Write down the equation of f^{-1}, the inverse graph of f.
Leave your answer in the form $y = ...$ (2)

5.3 Sketch the graphs of f and f^{-1} on the axes provided on the ANSWER SHEET. (3)

5.4 For which values of x is $f(x) < 1$? (1)

5.5 Write down the equation of h if h is a reflection of f about the x-axis. (1)

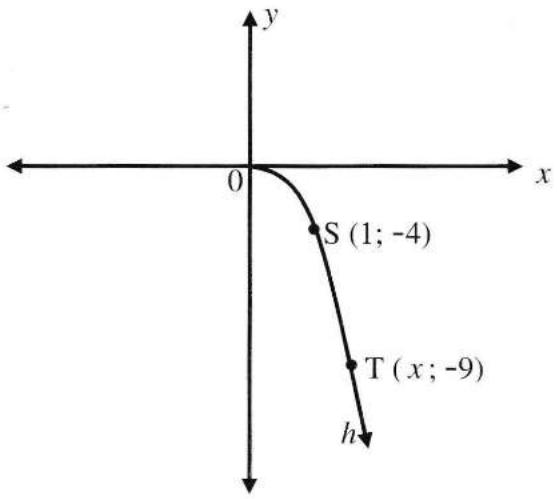
5.6 Write down the equation of a line that f has to be reflected about in order to obtain the graph of f^{-1}. (1)

5.7 Determine the range of the graph of $f(x) - 1$. (1)

[11]

QUESTION 6

The graph of h is sketched below, with $x \geq 0$.



6.1 Determine the equation of h. (3)

6.2 Determine the x-coordinate of point T if the equation of $h(x) = -4x^2$. (2)

6.3 Calculate the average gradient of ST if the coordinates of point T are $\left(\frac{3}{2}; -9\right)$. (2)

6.4 Determine the equation of $h^{-1}(x)$ in the form $y = ...$ (4)

6.5 Sketch the graph of h^{-1} on the axes provided on the ANSWER SHEET. Clearly indicate the axis of symmetry of h and h^{-1} (3)

[14]

QUESTION 7

7.1 Johan purchased a new car. The bank offered him a loan at an effective interest rate of 16,4% p.a. Determine the nominal interest rate, compounded monthly, that he is required to pay. (3)

7.2 A farmer sets up a sinking fund. He plans to accumulate R2,3 million in the fund at the end of 8 years by making equal quarterly payments into the fund. The interest rate on money accumulated is 12% p.a. compounded quarterly.
Calculate the quarterly payments into the fund if his first payment is made in 3 months' time. (4)

7.3 Emily plans to attend the Summer Olympics in Rio de Janeiro. She books an all-inclusive Olympic Package through a travel agency at the price of R135 000.
She acquires a personal loan to cover the costs at an interest rate of 14,75% p.a. compounded monthly and wishes to pay back the loan in 18 equal monthly instalments starting three months after receiving the loan.

7.3.1 Calculate the total amount owing, two months after the loan is granted. (2)

7.3.2 Calculate the monthly instalments on the loan. (4)
[13]

QUESTION 8

8.1 Given $f(x) = 4 - 3x^2$, determine $f'(x)$, using FIRST PRINCIPLES. (5)

8.2 Determine:

8.2.1 $D_x\left[x^4 - 2x + \frac{1}{x^2}\right]$ (3)

8.2.2 $\frac{dy}{dx}$ if $y = \frac{2x-3}{\sqrt[4]{x}}$ (2)

8.3 Calculate the x-coordinates of the points on the graph of $f(x) = x^3 - 7x^2$ at which the gradient of the tangent to f is equal to 5. (4)
[14]

QUESTION 9

A cubic graph f is defined by the following properties:

- $f(-2) = f(1) = f(6) = 0$
- $f(0) = 12$
- $a > 0$

9.1 By using the given information above, sketch a basic graph clearly indicating the intercepts with the axes. (3)

9.2 Show that $f(x) = x^3 - 5x^2 - 8x + 12$. (4)

9.3 Calculate the coordinates of the turning points of f. (5)

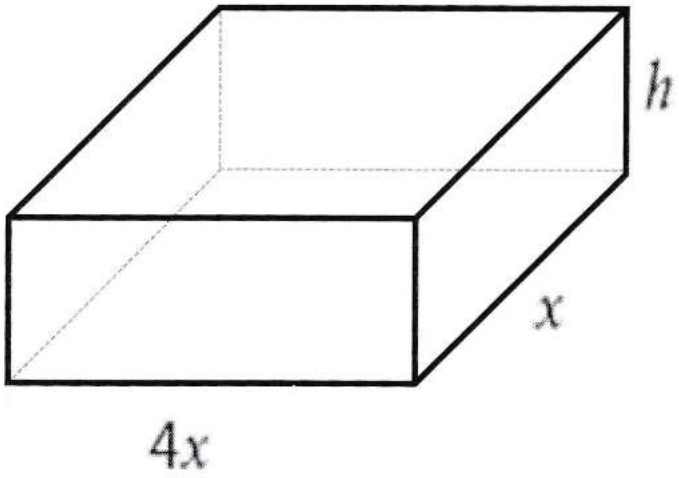
9.4 Determine the values of x for which $f'(x) < 0$. (1)

9.5 Calculate the value of x for which f is concave up. (3)

[16]

QUESTION 10

An open rectangular box is made of a very thin sheet of metal. The volume is $128\,cm^3$ and the base of the box has a width of $x\,cm$ and a length of $(4x)\,cm$.



10.1 Determine an expression for the height of the box in terms of x. (2)

10.2 Show that the total surface area of the box can be written as $\left(4x^2+\frac{320}{x}\right)cm^2$. (3)

10.3 Calculate the height of the box for which the surface area is a minimum. (4)

[9]

QUESTION 11

A survey of 80 participants at the 2016 Olympic Games was taken. Athletes' participation was recorded as follows:

- 44 participated in swimming;
- 33 participated in gymnastics;
- 39 participated in athletics;
- 23 participated in both swimming and athletics;
- 19 participated in both gymnastics and athletics;
- 9 participated in all three events;
- 69 participated in at least one event and
- 11 participants remained as reserves for the events (did not participate).

11.1 Let the number of participants in both swimming and gymnastics, but not in athletics, be represented by x.
Draw a Venn diagram to represent the survey. (4)

11.2 Hence show that $x = 5$. (2)

11.3 What is the probability that a participant chosen at random will participate in at least two of the three events. (3)
[9]

QUESTION 12

Given the word: E D U C A T I O N

12.1 In how many unique ways can all the letters in the word above be arranged? (1)

12.2 If 5 letters are randomly chosen from the word "EDUCATION", determine how many unique 5-letter arrangements can be formulated? (3)
[4]

TOTAL: 150

END

INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1+ni) \qquad A = P(1-ni) \qquad A = P(1-i)^n \qquad A = P(1+i)^n$$

$$\sum_{i=1}^{n} 1 = n \qquad \sum_{i=1}^{n} i = \frac{n(n+1)}{2} \qquad T_n = a + (n-1)d \qquad S_n = \frac{n}{2}\left(2a + (n-1)d\right)$$

$$T_n = ar^{n-1} \qquad S_n = \frac{a\left(r^n - 1\right)}{r-1} \;; \quad r \neq 1 \qquad S_\infty = \frac{a}{1-r} \;; -1 < r < 1$$

$$F = \frac{x\left[(1+i)^n - 1\right]}{i} \qquad P = \frac{x[1-(1+i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad \mathrm{M}\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c \qquad y - y_1 = m(x - x_1) \qquad m = \frac{y_2 - y_1}{x_2 - x_1} \qquad m = \tan\theta$$

$$(x-a)^2 + (y-b)^2 = r^2$$

In ΔABC: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$ $\qquad a^2 = b^2 + c^2 - 2bc.\cos A$ $\qquad$ *area* $\Delta ABC = \dfrac{1}{2}ab.\sin C$

$$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta \qquad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$$

$$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta \qquad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases} \qquad \sin 2\alpha = 2\sin\alpha.\cos\alpha$$

$$(x\,;\,y) \to (x\cos\theta - y\sin\theta\,;\; y\cos\theta + x\sin\theta)$$

$$\bar{x} = \frac{\sum fx}{n} \qquad \sigma^2 = \frac{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)} \qquad P(A\ of\ B) = P(A) + P(B) - P(A\ en\ B)$$

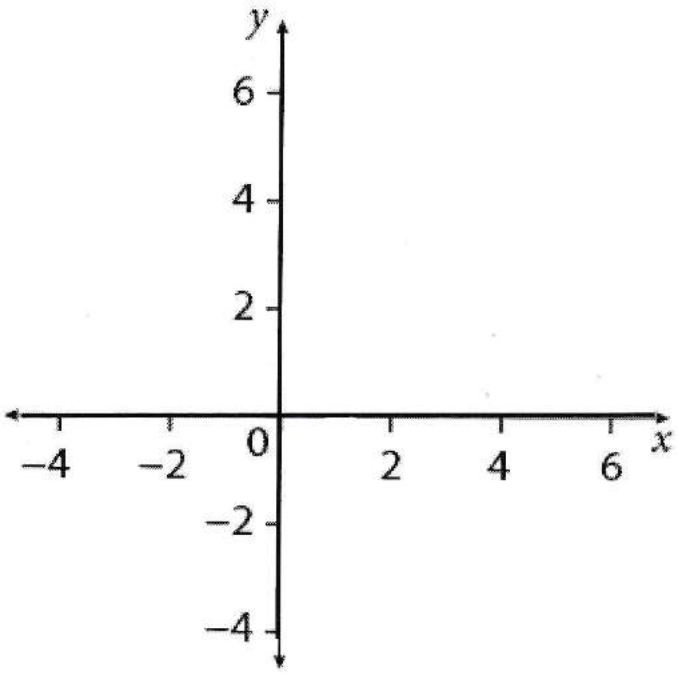
$$\hat{y} = a + bx \qquad b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$$

NAME OF LEARNER: _____

GRADE: _____

ANSWER SHEET

QUESTION 5.3



QUESTION 6.5