Basic Education

KwaZulu-Natal Department of Basic Education

REPUBLIC OF SOUTH AFRICA

MATHEMATICS P1

PREPARATORY EXAMINATION

SEPTEMBER 2016

NATIONAL SENIOR CERTIFICATE

GRADE 12

Marks: **150**

Time: **3 hours**

N.B: This question paper consists of 9 pages and 1 information sheet

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 12 questions.

2. Answer ALL the questions.

3. Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining your answers.

4. Answers only will not necessarily be awarded full marks.

5. An approved scientific calculator (non-programmable and non-graphical) may be used, unless stated otherwise.

6. If necessary, answers should be rounded off to TWO decimal places, unless stated otherwise.

7. Diagrams are NOT necessarily drawn to scale.

8. Number the answers correctly according to the numbering system used in this question paper. Write neatly and legibly.

QUESTION 1

1.1 Solve for x in each of the following:

1.1.1 $2x(3x-5)=0$ (2)

1.1.2 $x^2-3x=7$ (Give answer correct to TWO decimal places) (4)

1.1.3 $2x-5\sqrt{x}=3$ (6)

1.1.4 $2^x(3x+1)<0$ (3)

1.2 Calculate, **without using a calculator** $2^{100}-2^{99}$ (3)

1.3 Solve for x and y simultaneously:

$2x-y=3$ and
$x^2+5xy+y^2=15$ (6)

[24]

QUESTION 2

Given the quadratic sequence: 4; 4; 8; 16; . . .

2.1 Calculate the n^{th} term of the quadratic sequence. (4)

2.2 Between which two consecutive terms of the quadratic sequence, will the first difference be equal to 28088? (4)

[8]

QUESTION 3

3.1 Given the combined arithmetic and constant sequences:

6 ; 2 ; 10 ; 2 ; 14 ; 2 ; ...

3.1.1 Write down the next **TWO** terms in the sequence. (2)

3.1.2 Write down the sum of the first 50 terms of the constant sequence. (1)

3.1.3 Calculate the sum of the first 100 terms of the sequence. (4)

3.2 Prove that: $a+ar+ar^2+...(to\ n\ terms)=\dfrac{a(r^n-1)}{r-1};\quad r\neq 1$ (4)

[11]

QUESTION 4

4.1 Evaluate $\sum_{p=1}^{\infty}\left(\frac{2}{3}\right)^{p-1}$ (3)

4.2 In a series, $S_n = 3n^2 - n$, calculate the value of the fourth term in the series. (4)

[7]

QUESTION 5

5.1 If a car valued at R255 000 depreciates on a reducing balance method at an interest rate of 12,5 % p.a., calculate the book value of the car after 7 years. (3)

5.2 A loan of R10 000, taken on 1 February 2016, is to be repaid in regular fixed instalments of R450 on the first day of each month. Interest is charged on the loan at 9,5 % p.a. compounded monthly. The first instalment is paid on 1 August 2016.

Calculate:

5.2.1 the total amount payable on 1 July 2016. (2)

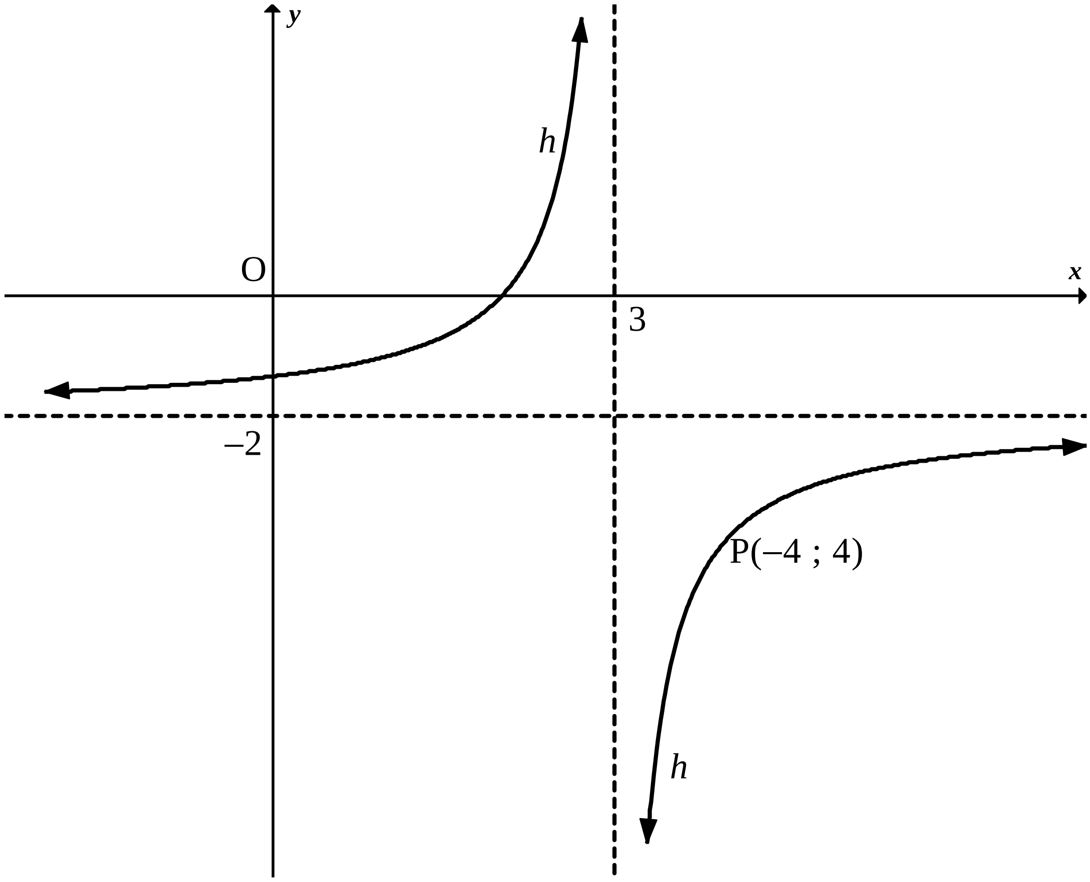
5.2.2 the number of payments that will be needed to settle the loan. (5)

5.2.3 the balance outstanding on the loan after the 25^{th} payment has been made. (4)

[14]

QUESTION 6

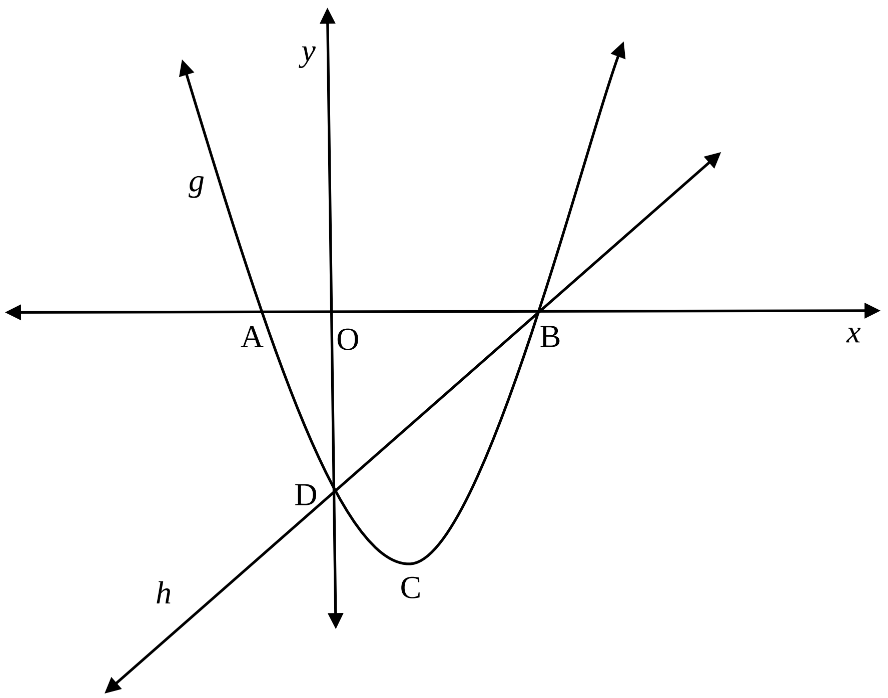
The diagram below shows the graph of $h(x)=\dfrac{a}{x+p}+q$. The lines $x = 3$ and $y = -2$ are asymptotes of h. $\text{P}(4;-4)$ is a point on h.



6.1	Write down the values of p and q.	(2)
6.2	Calculate the value of a.	(2)
6.3	Calculate the coordinates of the y – intercept of h.	(2)
6.4	If $g(x)=h(x+2)$, write down the equation of the vertical asymptote of g.	(2)
6.5	If the graph of h is symmetrical about the line $y=-x+c$, determine the value of c.	(2)
		[10]

QUESTION 7

The sketch below shows the graphs of $g(x) = x^2 - 3x - 10$ and $h(x) = ax + q$. The graphs intersect at B and D. The graph of g intersects the x – axis at A and B and has a turning point at C. The graph of h intersects the y – axis at D and the x – axis at B.



7.1 Write down the coordinates of D. (1)

7.2 Determine the coordinates of A and B. (4)

7.3 Write down the values of a and q. (2)

7.4 Calculate the coordinates of C, the turning point of g. (3)

7.5 Write down the turning point of t, if $t(x) = g(-x) + 3$. (2)

7.6 For which values of x will $g'(x).h'(x) \geq 0$? (2)

[14]

QUESTION 8

Given $p(x) = 3^x$.

8.1 Write down the equation of p^{-1}, the inverse of p, in the form $y = \ldots$ (2)

8.2 Sketch in your ANSWER BOOK the graphs of p and p^{-1} on the same system of axes. Show clearly all the intercepts with the axes and at least one other point on each graph. (4)

8.3 Determine the values of x for which $p^{-1}(x) \leq 3$ (4)

[10]

QUESTION 9

9.1 Given $f(x) = x^3$. Determine the derivative of f from first principles. (5)

9.2 Determine:

9.2.1 $f'(x)$ if $f(x) = \sqrt{x} - \dfrac{4}{x^2}$ (4)

9.2.2 $\dfrac{dy}{dx}$ if $\dfrac{1}{2}x^2 - 3 = \sqrt{y}$ (4)

[13]

QUESTION 10

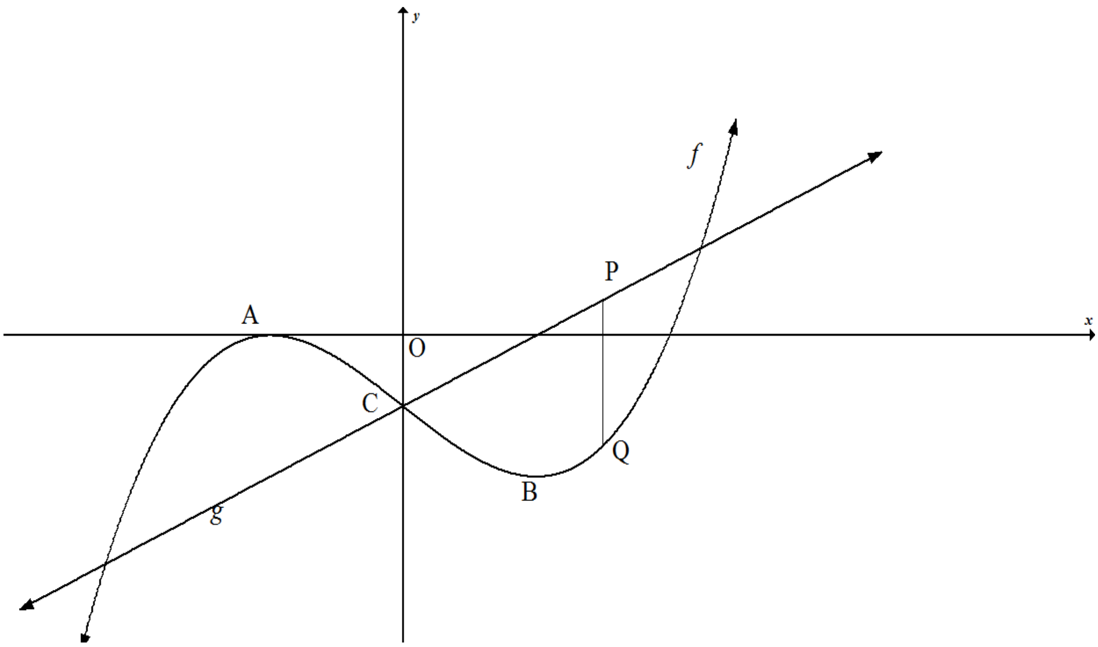
10.1 Given $f(x) = x^2 - 8$

10.1.1 Calculate $f(-3)$. (1)

10.1.2 Calculate $f'(-3)$. (1)

10.1.3 Determine the equation of the tangent to $f(x) = x^2 - 8$ at $x = -3$. (2)

10.2 The graph of a cubic function with equation $f(x) = x^3 - 3x - 2$ and $g(x) = 2x - 2$ is drawn. A and B are the turning points of f. P is a point on g and Q is a point on f such that PQ is perpendicular to the x – axis.



10.2.1 Calculate the coordinates of A and B. (4)

10.2.2 If PQ is perpendicular to the x – axis, calculate the maximum length of PQ, (4)

10.2.3 Determine the values of k for which $f(x) = k$ has only two real roots. (2)

10.2.4 Determine the values of x for which f is concave up. (3)

[17]

QUESTION 11

A car speeds along a 1 kilometre track in 25 seconds. Its distance (in metres) from the start after t seconds is given by
$s(t) = t^2 + 15t$

11.1 Write down an expression for the speed (the rate of change of distance with respect to time) of the car after t seconds. (1)

11.2 Determine the speed of the car when it crosses the finish line. (1)

11.3 Write down an expression for the acceleration (the rate of change of speed with respect to time) of the car after t seconds. (2)

11.4 Hence, or otherwise, calculate the acceleration of the car after 5 seconds. (1)

11.5 Calculate the speed of the car when it is 250 metres down the track. (4)

[9]

QUESTION 12

12.1 A study on eating chocolate and gender yielded the following results.

	Eating Chocolate	**Not Eating Chocolate**	**TOTAL**
Male	45	25	70
Female	35	45	80
TOTAL	80	70	150

12.1.1 How many people participated in this study? (1)

12.1.2 Calculate the following probabilities:

(a) P(male) (1)
(b) P(Eating Chocolate) (1)

12.1.3 Are the events being a male and eating chocolate independent? Justify your answer with relevant calculations. (3)

12.2 Four - digit codes (not beginning with 0), are to be constructed from the set of digits $\{1 ; 3 ; 4 ; 6 ; 7 ; 8 ; 0\}$.

12.2.1 How many four - digit codes can be constructed, if repetition of digits is allowed? (2)

12.2.2 How many four - digit codes can be constructed, if repetition of digits is not allowed? (2)

12.2.3 Calculate the probability of randomly constructing a four - digit code which is divisible by 5 if repetition of digits is allowed. (3)

[13]
[150]

INFORMATION SHEET: MATHEMATICS

INLIGTINGSBLAD: WISKUNDE

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $A = P(1-ni)$ $A = P(1-i)^n$ $A = P(1+i)^n$

$T_n = a + (n-1)d$ $S_n = \frac{n}{2}\left(2a + (n-1)d\right)$

$T_n = ar^{n-1}$ $S_n = \frac{a\left(r^n - 1\right)}{r-1}$; $r \neq 1$ $S_\infty = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x\left[(1+i)^n - 1\right]}{i}$ $P = \frac{x[1-(1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $M\left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $y - y_1 = m(x - x_1)$ $m = \frac{y_2 - y_1}{x_2 - x_1}$ $m = \tan\theta$

$$(x-a)^2 + (y-b)^2 = r^2$$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $a^2 = b^2 + c^2 - 2bc.\cos A$ $area\,\Delta ABC = \frac{1}{2}ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$ $\sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum x}{n}$ $\sigma^2 = \frac{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$\hat{y} = a + bx$ $b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$