education

DEPARTMENT: EDUCATION
MPUMALANGA PROVINCE

NATIONAL SENIOR CERTIFICATE EXAMINATION

MATHEMATICS P1

SEPTEMBER 2016

GRADE 12

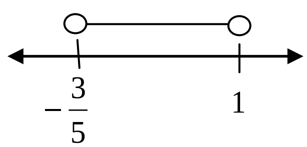
MEMORANDUM

MARKS: 150

TIME: 3 HOURS

This MEMORANDUM consists of 12 pages

QUESTION 1

1.1.1	$x^2 = 3x - 2$ $x^2 - 3x + 2 = 0$ $(x-2)(x-1) = 0$ $x = 2$ or $x = 1$	✓standard form ✓ factors ✓ answers (3)
1.1.2	$2x^2 + 5x - 8 = 0$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $x = \frac{-5 \pm \sqrt{5^2 - 4(2)(-8)}}{2(2)}$ $x = \frac{-5 \pm \sqrt{89}}{4}$ $x = 1{,}1$ or $x = -3{,}6$	✓substitution into correct formula ✓ $\sqrt{89}$ ✓ answers (3)
1.1.3	$5x^2 - 3 < 2x$ $5x^2 - 2x - 3 < 0$ $(5x+3)(x-1) < 0$  $-\frac{3}{5} < x < 1$	✓standard form ✓ factors ✓ method ✓ answers (4)
1.1.4	$3^{x+1} + 3^{x-1} + 3^x = 39$ $3^x(3^1 + 3^{-1} + 1) = 39$ $3^x\left(\frac{13}{3}\right) = 39$ $3^x = 9$ $3^x = 3^2$ $x = 2$	✓common factor ✓ Simplification ✓ exponential law ✓ answers (4)

1.2.1	$\left(3^{-4}\right)^{-x} = \left(3^{2}\right)^{y+3}$ $4x = 2y + 6$ $2x = y + 3$ $y = 2x - 3$	✓✓ Exponential Law ✓ Subject of the formula (3)
1.2.2	$x^2 + y^2 - 3x = -1$ $x^2 + (2x-3)^2 - 3x = -1$ $x^2 + 4x^2 - 12x + 9 - 3x + 1 = 0$ $5x^2 - 15x + 10 = 0$ $x^2 - 3x + 2 = 0$ $(x-2)(x-1) = 0$ $x = 2$ or $x = 1$ $y = 2(2) - 3$ $y = 2(1)$-1 $= -1$ $= 1$	✓ substitution ✓standard form ✓factors ✓ x-values ✓ y-values (5)
1.3	$x^2 + x(2p + 4) + 9p = 0$ $\Delta = b^2 - 4ac$ $= (2p+4)^2 - 4(1)(9p)$ $= 4p^2 + 16p + 16 - 36p$ $= 4p^2 - 20p + 16$ $\Delta = 0$ $4p^2 - 20p + 16 = 0$ $p^2 - 5p + 4 = 0$ $(p-1)(p-4) = 0$ p = 1 or $p = 4$	✓ substitution in Δ ✓ = 0 ✓ factors ✓ answer (4)

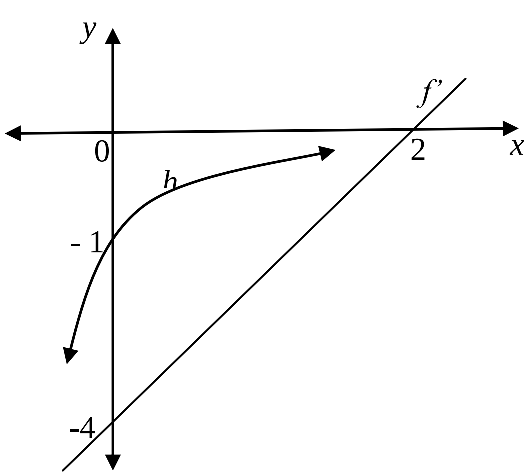
QUESTION 2

2.1	$1 ; -5 ; -13 ; -23$......	
2.1.1	$1 ; -5 ; -13; -23$ $-6 \quad -8 \quad -10$ $-2 \quad -2$ $2a = -2$ $3a + b = -6$ $a + b + c = 1$ $a = -1$ $3(-1) + b = 5$ $-1 - 3 + c = 1$ $b = -3$ $c = 5$ $T_n = -n^2 - 3n + 5$	✓ Value of a ✓ Value of b ✓ Value of c ✓ General term (4)

2.1.2	$-n^2 - 3n + 5 = -643$ $n^2 + 3n - 648 = 0$ $(n + 27)(n - 24) = 0$ $n = -27$ or $n = 24$ $\therefore T_{24} = -643$	✓ equating ✓ factors ✓ answer (3)
2.2.1	$3k-(2k+1)=5k-5-3k$ $k-1 = 2k-5$ $k = 4$	✓ Difference ✓ answer (2)
2.2.2	9,12,15 $T_n = 3n+6$ $\sum_{n=1}^{20}(3n+6)$	✓ Sequence ✓ General term ✓ Sigma Notation (3)
2.3.1	$r = \frac{4x^4}{8x^3}$ $r = \frac{x}{2}$ $-1 < r < 1$ $-1 < \frac{x}{2} < 1$ $-2 < x < 2$	✓ value of r ✓ Condition for converging series ✓ answer (3)
2.3.2	$S_\infty = \frac{a}{1-r}$ $S_\infty = \frac{8x^2}{1-\frac{x}{2}}$ $\frac{8}{3}\left(1-\frac{x}{2}\right) = 8x^2$ $\frac{8}{3}-\frac{8x}{6} = 8x^2$ $16 - 8x = 48x^2$ $48x^2 + 8x - 16 = 0$ $6x^2 + x - 2 = 0$ $(3x+2)(2x-1) = 0$ $x = -\frac{2}{3}$ or $x = \frac{1}{2}$	✓ Substitution into the correct formula ✓ Standard form ✓ Factors ✓ Answers (4)

2.4.1	$a + 2d = ar$ $\qquad$ $a + 12d = ar^2$ $1 + 2d = 1r$ $\qquad$ $1 + 12d = 1r^2$ $d = \frac{r-1}{2}$ $\longrightarrow$ $1 + 12\left(\frac{r-1}{2}\right) = r^2$ $1 + 6r - 6 = r^2$ $r^2 - 6r + 5 = 0$ $(r - 1)(r - 5) = 0$ $r = 1$ or $r = 5$ 1; 5; 25 OR $r = 1 + 2d$ $\longrightarrow$ $1 + 12d = r^2$ $1 + 12d = (1 + 2d)^2$ $= 1 + 4d + 4d^2$ $4d^2 - 8d = 0$ $4d(d - 2) = 0$ $d = 0$ or $d = 2$ $r = 1 + 2(0)$ $\quad$ $r = 1 + 2(2)$ $= 1$ $\quad$ $= 5$ 1; 5; 25	✓ $a + 2d = ar$ ✓ $a + 12d = ar^2$ ✓ values of d ✓ standard form ✓ factors ✓ values of r ✓ sequence (6) ✓ $a + 2d = ar$ ✓ $a + 12d = ar^2$ ✓ values of r ✓ standard form ✓ factors ✓ values of d ✓ sequence
2.4.2	$S_n = \frac{a(r^n - 1)}{r - 1}$ $S_7 = \frac{1(5^7 - 1)}{5 - 1}$ $= 19531$	✓ substitute in correct formula ✓ answer (2)
QUESTION 3		
3.1	$y = \frac{6}{x-2} + 1$ $0 = \frac{6}{x-2} + 1$ $-1(x - 2) = 6$ $x = -4$ A $(-4 : 0)$ $y = \frac{6}{0-2} + 1$ $= -2$ B $(0 ; -2)$ E$(4 ; 0)$ Alternative for E	✓ substitute x = 0 and y = 0 into correct formula ✓ $x = -4$ ✓ $y = -2$ ✓ Symmetry E(4 ; 0) (4)

	For E (x – intercept g) $-2x+8 = 0$ $-2x = -8$ $x = 4$ $\therefore E(4,0)$	✓ Answer
3.2	$f(x) = x^2 + c$ $20 = (-6)^2 + c$ $c = -16$	✓ substitute in $f(x)$ ✓value of c (2)
3.3	$g(x) \geq f(x)$ $-6 \leq x \leq 4$	✓inequality ✓answer (2)
3.4	$y = -mx + c$ $1 = -1\ (2) + c$ $c = 3$ $\therefore y = -x + 3$	✓ $m = -1$ ✓ $c = 3$ (2)
3.5	$BE^2 = (-2)^2 + (4)^2$ $= 20$ $BE = 2\sqrt{5}$	✓ Pythagoras ✓ $\sqrt{20}$ (2)
3.6	$y \in R\ ;\ y \neq 3$ OR $y \in (-\infty\ ; 3) \cup (3\ ; \infty)$	✓ 3 ✓answer (2)
3.7	$f'(x) = 2x$ $2x = -2$ $x = -1$ $f(-1) = -15$ $y - y_1 = m(x - x_1)$ $y - (-15) = -2(x - (-1))$ $y = -2x - 17$	✓ Derivative ✓ Value of x ✓ $f(-1) = -15$ ✓ $m = -2$ ✓ answer (5)

QUESTION 4		
4.1	$g^{-1}(x) = \left(\frac{1}{2}\right)^x$ or $g(x) = 2^{-x}$	✓✓ answer (2)
4.2	$f(x) = 2(x^2 - 2x) - 6$ $f(x) = 2(x^2 - 2x + 1 - 1) - 6$ $f(x) = 2(x-1)^2 - 2 - 6$ $f(x) = 2(x-1)^2 - 8$ OR $x = -\frac{b}{2a}$ Subst. $x = 1$ in $y = 2x^2 - 4x - 6$ $x = -\frac{4}{2(2)}$ $y = 2(1)^2 - 4(1) - 6$ $x = 1$ $y = -8$ $\therefore f(x) = 2(x-1)^2 - 8$	✓ common factor ✓ Adding and Subtracting 1 ✓ Simplification ✓ answer (4) ✓ Formula for x value at the turning point ✓ Value of x ✓ Value of y ✓ Answer
4.3	$p(x) = 2(x - 1 + 2)^2 - 8$ $p(x) = 2(x + 1)^2 - 8$	✓ Substitution ✓ Answer (2)
4.4	$0 < x \leq 1$	✓✓ answer (2)
4.5	$g^{-1}(x) = 2^{-x}$ $h(x) = -2^{-x}$ $f'(x) = 2x - 4$ $-2^{-x} - (2x - 4) = 3$ Graph Used  $\therefore x = 0$	✓ Shape of the exponential Graph ✓ Graph of the derivative of *f(x)* ✓ Answer (3)

QUESTION 5		
5.1	$2P = P \quad P\left(1+\frac{0,0875}{12}\right)^n$ $2 = \left(1+\frac{0,0875}{12}\right)^n$ $\log 2 = n \log\left(1+\frac{0,0875}{12}\right)^n$ $n = \frac{\log 2}{\log\left(1+\frac{0,0875}{12}\right)}$ n = 95,4063387 n = 95,41 months	✓ substitute into correct formula ✓ simplify ✓ correct use of logs ✓ answer (4)
5.2.1	$A = P(1 + i)^n$ $= 2\ 400\ 000\left(1+\frac{0,07}{2}\right)^{10}$ = R3 385 437,03 Need to pay R2 585 437,03	✓ substitution into the correct formula ✓ answer ✓ answer (3)
5.2.2	$F = \frac{x\,[(1+i)^n - 1]}{i}$ $2\,585\,437{,}03 = \frac{x\left[\left(1+\frac{0,16}{12}\right)^{60} - 1\right]}{\frac{0.16}{12}}$ x = R28 400,31	✓ $i = \frac{0,16}{12}$ ✓n=60 ✓ substitution in correct Formula ✓ answer (4)
5.2.3	$P = \frac{x\left[1-(1+i)^{-n}\right]}{i}$ $= \frac{28\,400{,}31\left[1-\left(1+\frac{0,16}{12}\right)^{-24}\right]}{\frac{0,16}{12}}$ = R 580 034,84	✓correct formula ✓ $i = \frac{0,16}{4}$ ✓ n = −24 ✓ answer (4)

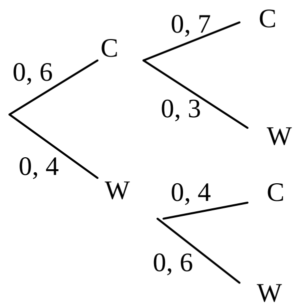
QUESTION 6

6.1	$f(x) = x^2 - 3x$ $f(x+h) = (x+h)^2 - 3(x+h)$ $= x^2 + 2xh + h^2 - 3x - 3h$ $f'(x) = \lim_{h\to 0}\frac{f(x+h)^2 - f(x)}{h}$ $= \lim_{h\to 0}\frac{x^2+2xh+h^2-3x-3h-(x^2-3x)}{h}$ $= \lim_{h\to 0}\frac{x^2+2xh+h^2-3x-3h-x^2+3x}{h}$ $= \lim_{h\to 0}\frac{h(2x+h-3)}{h}$ $= 2x-3$	✓ find $f(x+h)$ ✓ correct substitution into formula ✓ simplification ✓ factorise ✓ answer (5)
6.2.1	$y = (x+1)(2-3x)$ $y = 2x - 3x^2 + 2 - 3x$ $y = -3x^2 - x + 2$ $\frac{dy}{dx} = -6x - 1$	✓ Standard form ✓✓ Derivative (3)
6.2.2	$y = -2\sqrt{x} + x - \frac{1}{\sqrt{x}}$ $= -2x^{\frac{1}{2}} + x - x^{\frac{-1}{2}}$ $\frac{dy}{dx} = -x^{\frac{-1}{2}} + 1 + \frac{1}{2}x^{\frac{-3}{2}}$	✓ simplify ✓✓ ✓ Derivative (4)

QUESTION 7		
7.1	A (-2 ; 0) D (3 ; 0)	✓✓ answer (2)
7.2	$g(x) = x^3 + x^2 - 8x - 12$ $g'(x) = 3x^2 + 2x - 8$ $3x^2 + 2x - 8 = 0$ $(3x - 4)(x + 2) = 0$ $x = \frac{4}{3}$ or $x = -2$ $g(x) = x^3 + x^2 - 8x - 12$ $= \left(\frac{4}{3}\right)^3 + \left(\frac{4}{3}\right)^2 - 8\left(\frac{4}{3}\right) - 12$ $= \frac{-500}{27}$ $C\left(\frac{4}{3} ; \frac{-500}{27}\right)$	✓ $g'(x) = 0$ ✓ factors ✓ substitute correct x- value ✓ y- value (4)
7.3	$f(x) = 2x - 6$	✓ answer (1)
7.4	$g''(x) = 6x + 2$ $6x + 2 = 0$ $6x = -2$ $x = -\frac{1}{3}$	✓ Inflection ✓answer (2)
7.5	t > 0 or t $< \frac{-500}{27}$	✓ answer ✓ answer (2)
7.6	$f(x) \cdot g'(x) < 0$ $\frac{4}{3} < x < 3$ $x < -2$	✓answer ✓ answer (2)

QUESTION 8

8.1	$B = 60^o$ In Δ BED : $\frac{DE}{BE} = \tan 60^o$ $DE = BE \tan 60^o$	✓$B = 60^o$ ✓ $\frac{DE}{BE} = \tan 60^o$ ✓ $DE = BE \tan 60^o$ (3)
8.2	$EF = 3 - 2x$ Area DGEF = DE x EF $= x \tan 60^o (3 - 2x)$ $= x.\sqrt{3}.(3 - 2x)$ $= \sqrt{3}.x(3 - 2x)$	✓ $EF = 3 - 2x$ ✓ $x \tan 60^o (3 - 2x)$ (2)
8.3	$A(x) = \sqrt{3}.x(3 - 2x)$ $= 3.\sqrt{3}.x - 2.\sqrt{3}.x^2$ $A'(x) = 0$ $3.\sqrt{3} - 4.\sqrt{3}x = 0$ $x = \frac{3}{4}$ max area: $3.\sqrt{3}.\frac{3}{4} - 2.\sqrt{3}.(\frac{3}{4})^2$ $= \frac{9\sqrt{3}}{8}$ square units $= 1{,}95$ square units	✓ $A(x)=3.\sqrt{3}.x - 2.\sqrt{3}.x^2$ ✓ $A'(x) = 0$ ✓ $x = ¾$ ✓ $3.\sqrt{3}.¾ - 2.\sqrt{3}.(¾)^2$ ✓ $\frac{9\sqrt{3}}{8}$ (5)

QUESTION 9		
9.1.1	P(A or B) = P(A) + P(B) – P(A and B) $= \frac{3}{8} + \frac{1}{4} - 0$ $= \frac{5}{8}$ Accept 0,625 or 0,63	✓formula ✓ P(A and B) = 0 ✓answer (3)
9.1.2	P(A or B) = P(A) + P(B) – P(A and B) = P(A) + P(B) - P(A) x P(B) $= \frac{3}{8} + \frac{1}{4} - \frac{3}{8} \times \frac{1}{4}$ $= \frac{17}{32}$ Accept 0,53	✓ P(A and B) = P(A) x P(B) ✓substitute ✓answer (3)
9.2	 P(CC + WC) = 0,6 . 0,7 + 0,4 . 0,4 = 0,58	✓ tree diagram ✓ values on tree diagram ✓ calculations ✓answer (4)
9.3.1	6! = 720	✓6! ✓720 (2)
9.3.2	2! x 5! = 240 P(sitting together) = $\frac{240}{720} = \frac{1}{3}$ OR P(sitting together) = $\frac{2!4!5}{6!} = \frac{1}{3}$	✓2! ✓ 5! ✓ $\frac{1}{3}$ (3)

TOTAL/TOTAAL: 150