education

DEPARTMENT: EDUCATION
MPUMALANGA PROVINCE

NATIONAL SENIOR CERTIFICATE EXAMINATION

MATHEMATICS P1

SEPTEMBER 2016

GRADE 12

MARKS: 150

TIME: 3 HOURS

This question paper consists of 9 pages and 1 information sheet,

INSTRUCTIONS TO CANDIDATES

Read the following instructions carefully before answering the questions.

1. This question paper consists of **9** questions.
   Answer ALL the questions.

2. Show clearly ALL the calculations, diagrams, graphs, etcetera, which you have used in determining the answers.

3. An approved scientific calculator (non-programmable and non-graphical) may be used, unless stated otherwise.

4. If necessary, answers should be rounded off to TWO decimal places, unless stated otherwise.

5. Number the answers correctly according to the numbering system used in this question paper.

6. Diagrams and graphs are not necessarily drawn to scale.

7. Write neatly and legibly.

8. A formula sheet is attached.

QUESTION 1

1.1 Solve for x correct to ONE decimal places if necessary:

1.1.1 $x^2 = 3x - 2$ (3)

1.1.2 $2x^2 + 5x - 8 = 0$ (3)

1.1.3 $5x^2 - 3 < 2x$ (4)

1.1.4 $3^{x+1} + 3^{x-1} + 3^x = 39$ (4)

1.2 Consider $\left(\frac{1}{81}\right)^{-x} = 9^{y+3}$ and $x^2 + y^2 - 3x = -1$

1.2.1 Show that $y = 2x - 3$ (3)

1.2.2 Solve for x and y simultaneously (5)

1.3 For which values of p will the equation $x^2 + x(2p + 4) + 9p = 0$ has equal roots? (4)

[26]

QUESTION 2

2.1 The first four terms of a quadratic sequence are: $1\ ;\ -5;\ -13;\ -23\ \ldots.$

2.1.1 Calculate the general term of the sequence. (4)

2.1.2 Which term has a value of -643? (3)

2.2 $2k+1\ ;\ 3k\ ;\ 5k-5$ are the first three terms of an arithmetic sequence

2.2.1 Calculate the value of *k.* (2)

2.2.2 Write down this sequence in sigma notation for the first 20 terms. (3)

2.3 Given the geometric series: $8x^2+4x^3+2x^4+\ldots$

2.3.1 For what value(s) of *x* will the series converge? (3)

2.3.2 Calculate the values of x if $S_\infty = \frac{8}{3}$. (4)

2.4 The first, third and thirteenth terms of arithmetic sequence are the first 3terms of a geometric sequence. If the first term of both sequences is 1, determine:

2.4.1 the first three terms of the geometric sequence if r > 1 (6)

2.4.2 the sum of 7 terms of the geometric sequence if the sequence is $1\ ;\ 5\ ;\ 25$. (2)

[27]

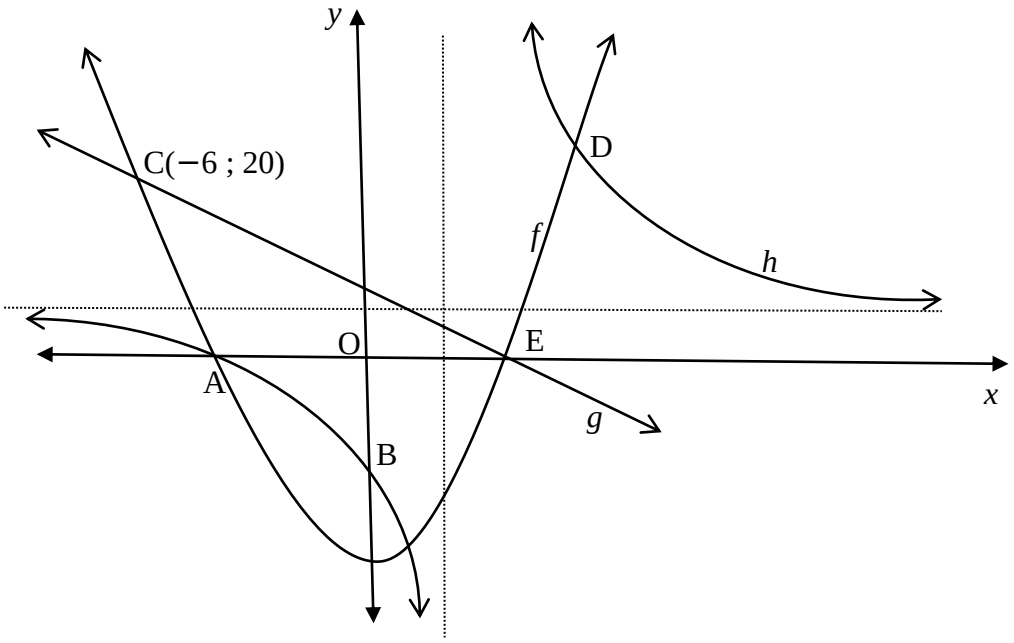
QUESTION 3

Sketched below are the graphs of: $g(x) = -2x + 8$

$$f(x) = x^2 + c$$

and $$h(x) = \frac{6}{x-2} + 1$$

A and B are the x- and y - intercepts of h respectively, C $(-6 ; 20)$ and E are the points of intersection of f and g .



3.1 Calculate the coordinates of A, B and E. (4)

3.2 Show that the value of $c = -16$ (2)

3.3 Write down the values of x for which $g(x) - f(x) \geq 0$ (2)

3.4 Determine the equation of the symmetry axis of h if the gradient is negative. (2)

3.5 Show that the length of BE = $2\sqrt{5}$ (2)

3.6 Write down the range of s, if $s(x) = h(x) + 2$. (2)

3.7 If t is a tangent to f and parallel to g, determine the equation of the tangent, t in the form $y = mx + c$. (5)

[19]

QUESTION 4

Given: $g(x) = \log_{\frac{1}{2}} x$ and $f(x) = 2x^2 - 4x - 6$

4.1 Determine the equation of g^{-1}, the inverse of g, in the form $g^{-1}(x) = \ldots$ (2)

4.2 Write f in the form $f(x) = a(x+p)^2 + q$ (4)

4.3 The graph of f is shifted 2 unit to the left to form p. Write down the equation of the new graph in the form $p(x) = \ldots\ldots\ldots$ (2)

4.4 Write down the values of x for which $\log_2 x \leq 0$ (2)

4.5 If h is the reflection of g^{-1} about the x-axis, for which value of x will $h(x) - f'(x) = 3$ (3)

[13]

QUESTION 5

5.1 Read the advertisement carefully. The question below refers to the advertisement.

BANK WITH THE
REFORMED BANK
8,75% interest compounded
Monthly

How many months will it take any investment to double if the money is invested in the account as advertised above ? (4)

5.2 John, the owner of a mine, has just bought a new excavator for R2 400 000. He has decided to replace the excavator in 5 years' time, when its trade-in value will be R800 000. The replacement cost of the excavator is expected to increase by 7% per annum, compounded semi-annually.

5.2.1 John wants to pay cash for the new excavator after trading in his current excavator in 5 years' time.
How much will he need to pay? (3)

5.2.2 One month after purchasing the present excavator, John deposited *x* rand into an account that pays interest at a rate of 16% per annum, compounded monthly. He continues to deposit the same amount at the end of each month for a total of 60 months. After 5 years he has the exact amount that is needed to purchase the new excavator after trading in his current one.
Calculate the value of *x*. (4)

5.2.3 John's brother, Micheal, wants to invest in the mine 3 years after John has bought the mine. They agree that Micheal pays the rest of the monthly instalments in a lump sum. What is the amount that Micheal has to pay, in order to still buy the new excavator in 5 years' time? (4)

[15]

QUESTION 6

6.1 Given: $f(x) = x^2 - 3x$

Determine $f'(x)$ from FIRST PRINCIPLES. (5)

6.2 Determine:

6.2.1 $\frac{dy}{dx}$ if $y = (x+1)(2-3x)$ (3)

6.2.2 $\frac{dy}{dx}$ if $y = -2\sqrt{x} + x - \frac{1}{\sqrt{x}}$ (4)

[12]

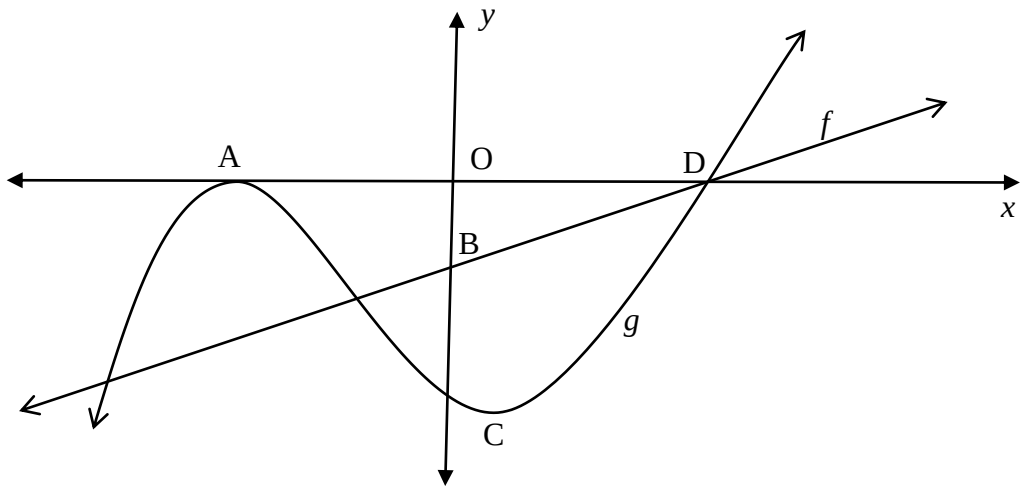
QUESTION 7

Sketched below is the graph of $g(x) = x^3 + x^2 - 8x - 12$.

$$= (x+2)^2(x-3)$$

and $f(x) = 2x + c$

A and D are the *x*-intercepts of *g*. OB is 6 units . A and C are the turning points of *g*.



7.1 Write down the coordinates of A and D. (2)

7.2 Determine the coordinates of C. (4)

7.3 Write down the equation of f. (1)

7.4 Calculate the *x*-value of the point of inflection of *g*. (2)

7.5 Use the graph to determine the value(s) of *t* for which $g(x) = t$ will have only one real root. (2)

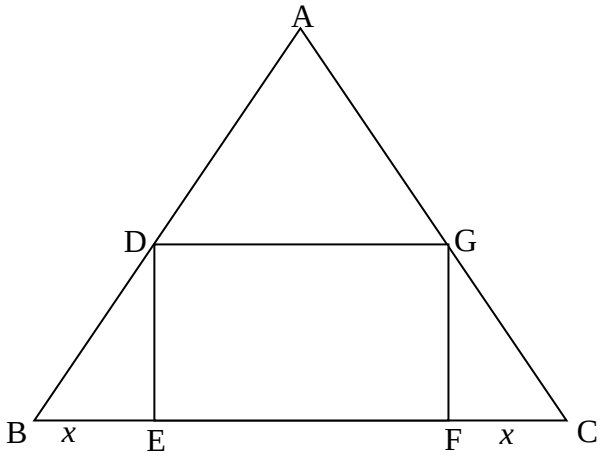
7.6 For which value(s) of x is $f(x) \cdot g'(x) < 0$? (2)

[13]

QUESTION 8

In the diagram ABC is an equilateral triangle whose sides have a length of 3 units. DEFG is a rectangle inside the triangle as indicated, D lies on AB, G lies on AC and EF lies on BC.

BE = FC = x



8.1 Show that DE = $x \tan 60^\circ$ (3)

8.2 Prove that the area (A) of the rectangle is given by A = $\sqrt{3}\,x(3 - 2x)$ (2)

8.3 Calculate the maximum area that the rectangle can be. (5)

[10]

QUESTION 9

9.1 If P(A) = $\frac{3}{8}$ and P(B) = $\frac{1}{4}$, calculate P(A or B) if :

9.1.1 A and B are mutually exclusive events. (3)

9.1.2 A and B are independent events. (3)

9.2 The probability of getting the first answer in a quiz correct, is 0,6. If the first answer is correct, the probability of getting the second answer correct rises to 0,7. However, if the first answer is incorrect, the probability of getting the second answer correct, is only 0,4. Determine the probability of getting the second answer correct. (4)

9.3 Three married couples Mr and Mrs Small, Mr and Mrs Beaver and Mr and Mrs Khoza are to be seated on a bench.

9.3.1 How many different ways can the people be arranged? (2)

9.3.2 Calculate the probability that Mr and Mrs Khoza are seated next to each other on the bench. (3)

[15]

TOTAL: 150

FORMULA SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1 + in) \qquad A = P(1 - in) \qquad A = P(1 - i)^n \qquad A = P(1 \pm i)^n$$

$$T_n = a + (n-1)d \qquad S_n = \frac{n}{2}[2a + (n-1)d] \qquad T_n = ar^{n-1}$$

$$S_n = \frac{a(r^n - 1)}{r - 1};\ r \neq 1 \qquad S_\infty = \frac{a}{1 - r};\ -1 < r < 1 \qquad F = \frac{x[(1+i)^n - 1]}{i}$$

$$P = \frac{x[1 - (1+i)^{-n}]}{i} \qquad f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c \qquad y - y_1 = m(x - x_1) \qquad m = \frac{y_2 - y_1}{x_2 - x_1} \qquad m = \tan\theta$$

$$(x-a)^2 + (y-b)^2 = r^2 \qquad \text{In } \Delta ABC: \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \qquad a^2 = b^2 + c^2 - 2bc.\cos A$$

$$\text{Area of } \Delta ABC = \frac{1}{2}ab.\sin C$$

$$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha\sin\beta \qquad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha\sin\beta$$

$$\cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta \qquad \cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$$

$$\cos 2A = \begin{cases} \cos^2 A - \sin^2 A \\ 1 - 2\sin^2 A \\ 2\cos^2 A - 1 \end{cases} \qquad \sin 2A = 2\sin A.\cos A$$

$$P(A) = \frac{n(A)}{n(S)} \qquad P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\bar{x} = \frac{\sum x}{n} \qquad y = a + bx$$

$$b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})} \qquad \sigma^2 = \frac{\sum_{t=1}^{n} (x_1 - \bar{x})^2}{n}$$