YOUR SCHOOL NAME

NATIONAL
SENIOR CERTIFICATE

GRADE 12

MATH.1

MATHEMATICS P1

SEPTEMBER 2016

MARKS: 150

TIME: 3 hours

This question paper consists of 8 pages and an information sheet

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1.   This question paper consists of 11 questions.

2.   Answer ALL the questions.

3.   Number the answers correctly according to the numbering system used in this question paper.

4.   Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining your answers.

5.   Answers only will NOT necessarily be awarded full marks.

6.   You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.

7.   If necessary, round off answers to TWO decimal places, unless stated otherwise.

8.   Diagrams are NOT necessarily drawn to scale.

9.   An information sheet with formulae is included at the end of the question paper.

10.   Write neatly and legibly.

QUESTION 1

1.1   Solve for x.

        1.1.1 $x(5x + 2) = 0$ (2)

        1.1.2 $x(2x - 3) = 4$ (correct to TWO decimal places)   (4)

        1.1.3 $x^2 - x - 6 \geq 6$ (4)

1.2   Solve for x and y simultaneously if:

        $x - y = 3$ and $x^2 + xy - 2y^2 = 0$ (5)

1.3   The solution of a quadratic equation is given by:

        $$x = \frac{-2 \pm \sqrt{13 - 2k}}{3}$$

        Determine the largest integral value of k for which these x-values will be rational.   (3)

1.4   Determine the value(s) of a for which the graphs of $f(x) = x^2 - 2x - 3$ and
        $g(x) = 2x + a$ will not intersect each other.   (5)
                                                                                                         [23]

QUESTION 2

2.1   Given:

        $$\frac{3x - 1}{4} ; \frac{2x - 1}{3} ; \frac{7x - 5}{12}$$

        2.1.1   If $x = 5$, determine the values of the first three terms.   (1)

        2.1.2   What type of sequence is this? Give a reason for your answer.   (2)

        2.1.3   Which term will be equal to $-44{,}5$?   (3)

2.2   Given the series:

        $18 + 6 + 2 + \cdots$

        2.2.1   What is the value of the first negative term, if any? Explain your answer.   (2)

        2.2.2   Determine the tenth term, T_{10}.   (2)

        2.2.3   Determine $S_\infty - S_{10}$ (5)
                                                                                                         [15]

QUESTION 3

3.1   Determine the value of:

$$\sum_{k=2}^{33}(1-2k)$$

(3)

3.2 $6\,;5+x\,;-6$ and $6x$ form the first 4 terms of a quadratic sequence.

        3.2.1   Show that $x=-3$.   (4)

        3.2.2   Determine an expression for the general term of the sequence.   (4)

[11]

QUESTION 4

The diagram shows the graphs of $f(x)=-2x^2-4x+6$ and $g(x)=mx+c$. A, B and C are the intercepts of f with the axes. T is the turning point of the graph of f. The graph of g is a straight line parallel to AC, and is a tangent to the graph of f at D.
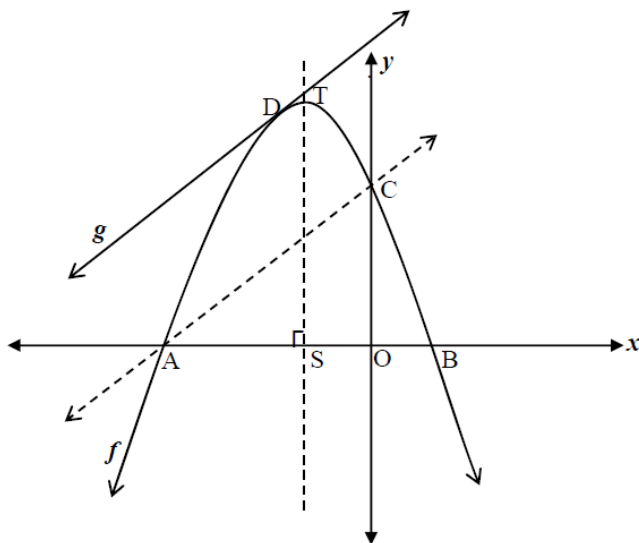
4.1   Determine the lengths of OC and AB.   (5)

4.2   Determine the equation of the axis of symmetry of the graph of f.   (2)

4.3   Show that the length of ST = 8 units.   (2)
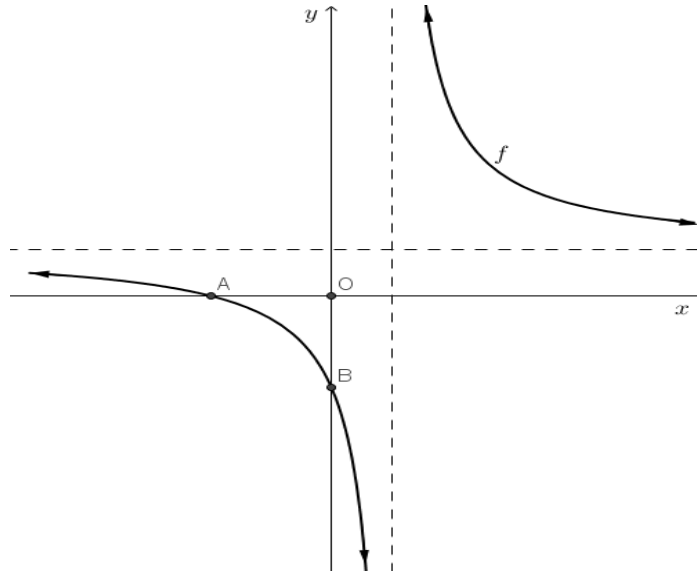
4.4   Calculate the gradient of AC.   (2)

4.5   Hence, or otherwise, calculate the coordinates of D.   (5)

4.6   For which value(s) of a will $f(a+t)=f(a-t)$ for all values of t ?   (2)

[18]

QUESTION 5

The sketch of $f(x) = \frac{2+x}{x-1}$ is drawn below.

5.1 Write down the equation of the vertical asymtote of f. (1)

5.2 Determine the coordinates of A, the x-intercept of the hyperbola. (2)

5.3 Calculate the area of $\triangle$AOB. (3)

5.4 Show that $f(x)$ can be rewritten as $f(x) = \frac{3}{x-1} + 1$ (2)

5.5 The graph of f is shifted such that point A lies on the origin. What are the coordinates of the point of intersection of the asymptotes of the new graph? (2)

[10]

QUESTION 6

6.1 Given: $f(x) = 2.2^x - 1$

    6.1.1 Write down the range of f. (2)

    6.1.2 $g(x) = f(x-1) + 1$. Write down the equation of g^{-1}, the inverse of g in the form $y = ...$ (2)

6.2 Given: $h(x) = -\sqrt{\frac{x}{3}}$; $x \geq 0$

    6.2.1 If $k(x)$ is the inverse of h, give the equation of $k(x)$ (2)

    6.2.2 Give the coordinates of the point of intersection of $h(x)$ and $k(x)$ (2)

[8]

QUESTION 7

7.1   A car, bought for R128 000, depreciates annually at a compound rate. After 6 years it is worth R45 500. At what rate did the value depreciate?   (4)

7.2   Keith sold his house for R250 000 and invested the money at 9,5% p.a., compounded quarterly.
Twelve years later he used the proceeds of the investment to buy another house for R2 920 000 and obtained a mortgage bond for the remaining amount. The bond was granted for 20 years at 10,3% interest p.a. compounded monthly.

         7.2.1   Calculate the value of Keith's original investment after 12 years.   (3)

         7.2.2   Determine the value of the bond that Keith obtained.   (1)

         7.2.3   Calculate the monthly payment that he has to make to pay off the bond.   (4)

         7.2.4   He paid off the bond in 20 years. How much interest did he pay on the bond?   (2)

                 [14]

QUESTION 8

8.1   Determine $f'(x)$ using first principles if $f(x) = \frac{2}{x}$.   (5)

8.2   Determine the following:

         8.2.1 $h'(x)$ if $h(x) = 2x^3 - \dfrac{4}{x} + 3\sqrt{x}$ (4)

         8.2.2 $D_x\left(\dfrac{8x^3 - 27}{2x - 3}\right)$ (3)

8.3   Explain, by doing the necessary calculations, why the tangent to the curve of $f(x) = -x^3 - 2x$ will never have a positive gradient.   (3)

                 [15]

QUESTION 9

The diagram below shows the graph of : $f(x) = x^3 + x^2 - x - 1$
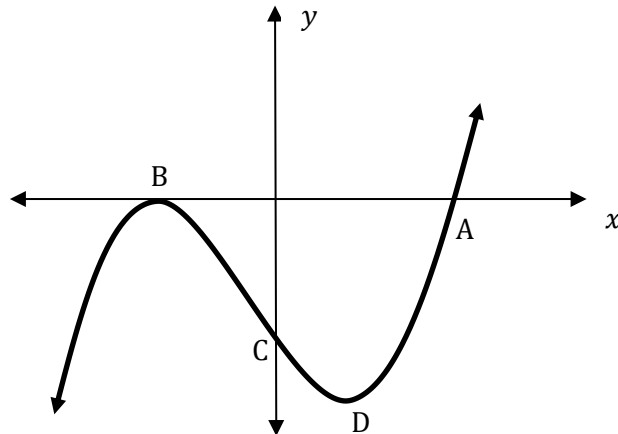
9.1   Calculate the distance between A and B, the x-intercepts.   (5)

9.2   Calculate the coordinates of D, a turning point of f.   (3)

9.3   Show that the concavity of f changes at $x = -\frac{1}{3}$ (3)

9.4   For which values of x is:

         9.4.1 $f(x) > 0$ (2)

         9.4.2 $f(x).f'(x) < 0$ (2)

                                                        [15]

QUESTION 10

A water tank with an inlet and an outlet is used to water a garden. The equation
$D = 3 + \frac{1}{2}t^2 - \frac{1}{4}t^3$ gives the depth of water in metres where t is the time in hours that has elapsed since 09:00.

10.1   What is the depth of the water at 11:00 (2 hours later)?   (1)

10.2   At what rate does the depth of the water change at 12:00?   (2)

10.3   At what time will the inflow of the water be the same as the outflow of water?   (4)

                                                        [7]

QUESTION 11

11.1   A group of 540 people with green or blue eyes were randomly selected in order to determine whether or not green or blue eyes are dependent on gender. The results are tabulated below:

	Male	**Female**	**Total**
Green eyes	183	147	**330**
Blue eyes	117	93	**210**
Total	**300**	**240**	**540**

        11.1.1   If a person is selected at random, determine the probability that it will be a female with green eyes. (2)

        11.1.2   After analysing the results, a grade 11 learner concludes that the probability of having green eyes is independent of gender. Is he correct? Substantiate your answer with relevant calculations. Give all answers correct to 2 decimal places. (5)

11.2   The letters in the word CURRICULUM are rearranged to form other words. Assume that all words have meaning.

        11.2.1   How many different letter arrangements (words) are possible? (3)

        11.2.2   What is the probability that a word will start and end with the letter U? (4)

                                                              [14]

                                                          TOTAL: 150

INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1 + ni) \qquad A = P(1 - ni) \qquad A = P(1 - i)^n \qquad\qquad A = P(1 + i)^n$$

$$T_n = a + (n-1)d \qquad\qquad S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$T_n = ar^{n-1} \qquad\qquad S_n = \frac{a(r^n - 1)}{r - 1} \ ; \ r \neq 1 \qquad\qquad S_\infty = \frac{a}{1 - r} \ ; \ -1 < r < 1$$

$$F = \frac{x\left[(1 + i)^n - 1\right]}{i} \qquad\qquad P = \frac{x[1 - (1 + i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad\qquad M\left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c \qquad\qquad y - y_1 = m(x - x_1) \qquad\qquad m = \frac{y_2 - y_1}{x_2 - x_1} \qquad\qquad m = \tan\theta$$

$$(x - a)^2 + (y - b)^2 = r^2$$

In $\triangle ABC$: $\qquad\qquad \dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

$$a^2 = b^2 + c^2 - 2bc.\cos A$$

$$area\,\triangle ABC = \frac{1}{2}ab.\sin C$$

$$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta \qquad\qquad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$$

$$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta \qquad\qquad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases} \qquad\qquad \sin 2\alpha = 2\sin\alpha.\cos\alpha$$

$$\bar{x} = \frac{\sum fx}{n} \qquad\qquad\qquad \sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)} \qquad\qquad\qquad P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\hat{y} = a + bx \qquad\qquad\qquad b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$$