Possible solutions to the DBE Core Maths 2019 Paper 2

Please note that this is **NOT** an official document. It has been created mainly as a service to IEB pupils as another source of practice ahead of their upcoming examination.

It is **NOT** a marking memo in the sense that it does not contain mark allocations. Nor does it contain alternate methods. It is likely not even fully correct.

For all the above reasons I would caution against sharing it with pupils in the DBE who, having seen it may be concerned about how they have fared if their answers don't match these. This will only serve to stress them out when they should be focused on their upcoming examinations. As we know, one can get plenty of part marks despite getting the final answer wrong. However, pupils won't necessarily appreciate that.

**QUESTION 1**

1.1

$$y = 0.4053x - 1946.9$$

1.2

$$\begin{aligned} y &= 0.4053(14000) - 1946.9 \\ &= R3727.30 \end{aligned}$$

1.3

$$r = 0.947$$

1.4

D

**QUESTION 2**

2.1

19

2.2

$$\frac{50\times7+150\times12+250\times a+350\times35+450\times b+550\times6}{100}=309\ \ (1)$$

$$and\ 7+12+a+35+b+6=100\ \ (2)$$

$$from\ (2)\ \ a=40-b$$

$$substituting\ this\ into\ (1):$$

$$350+1800+250(40-b)+12250+450b+3300=30900$$

$$\therefore -250b+450b=3200$$

$$\therefore 200b=3200$$

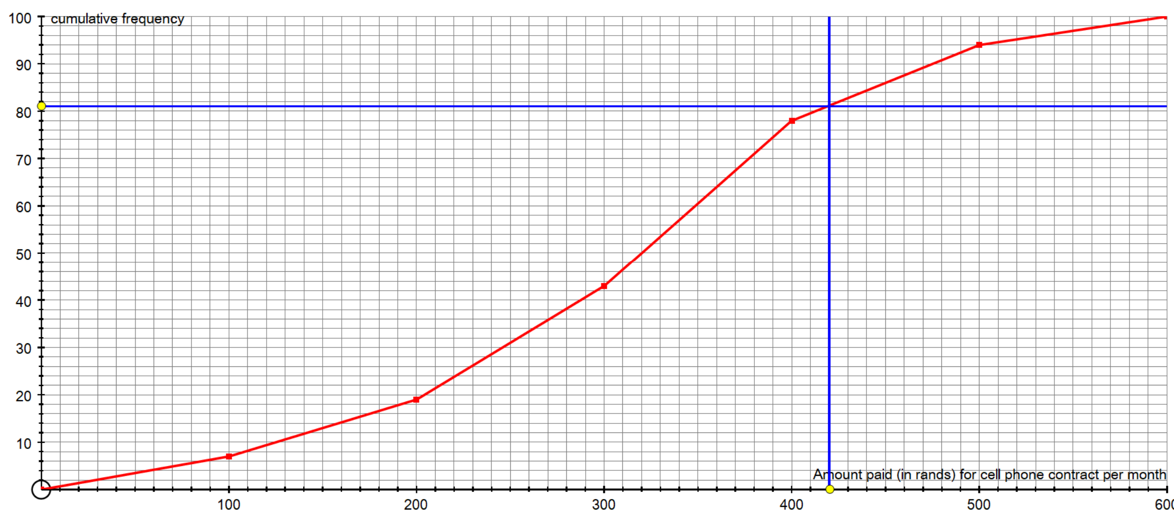
$$\therefore b=16$$

$$and\ a=4$$

2.3

$(300;400]$ is the modal class

2.4




2.5

19

**QUESTION 3**

3.1

$y=5$

3.2

3.2.1

$$_{m}RS=\frac{12}{6}=2$$

3.2.2

$\theta=\tan^{-1}2=63.4^{\circ}$

3.2.3

*equation of RS is*:

$y-5=2(x-3)$

$y-$*intercept of RS is found by letting* $x=0$

$\therefore y=2(-3)+5$

$\therefore D(0\,;-1)$

3.3

$(-5--3)^2+(k--7)^2=\left(2\sqrt{5}\right)^2$

$\therefore 4+(k+7)^2=20$

$\therefore (k+7)^2=16$

$\therefore k+7=\pm 4$

$\therefore k=-7\pm 4$

$\therefore k=-11$ *or* $-3$

*clearly* $k=-3$

3.4

The same translation from T to S will apply from D to N

$\therefore N(2\,;-5)$

3.5

It will be the same size as $P\hat{R}D$
which is $63.4^{\circ}$ due to opposite angles of parallelogram PEFR.

## QUESTION 4

4.1

$$(x+1)^2+(y-1)^2=1$$

4.2

$$C(-1;2)$$

4.3

$$m_{CN}=-1$$
$$\therefore m_{CD}=1 \ (radius \perp tangent)$$
$$\therefore y-2=1(x--1)$$
$$\therefore y=x+3$$
$$or \ y-x=3$$

4.4

$$t>3 \ \ or \ \ t<1$$

4.5

$$D(-3;0)$$
*so, circle has been moved 2 left and 2 down*
$$so \ E\left(-\frac{5}{2};-\frac{1}{2}\right)$$

4.6

$$Area \ OBCD = area \ \Delta CAO + area \ trap. \ ACBO$$
$$\therefore 2a^2=\frac{1}{2}\times base\times ht+ht\times average \ of \ parallel \ sides$$
$$\therefore 2a^2=\frac{1}{2}\times 2\times 2+1\times\left(\frac{1+2}{2}\right)$$
$$\therefore 2a^2=3\frac{1}{2}$$
$$\therefore 2a^2=\frac{7}{2}$$
$$\therefore a^2=\frac{7}{4}$$
$$\therefore a=\frac{\sqrt{7}}{2}$$

**QUESTION 5**

5.1

$$\frac{\sin x}{\cos x \tan x}+\sin(180^\circ+x)\cos(90^\circ-x)$$

$$=\frac{\sin x}{\cos x\times\frac{\sin x}{\cos x}}-\sin x\times\sin x$$

$$=1-\sin^2 x$$

$$=\cos^2 x$$

5.2

$$\frac{\sin^2 35^\circ-\cos^2 35^\circ}{4\sin 10^\circ\cos 10^\circ}$$

$$=\frac{-\left(\cos^2 35^\circ-\sin^2 35^\circ\right)}{2(2\sin 10^\circ\cos 10^\circ)}$$

$$=\frac{-\left(\cos 2(35^\circ)\right)}{2\left(\sin 2(10^\circ)\right)}$$

$$=\frac{-\cos 70^\circ}{2\sin 20^\circ}$$

$$=\frac{-\cos 70^\circ}{2\cos 70^\circ}$$

$$=-\frac{1}{2}$$

5.3

$$\cos 154^\circ=2\cos^2 77^\circ=1-2\sin^2 77^\circ$$

$$\therefore 2\sin^2 77=1-\cos 154^\circ$$

$$\therefore 2\sin^2 77=1-\left(\cos(180^\circ-26^\circ)\right)$$

$$\therefore 2\sin^2 77=1+\cos 26^\circ$$

$$\therefore 2\sin^2 77=1+m$$

5.4

5.4.1

$$f\left(x\right)=\sin\left(x+25^\circ\right)\cos 15^\circ-\cos\left(x+25^\circ\right)\sin 15^\circ$$

$$\therefore f\left(x\right)=\sin\left(x+25^\circ-15^\circ\right)=\sin\left(x+10^\circ\right)$$

$$\therefore \sin\left(x+10^\circ\right)=\tan 165^\circ$$

$$\therefore \sin\left(x+10^\circ\right)=-2+\sqrt{3}$$

$$\textit{so, key angle is } \sin^{-1}\left(2-\sqrt{3}\right)=15.5^\circ$$

$$\therefore x+10^\circ=195.5^\circ+360k$$

$$\textit{or } x+10^\circ=344.5^\circ+360k$$

$$\therefore x=185.5^\circ+360k$$

$$\textit{or } x=334.5^\circ+360k \quad \textit{both with } k\in\mathbb{Z}$$

5.4.2

$$y=\sin\left(x+10^\circ\right) \textit{ will have a minimum value when } x=260^\circ$$

$$\begin{pmatrix} \sin x \textit{ is normally at a minimum} \\ \textit{at } x=270^\circ \textit{ but } \sin\left(x+10^\circ\right) \textit{ is} \\ \textit{a regular sin curve shifted } 10^\circ \textit{ left} \end{pmatrix}$$

**QUESTION 6**

6.1

$y \in [-2\,;0]$

6.2

$x \in (90^\circ\,;270^\circ)$

6.3

$$\textit{if the } x-\textit{value of point } P \textit{ and } Q \textit{ is } \theta \textit{ then}$$

$$PQ = \cos 2\theta - (\sin\theta - 1)$$

$$\therefore PQ = 1 - 2\sin^2\theta - \sin\theta + 1$$

$$= -2\left[\sin^2\theta + \frac{1}{2}\sin\theta - 1\right]$$

$$= -2\left[\left(\sin\theta + \frac{1}{4}\right)^2 - 1 - \frac{1}{16}\right]$$

$$= -2\left(\sin\theta + \frac{1}{4}\right)^2 + \frac{17}{8}$$

*so PQ has a* max *value of* $\frac{17}{8}$ *when* $\theta = 180^\circ + \sin^{-1}\frac{1}{4} = 194.5^\circ$

**QUESTION 7**

7.1

$$\sin 60^\circ = \frac{AK}{x}$$
$$\therefore AK = x\sin 60^\circ$$
$$\therefore AK = \frac{\sqrt{3}x}{2}$$

7.2

$$K\hat{C}F = 120^\circ$$

7.3

$$DC = x\ (\textit{all sides of rhombus are equal})$$
$$\therefore CK = \frac{x}{2}$$
$$\textit{now } CF = x\ (\textit{all sides of rhombus are equal})$$
$$\therefore KF = \sqrt{\left(\frac{x}{2}\right)^2 + x^2 - 2\left(\frac{x}{2}\right)(x)\cos 120^\circ}\quad (\cos \textit{ rule in } \Delta CKF)$$
$$\therefore KF = \sqrt{\frac{x^2}{4} + x^2 + \frac{x^2}{2}}$$
$$\therefore KF = \sqrt{\frac{7x^2}{4}} = \frac{\sqrt{7}x}{2}$$
$$\textit{now Area } \Delta AKF = \frac{1}{2}(AK)(FK)\sin y$$
$$\therefore \textit{Area } \Delta AKF = \frac{1}{2}\left(\frac{\sqrt{3}x}{2}\right)\left(\frac{\sqrt{7}x}{2}\right)\sin y$$
$$= \frac{\sqrt{21}x^2\sin y}{8}$$

**QUESTION 8**

8.1

8.1.1

$\hat{R} = 80° \ (co\text{-}int.\ \angle s \text{ on } || \text{ lines})$

8.1.2

$\hat{P} = 100° \ (opp.\ \angle s \text{ of cyclic quad})$

8.1.3

$P\hat{Q}R = 136° \ (ext.\ \angle \text{ of cyclic quad})$

$\therefore P\hat{Q}W = 36°$

8.1.4

$$\hat{U}_2 = 136° \begin{pmatrix} alt.\ \angle s \text{ on } || \text{ lines} \\ OR \\ ext.\ \angle \text{ of } \Delta UPQ \end{pmatrix}$$

8.2

8.2.1

$$\frac{EF}{DC} = \frac{FT}{CT} = \frac{ET}{DT} = \frac{1}{2}$$

$\therefore \Delta EFT \ ||| \ \Delta DCT \ (corr.\ sides \text{ in proportion})$

$\therefore E\hat{F}T = T\hat{C}D \ (||| \ \Delta s)$

$\therefore E\hat{F}D = E\hat{C}D$

8.2.2

$E\hat{F}D = E\hat{C}D \ (proved \text{ in } 8.2.1)$

$\therefore FEDC \text{ is cyclic } (converse\ \angle s \text{ in same segment})$

$\therefore D\hat{F}C = D\hat{E}C \ (\angle s \text{ in same segment})$

## QUESTION 9

$\hat{O}_2 = 360° - x$ ($\angle s$ *round a pt*)

$\therefore \hat{M} = 180° - \frac{x}{2}$ ($\angle$ *at centre*)

$$\therefore \hat{P}_1 = \frac{180° - \left(180° - \frac{x}{2}\right)}{2} \quad (\angle \text{ sum of isos. } \Delta)$$

$\therefore \hat{P}_1 = \frac{x}{4} = \frac{1}{4}x$

*but* $S\hat{T}M = \hat{P}_1$ (tan *chord*)

$\therefore S\hat{T}M = \frac{1}{4}x$

## QUESTION 10

10.1

$$\frac{\Delta ADE}{\Delta DBE} = \frac{AD}{DB} \quad (\text{shared heights so areas in same ratio as bases}) \quad (1)$$

*likewise* $\frac{\Delta ADE}{\Delta DEC} = \frac{AE}{EC}$ (2)

*now* $\Delta ABE = \Delta DEC$ $\begin{pmatrix}\text{shared base DE and between parallel lines} \\ \text{so equal height}\end{pmatrix}$

$\therefore \frac{\Delta ADE}{\Delta DBE} = \frac{\Delta ADE}{\Delta DEC}$

$\therefore \frac{AD}{DB} = \frac{AE}{EC}$ (*equating the right hand sides of* (1) *and* (2))

10.2

10.2.1

$\hat{V}_3$ (*isos.* $\Delta$, *tangents from a common pt.*)

$\hat{R}$ (*tan chord*)

$\hat{W}_3$ (*corr.* $\angle s$ *on* || *lines*)

10.2.2

(a)

$\hat{W}_3 = \hat{V}_3 \ (both = x, from\ 10.2.1)$

$\therefore WSTV\ is\ cyclic\ (converse\ \angle s\ same\ segment)$

(b)

$\hat{W}_2 = x\ \ (\angle s\ in\ same\ segment)$

$\therefore \hat{V}_1 = x\ (alt.\ \angle s\ on\ ||\ lines)$

$but\ \hat{R} = x\ (from\ 10.2.1)$

$\therefore \Delta WRV\ is\ isosceles\ (2\ equal\ angles)$

(c)

$\hat{R} = \hat{S}_2\ (both = x\ from\ 10.2.1)$

$\hat{V}_1 = \hat{V}_3\ \ (both = x\ from\ above)$

$and\ \hat{W}_1 = S\hat{T}V \begin{pmatrix} ext.\ \angle\ of\ cyclic\ quad \\ OR\ \angle\ sum\ of\ \Delta \end{pmatrix}$

$\therefore \Delta WRV\ |||\ \Delta TSV\ (AAA)$

(d)

$\frac{RV}{SV} = \frac{WR}{TS}\ \ (|||\ \Delta s)$

$so\ SV.WR = RV.TS\ \ (1)$

$but\ \frac{SV}{KV} = \frac{SR}{WR}\ \ (prop.\ int.\ theorem)$

$\therefore SV.WR = KV.SR\ \ \ (2)$

$combining\ (1)\ and\ (2):$

$RV.TS = KV.SR$

$\therefore \frac{RV}{SR} = \frac{KV}{TS}$